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Making Number Sense through Relevancy: Place Value, Adding and Subtracting, Decimals, and Estimation

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Introduction

Our school is different from the usual public-school setting because our population of students is 98% Native American and predominantly English Language Learners. At the beginning of each year I have students complete an activity like a scavenger hunt, which gives me a chance to ask questions about their household. One of the questions asks the student what languages people speak in their home. I use this activity to obtain background information of my students. This year the information I obtained regarding the students' background indicates some students have parents or grandparents that are fluent Navajo speakers. About a fourth of the students understand the Navajo language because they are exposed to both Navajo and English language, but they cannot speak the Navajo language fluently. Only four students are capable of orally communicating fluently in Navajo and English. This is important information because now I know our students have communication issues due to their dual language exposure, so I need to incorporate English Language Learner (ELL) strategies into my lessons.

Another important factor is their home environment. It seems that each year, there is an increase in dysfunctional family issues. The most challenging issue our students deal with is poverty, which is shared across the reservation. Understanding poverty, especially as one works to dismantle its negative effects, is too complex to reduce to just economics and must include a discussion of cultural relevance beyond just material wealth. Poverty continues to generate other problems that have been escalating in our communities, such as alcoholism, which is a constant issue, but recently there is an increase also in drug abuse on the reservation. Due to these issues with parents and guardians, our students are not being taught their traditional cultural values, which are important components in the development of self-identity. Students need self-identity to support their motivational drive for success. These are a few factors that hinder our students' motivation toward their educational endeavors.

This year's math score is like those of previous years: the NWEA beginning of the year (BOY) math assessment results indicate 72% (49 students) of my students in sixth grade were below average in all five areas of the math goal indicators (Number System, Expression and Equation, Geometry and Probability/Statistics); 25% (17 students) are nearing proficiency; and 3% (2 students) are proficient. Overall, our school's math scores are still below the average range at the beginning of the school year, but the data does show that our math scores

have been improving. Although, we are improving in our assessment scores, it's a slow growth and so we are still considered a school "In Need of Improvement". As I reflect on this information it is obvious our students need to build a solid understanding of number sense to support their academic growth in math concepts. Furthermore, with all the information I gathered regarding my students' background, home environment and assessment data, I knew I had to create the best educational setting and instruction to support the growth of their math comprehension. By reflecting on this data, I have come to believe that my students need the fundamental understanding of numbers which begins with knowing place value, and to broaden their understanding of small numbers and large numbers.

(This part of my curriculum can be adapted to other languages as well or it can be disregarded.)

Navajo language extinction has been a concern for many educators on the Navajo Reservation for the past decade and more so in the past few years. We have just recently realized

how important language is to every culture. It's a blessing that we have not fully lost our Dine language as in some cases of other Native American Tribal Languages. We still have hope in continuing our native dialect for future generations. In this regard, it is imperative that we, the Dine People, come together with plans in preserving our language and culture. Therefore, using background information and my understanding of our Navajo students, I incorporate Navajo language into my math lessons on decimals using speaking and writing.

Content

This unit is created to help students develop an authentic understanding of the basic number system through applicable hands-on activities and content relevancy. It is crucial that they construct meaning for themselves using engaging teaching strategies. This understanding, needs to begin with repeated and prolonged exposure to the base ten system and place value It is designed to enhance or present an in-depth understanding of basic number relations for my 6th grade math class. It is also designed using English as a Second Language (or ELL) strategies, and emphasizes relevant content to support the understanding of the foundational math concepts. The unit addresses place value, procedural computation, decimals and estimation. Here are more details about the topics involved:

- Place Value – whole numbers, place value, and decimals to the hundredth. To develop further understanding of place value concepts, my activities will involve concrete models (place value mats, place value chart, and number lines), and practice using place value language orally; and illustrations; and will develop connections to real world situations with numbers at school, home, and community to make learning meaningful.
- Procedural computation of addition and subtraction – using the strategy for adding and subtracting using expanded base 10 form.
- Decimal fractions – Using place value and number lines to support student's conceptual understanding of decimal fractions.
- Connecting Decimals to money – Using real world context to help students understand the concept of money.
- Estimation and basic error in connection to the concept of money –Using real world situations with money, students will have a better understanding of the amount of error when rounding to the nearest

whole number and to the nearest tenth.

Place Value

Place value is an important foundational concept in the development of math skills. It is essential to have a strong foundation in place value in order to achieve success in making sense of the number system for counting, adding-subtracting-multiplying-dividing multi-digit numbers, estimating, decimals, and other mathematical skills. Place value is a key concept for students to learn and have a better understanding of numbers greater than ten. For this reason, it is important to teach it, in isolation at first, but also reviewing and integrating it into other concepts throughout the year. I believe helping my students build understanding of place value through base-ten and interesting problems, will help them through their mathematical journey. Recognizing that numbers can be broken apart, rearranged, and re-grouped, gives students a better understanding of how addition, subtraction, multiplication, and division work. This is especially true when my students have a sound understanding of what each part of a whole number represents

An introductory review of place value will use modeling of hundreds, tens, ones on a place value mat using base 10 blocks, using these simple tools to develop conceptual understanding. I call it a review, since this method is usually taught at the elementary level.

1. Review a one-digit number with the student. Write the number and place the corresponding number of units on a place-value mat. For example, the number 7 has 7 units blocks in the ones place. We use little blocks, called units or unit cubes, to show the number of ones. The number 7 does not have any tens or any hundreds, so we leave those columns empty: 7 = 7 ones.
2. Review the number 10. Write the number 10. Place 10 units in the ones column. Line them up to show 10 units to equal 1 rod. Exchange the 10 ones (unit blocks) for 1 ten (rod) and place the rod in the tens column.
3. Point to the 1 in the tens place of the written number (10) and the 1 rod in the tens column on the place-value mat. Emphasize that 0 in the ones place of a written number corresponds to an empty ones column on the place-value mat. It represents 1 ten and zero ones.
4. Explain the same details for the number (100) which is represented by what we call a flat or 10 rods put together to produce a 10x10 square. Discuss larger number representation, simultaneously show visuals of the object on the promethean board to aid in their understanding. Emphasize that one hundred equals ten tens.
5. Write a three-digit number such as 142. Review that this means one hundred plus four tens plus two ones. Place two unit blocks, four rods, and one flat in their corresponding columns. Explain that the two units equal the number of ones in the number 2, the four rods equal the number of tens in the number 40, and the one flat equals the number of hundreds in the number 100. Draw a connection between the numbers of objects in each column of the place-value mat with the corresponding digits in the written number. The whole number is the sum of these pieces. In symbols, 142 = 100 + 40 + 2. Continue the lesson by discussing how other numbers are represented and have students use the place value mat to demonstrate understanding.

Then have students use a place value chart to illustrate the corresponding value of the digit. Using the chart emphasizes the value of a digit to the left of another digit is 10 times larger and the value of the digit to the right of that same digit is one tenth the value. For example, if we have the number 432, which is said “four hundred thirty-two”, the 4 means 4 hundreds, the 3 means 3 tens, and the 2 means 2 ones. In symbols,

$432 = 4 \acute{} 100 + 3 \acute{} 10 + 2 \acute{} 1 = 4 \acute{} (10 \acute{} 10) + 3 \acute{} 10 + 2 \acute{} 1.$

Place value is one of the key concepts in a mathematical curriculum and though it is only explicitly taught in the lower grades, understanding place value (or not understanding it) will follow students through their mathematics journey. It is essential that students understand the meaning of a number. For example, in the number 635, the 6 represents 600. Without this understanding, students often struggle with when to "borrow," or regroup. This hinders their understanding of algorithms for adding and subtracting multi-digit numbers.

	Billions	dímíil dímíiltsoh
	Hundred Millions	neeznádiin-dímíiltsoh
	Ten Millions	neeznáá-dímíiltsoh
	Millions	dímíiltsoh
	Hundred Thousands	neeznádiin-dímíil
	Ten Thousands	neeznáá-dímíil
	Thousands	dímíil
	Hundreds	neeznádiin
	Tens	neeznáá
	Ones	t'ááłá'í
●	AND	doo
	Tenths	neeznáá yistsoh
	Hundredths	neeznádiin yistoh
	thousandths	dímíil yistoh

(The place value chart is an instructional tool to support students' understanding of the number system.)

Using a number line is another useful strategy in strengthening my students' number sense. The use of a number line not only helps students use a tool to solve math situations, but also helps my students develop

the necessary number sense to construct mathematical meaning and numerical relationships. The number line is an effective teaching tool that can be integrated in many math lessons as a visual aid in strengthening my students' understanding of mathematical concepts.

Number Lines can be used in conjunction with Place Value charts, which are both effective teaching tools. The place value number system is built around ten symbols which we call digits (0,1,2,3,4,5,6,7,8,9). Using combinations of these digits, we can create an numbers as large or as small as we want. The value of a digit depends on its place, or position, in the number. Each place has a value of 10 times the place to its right and one tenth times its value to the left. A number in standard form is separated into groups of three digits using commas, to make it easier to read. Each of these groups is called a period. A number in standard form is understood to be the sum of the numbers represented by the individual digits, which gets ten times larger for each place from the right. For example, 435, is 400 + 30 + 5. Another example: 6,802 is the sum of 6,000 + 800 + 2. Notice there is a zero for the tens place, so the tens place has no value. The 6,802 is considered standard form which is the sum of 6,000 + 800 + 2 in expanded form. The 6 is in the thousands place, and represents 6 thousands. The 8 is in the tens place, and represents 8 hundreds. The 0 is a place holder for the tens place, and represents zero tens. The 2 is in the ones place, and represents 2 ones. Once this fundamental concept is understood, we will give the pieces represented by the digits a name: place value pieces. (a single digit times a product of some 10s, also called a power of 10). This gives us five increasingly explicit ways to write a base ten number. We call these the *five stages of place value.*(Howe-Reiter). (1)

For example using the standard representation of 4,769:

The five Stages of Place Value

4,769 (standard form)

$= 4{,}000 + 700 + 60 + 9$ (2^{nd} stage)

$= 4 \times 1{,}000 + 7 \times 100 + 6 \times 10 + 9 \times 1$ (3^{rd} stage)

$= 4 \times (10 \times 10 \times 10) + 7 \times (10 \times 10) + 6 \times 10 + 9 \times 1$ (4^{th} stage)

$= 4 \times 10^3 + 7 \times 10^2 + 6 \times 10^1 + 9 \times 10^0$ (polynomial in 10)

In the five stages of place value the second stage is explained as an expanded form of the standard number. It shows that the number is the sum of its place value pieces. The 3^{rd} stage shows the same quantity but using the one digit multiplied by the base 10 units such as 4 x 1,000, 7 x 100, 6 x 10 and 9 x 1. It shows that each place value piece consists of the given digit times a base ten unit (1, 10, 100, etc.). The 4^{th} stage shows that each base ten unit is a product of tens, with one more ten for each place to the left of 1: $1{,}000 = 10 \times 100 = 10 \times (10 \times 10)$, which is the same as $1{,}000 = 10 \times 10 \times 10$. Similarly, 100 = 10 x 10. This concept is usually not taught in our elementary school, but it is an important step. It shows the relative values of the base ten units whereas each base 10 unit is 10 times larger than the one to the right, and only one tenth as large as the one to the left. So, using this concept you can see that it doesn't take very many digits before the numbers are very large or very small, when multiplying by 10. The last stage is the polynomial in 10, which deals with the order of magnitude. The order of magnitude of a single-place number is the number of zeroes used to write it, for instance using the standard number above would be $4{,}000 = 4 \times 10^3$, $700 = 7 \times 10^2$, $60 = 6 \times 10^1$ and $9 = 9 \times 10^0$. The order of magnitude is also the number of 10s multiplied together to make the base ten unit, and

it appears as the exponent in the polynomial form. Hence, the order of magnitude of 4,000 is 3, the order of magnitude of 700 is 2, the order of magnitude of 60 is 1 and the order of magnitude of 9 is 0. The first four stages are important for concepts for my students to comprehend so they have a better understanding of the computational components of adding and subtracting. We will not worry about the 5th stage, which involves concepts from later grades. In our 6th grade Common Core Standards, students are expected to fluently add and subtract multi-digit numbers, but some of my students still struggle with two-digit adding and subtracting, so I believe these stages will enlighten their understanding of numbers to support these two math concepts.

I find that using number lines helps my students build their mental math ability in creating models to defend their thinking, and to explore relationships between numbers and operations. As students use the number line they can visually see that there are many different ways in reaching the same number, but there is also an efficient strategy, which is the expanded form. In instructing place value I will use hands on materials such as base 10 blocks and number lines, which are effective, but I will also make sure to simultaneously connect these to the symbolic forms of the stages of place value.

The following is a collection of examples of the kinds I will use with my students. My goal is to help them see how the usual procedures for addition and subtraction are based on the expanded form of numbers.

Addition

The addition algorithm is a strategy for adding two numbers in base 10 form. It consists of three steps:

1. Break each of the numbers (the “addends”) into its single-place components.

2. Add the corresponding components for each order of magnitude. If the component for an order of magnitude of an addend is missing, it is treated as if it were zero.

3. Recombine the sums from step 2 into a number in base 10 form.

Here is an example procedural computation of addition:

345+621 = (300+40+5)+(600+20+1) by breaking each addend into its single-place components

= (300+600)+(40+20)+(5+1) by grouping single-place components by order of magnitude

= 900+60+6 by adding components for each order of magnitude

= 966 by recombining the single-place components into a base 10 number.

Sometimes step 3 entails regrouping:

437+254 = (400 + 30 + 7) + (200 + 50 + 4)breaking each addend into its single place component

= (400 + 200) + (30 + 50) + (7 + 4) group the single-place components by order of magnitude

= (600 + 80 + 11) adding components for each order of magnitude we see that we have 11 in the ones place so we need to regroup to move the 10

= (600 + 80 + 10 + 1) from the ones (10 + 1) which leaves 1 in the ones place and add one 10 to the tens place to make it

= (600 + 90 + 1) (80 + 10 = 90), or 9 tens.

= 691

Subtraction

Vocabulary for the subtraction concept:

15 - 10 = 5

Minuend − Subtrahend = Difference

Minuend: The number that is to be subtracted from.

Subtrahend: The number that is to be subtracted.

Difference: The result of subtracting one number from another.

Subtracting a number undoes adding a number, taught as fact family in the elementary grades. This supports the students' understanding that if, a – b = c then b + c = a, or using digits as an example is 15 - 10 = 5 then 10 + 5 = 15

When the subtraction a − b = c involves no regrouping or borrowing then the same orders of magnitude are affected in both computations. For example,

5631 - 3,221 = (5,000 + 600 + 30 + 1) - (3,000 + 200 + 20 + 1)

= (5,000 – 3,000) + (600 - 200) + (30 - 20) + (1 - 1)

= 2,000 + 400 + 10 + 0 = 2,410

Sometimes, however, there are not enough of some unit in the minuend to subtract the corresponding part of the subtrahend. In this case, one copy of a larger unit must be broken up to provide 10 extra copies of the deficient unit. This process is called *regrouping*.

For instance, in this example one 1,000 must be regrouped into ten 100s:

5431 - 3,721 = (5,000 + 400 + 30 + 1) - (3,000 + 700 + 20 + 1)

= (5,000 – 3,000) + (400 - 700) + (30 - 20) + (1 - 1)

Starting with the ones place we see this can be subtracted (1 – 1 = 0), then the tens place we can subtract (30 – 20 = 10), but when we get to the hundreds place the subtrahend is larger than the minuend, so we need to regroup or borrow from the thousands place. When we borrow the 5,000 becomes 4,000. The 1,000 that is borrowed is added to the hundreds place making it 1,400. Now we can complete the computation.

= (4,000 – 3,000) + (1,400 – 700) + (30 – 20) + (1 – 1)

= 1,000 + 700 + 10 + 0

= 1,710

Regrouping may be required for multiple base ten units. This next example involves three places of regrouping:

6,453 – 3,784 = (6,000 + 400 + 50 + 3) - (3,000 + 700 + 80 + 4)

= (6,000 – 3,000) + (400 – 700) + (50 – 80) + (3 – 4)

= (6,000 – 3,000) + (400 – 700) + (40 – 80) + (13 – 4)

= (6,000 – 3,000) + (300 – 700) + (140 – 80) + (13 – 4)

= (5,000 – 3,000) + (1300 – 700) + (140 – 80) + (13 – 4)

In this problem we start with the ones and see the subtrahend is larger than the minuend, so we need to regroup or borrow from the tens which adds 10 to the 3 in the ones place and now we can subtract (13 – 4 = 9). Since we borrowed from the tens the subtrahend for the tens is reduced to 40, so we regroup or borrow from the hundreds which adds 100 to make it 140 and now we can subtract (140 – 80 = 60). Next, since we borrowed again from the hundreds the subtrahend is reduced to 300, and we borrow from the thousands place which increases the subtrahend to 1,300 and we are able to subtract (1,300 – 700 = 600). Now we can continue to solve the problem:

= (5,000 – 3,000) + (1,300 – 700) + (140 – 80) + (13 – 4)

= 2,000 + 600 + 60 + 9 = 2, 669

Unfortunately for learners of subtraction, even understanding how to do multiple regroupings as above is not enough to solve all subtraction problems. In fact, unlike addition, there are not a fixed number of procedures that allow solving all subtraction problems. The most difficult part of subtraction for students is what is called “borrowing past a zero”. This can actually involve borrowing past arbitrarily many zeros. The basic problem can be seen when subtracting two place value pieces. For example, consider the following problems

70 – 3 = 60 + (10 – 3) = 60 + 7 = 67

700 – 3 = 600 + 100 – 3 = 600 + 90 + (10 – 3) = 600 + 90 +7 = 697

7,000 – 3 = 6,000 + 1,000 – 3 = 6,000 + 900 + 100 – 3

= 6,000 + 900 + 90 + (10 – 3) = 6,00 + 900 + 90 + 7 = 6,997

It can be necessary to decompose arbitrarily many base ten units to enable to perform subtraction at a single place. When a problem like this is embedded in a more complicated situation, students can find it confusing. I will try to help them understand subtraction better by having them work numerous examples with both minuend and subtrahend being place value pieces, to help them see the pattern as in the examples above. Then we will work on what to do when this situation appears inside a more general subtraction situation. The good news is, that if you have to borrow across one or more zeros, all the places in the minuend that were zeros become nines, so there will be no need to borrow at those places.

Decimals

Sets of numbers, like whole numbers and decimals, are all important concepts that my students need to learn,

and I believe it's easier for them to decipher when used in conjunction with place value chart and number lines. With decimals, it is important for my students to understand whether a number or digit is in the ones or tenths place. Conventionally, I use the standard practice of writing a decimal point (period) to the right of the ones, in between the ones and tenths place. One good strategy is to place these on a number line, so students see, as with whole numbers, the value of any digit is divided by 10 if it moves right one place. When 4 tenths is written as 0.4, the zero shows there are no ones, the point indicates that the whole numbers have ended, and the 4 means 4 tenths. It's important that my students understand place value with decimals: 0.4 is not to be said as "zero point four", but rather as "zero ones and four tenths" so that students will recall such knowledge and be able to understand decimal values as well as whole number values. Decimal Number lines will be used in order to display a variety of numbers that can be incorporated in accordance with the Curriculum using the Common Core Standards. If this area is neglected, not understood or not taught properly, it may hinder the ability of my students to master later mathematical concepts. Visual models, including number lines, would aid in the understanding of decimals and a simplified way of showing their relationships, and especially, that for each place you move to the right, the value of the digit gets divided by 10. One task would entail students to break down decimal places into tenths, hundredths, thousandths and so on, to show how the digits actually look. Another strategy I plan to use for a visual model is using place value in accordance with a number line to show that 9 tenths are greater than 7 tenths, and also that .9 > .72 > .651 and so on. They will be able to grasp this concept through using visual example models. I know that there are some common misconceptions that have been documented that can occur when teaching Place Value. For this reason, I want to make sure I am teaching the students correctly, so my students are able to use this concept of decimals and move on to the next concepts of comparing decimals and rounding. So, my next mini lessons will be comparing and ordering decimals and how to round decimals to the nearest hundreds, tens, tenths, and hundredths. These concepts are also important for my students to comprehend numbers both whole numbers and decimal numbers.

Connecting Decimals to Money

Procedural fluency is defined by the Common Core as "Skill in carrying out procedures flexibly, accurately, efficiently and appropriately." In 4th and 5th grades, students are supposed to add and subtract decimals. In 6th grade, students are expected to become fluent in the use of the standard algorithms of each of these operations, but my students still needed the basic understanding which I covered in the previous lessons. In fact, these algorithms are exactly the same as for whole numbers if you think in terms of base ten units and the relationship between two adjacent units. I will work hard to get my students to understand this idea.

The previous lessons, and especially understanding the size relationship between places, should help them understand the next mathematical concept of estimation. The use of estimation strategies should support my students understanding of decimal operations. Therefore, learning decimals is another important concept in understanding numbers. Since my students have already learned about whole numbers in the previous lessons I would like to introduce or review depending on my students' understanding, the same stages to teach about decimals. After my students have better understanding of whole numbers and decimal numbers they will be ready for the next lesson of connecting these two concepts into money, a very real-world topic that they need to understand. With this lesson I will use money as a connection to understanding decimal values and also connect Navajo language (speaking and writing of money) into the lesson. The reason I am using money for the relevancy connection is due to the fact that my students are naïve about it. Most of my students have not been exposed to budgeting and the value of money. I want them to understand that "money does not grow on trees", as the saying goes. Also, budgeting is a life skill that is important for our students to learn for their future.

So, when I start this lesson the students will practice writing decimal numbers in expanded form, standard form, and word form using English. Next, they will use the word form concept as a guide in understanding how to translate the English language into the Navajo language. I will introduce the Navajo words orally while the students have the written form (I will have the Navajo words listed on a sheet as a guide), and I will have students learn how to say the words by repeating after me. We will practice by using cooperative learning styles in partners or groups to sound out words. Then I will start having them write the words using the sheet as a guide, next they will continue to practice by using the worksheet shown below. In my opinion our Native American students are motivated to learn when their Navajo language and culture are part of the learning process.

Money Language: Write the money in English and translate into Navajo.

1) $ 52.01 - English _____

Navajo _____

2) $ 47.84 - English_____

Navajo_____

Once my students have a better understanding of place value, procedural computation of addition and subtraction, and decimals, our next step is to learn estimation, using money. Students will learn estimation and connect this concept using real world situations such as estimating the cost of a grocery bill or budgeting.

Estimation

First, students need to know estimation words and phrases, such as “about”, “close”, “just about”, “a little more or less than” and “between these two numbers. I will try to make these ideas meaningful for my students by having them plot some numbers and their rounded versions on number lines of a suitable size. Students should understand that they are trying to get as close as possible using quick and easy methods, but there is no actual correct estimate. An estimate depends on how close you want to get to the actual number, which connects to the concept of error.

By itself, the term “estimate” refers to a number that approximates an exact number given in a particular context. This concept of an estimate is applied not only to computation but also to other math concepts such as measures and quantities. For this unit we will discuss estimates for money.

Whenever we are faced with a computation in real-life, we have a variety of choices to make concerning how we will handle the computation. Do we need an exact answer, or will an approximate answer suffice? Sometimes, we do not need an exact answer and so we can use an estimate in real world context. Next, we need to know how close an estimate do we need? This also depends on the situation, which connects to error. Error in this unit context is the magnitude of the difference between the exact value and the approximation to the whole number or to the dime, which is called the “absolute error”. The following are three strategies that can be used in estimating money. Front-end method is a strategy that focuses on the leading or left most digits in numbers or whole (dollar amount) number and ignoring the cents. After an estimate is made on the basis of only these front-end digits, an adjustment can be made by noticing how much has been ignored. Usually the error is larger using this method in the context of money. One strategy to instruct this concept is to give them a list of numbers, of many different sizes (including some decimal fractions) and have them

compare the value of the leading digit with the rest of the number. Is it always bigger than the rest of the number? How often is it twice as big? Five times as big? Ten times? Then repeat the same question with the same numbers, but ask them to compare the value of the leading two digits with the rest of the number. Is it always10 times as big? How often is it 20 times as big? Fifty times? One hundred times? Then repeat everything with amounts of money. It doesn't need to lead to anything formal, but if you discuss it and ask what they learned from it, hopefully some students will say that the first digit is the most important, more than all the rest, and the first two digits tell you almost all (in fact, at least 90%, but that level of precision is not crucial) of the number.

Rounding methods is the familiar form of estimation and is a way of changing the problem to one that is easier to work. We can round to the nearest dime, dollar, ten dollars, hundred dollars and so on. Good estimators follow their mental computation with and adjustment to compensate for the rounding. When adding a long list of numbers, it is sometimes useful to use compatible numbers – that is, to look for two or three numbers that can be grouped to make benchmarks of 10, 100, 1,000 and so on.

I believe the best instructional approach to improve my students' estimation skills is to have them do a lot of estimating using the three methods. Finding real life estimating problems and discuss the strategies we need to use in which the computational estimations will suffice. Some simple examples include dealing with grocery store situations, adding up distances in planning a trip, determining approximate yearly or monthly totals of household bills and figuring the cost of an evening out. Discuss why exact answers are not necessary in some instances and why they are necessary in others. Using real life context to help with understanding estimates is a good strategy to help students understand that math is a part of our daily life. For instance ask students, "Would the cost of a car more likely be $950 or $9500?" Also ask, would the cost of a car more likely be $1,000 or $10,000, and then ask if there is a very big difference between these two questions. A simple computation can provide the important digits. An estimate is usually only about the first, or maybe the first two digits. Of course, there is no correct answer but a range of answers. I intend to focus on flexible methods of estimation. My primary concern is to help students develop strategies for making estimates. Reflection on strategy use will lead to strategy development and understanding of estimating in a real-life context. It's also good to discuss how different students made their estimates. This helps students understand that there is no single right way to estimate and reminds them of different approaches. If estimates differ in the first digit, it is important to figure out why. Maybe the true number was almost halfway in between two place value pieces. If estimates differ in the second digit, that is pretty normal, and not as important. Reiterate to students that estimates are not the exact answer but an educated guess using the understanding of numbers and error. Think in relative terms about what is a good estimate.

Mathematically, at this point, my students should have a better understanding of place value, whole numbers (addition, subtraction), and decimals. So, before I begin estimation with my students I will complete an informal pre-test by arranging approximately 10 items with their prices marked on a table in the classroom. I will give the class a minute or two to determine the total cost of the 10 items. The students should not write down anything while they are looking at the items. Next, I will ask the students to record their estimate of the price of the ten items on a sheet of paper. Also, I will ask them to briefly write down any impressions they had while making their guesses. We will debrief together and discuss their answers. This strategy will expose them to different methods of estimating, and by discussing their outcomes, they can comprehend that estimation is a flexible concept depending on the situation.

Students will use estimation in a real-life context using the concept of money.

Ex: Carrots cost $2.71, peppers cost $1.73 and broccoli cost $1.10. estimate the total cost of the vegetables but rounding to the nearest whole dollar and to the nearest dime

(have them underline each dollar and circle each tenths place to round to the nearest whole number or dollar, next have them underline the tenths place (dime) and circle the ones place (penny) which is rounding to the nearest dime.)

Dollar	Dime
$2.71 ~ 3.00	$2.71 ~ 2.70
$1.73 ~ $2.00	$1.73 ~ $1.70
$1.10 ~ $1.00	$1.10 ~ $1.10
Estimated cost of the vegetables is $6.00 (error = $0.46)	Estimated cost of the vegetables is $5.50 (error = $0.04)

The main reason I want to incorporate a lesson on money is because many of our students and their family sell goods as their sole source of income. Students as young as 7 years old are selling jewelry in restaurants, outside of businesses, and at swap meets. This is also a real world connection for our students. When the concept is relevant to them they tend to have a better understanding.

Strategies

To improve student achievement, I incorporate ELL strategies, appropriate modifications, and accommodations as listed in available Individualized Education Plans (IEP) into daily lessons. Students frequently work with a partner or as part of a cooperative group made up of 4 students. Students are expected to help each other whenever possible, whether completing a hands-on-activity or reading a section from a book; the learning styles of students in this class are primarily visual, tactile, and kinesthetic. To increase comprehension and retention, I try to plan hands on activities and incorporate visuals, props, and graphic organizers into my lessons. I use a variety of ELL strategies with all students. These include small group and whole group instruction, modeling, guided instruction, math tutorial videos (technology), demonstrations (powerpoint), visual aids (anchor charts), group and class discussions, think pair share, differentiated instruction/methods, hands on activities, math interactive notebook, math journals, presentations, content relevant activities and creating assessments aligned to my unit.

With the three types of groups I will differentiate my instruction to support the needs of the students. The proficient students, which are very few, will be challenged with higher expectations using critical thinking activities, such as a performance task. The nearing proficient group will still be challenged with high expectation but generally given more support and assistance when they are completing their class assignments. Then the basic group, which is the largest group, need more assistance and sometimes requires one-on-one support from me. Sometimes these students need step by step instructions or re-teaching to help them understand the concepts being taught. Instructing in ways that connect with students requires understanding of differences that may arise from culture, family experiences, assessment data, and academic skills. As a teacher, I need to be able to instruct using strategic, well planned lessons, listen carefully during discussions, and look thoughtfully at student work and assessments to determine their next level of instruction. My students get frustrated very easily when they are struggling, so by differentiating the

instruction, all students are learning at their level. Once students have a better understanding of the concepts learned, they can be challenged using performance task activities.

The formative and summative assessments are also important components of a unit and in our daily lessons. I will incorporate formative assessments in the form of exit tickets, thumbs up/down, reflective writing, parking lot, popsicle stick questioning, think pair share, class discussions, observations, and 4 question worksheets. A pre-test is a good assessment tool to find out student knowledge. Assessment has, for many years, been used as a fundamental tool in measuring academic growth for students. It plays a critical role in teaching because it aids in determining the extent to which students have mastered a specific concept. It gives guidance in determining the next step of instruction or if intervention is warranted. Assessments can be a very useful tool for instruction when it is implemented correctly and strategically, to gather information on student progress. So, I believe teacher created summative assessments can lead to deeper learning and more powerful changes in instructional climate.

I started the process in designing summative assessments that would benefit my student's educational growth in conjunction with their learning styles. I design assessments according to my students' needs and aligned with my unit. I find doing this challenging, but a good learning experience. This process of designing assessment is time consuming, I remind myself that it benefits me professionally and benefits my students as well. Occasionally, when I find an excellent assessment, I use backward design and revise my lessons according to the assessment. The backward design does take time to create, but once it is completed the lessons will transition smoothly toward achieving the outcome. Another type of assessment I will implement is the performance task assessments, which require a more hands on approach. The performance task assessments are project based and can be completed as a group or individually. After each assessment I will have students complete a reflective writing assignment to describe their learning experience and their learning outcome.

Another teaching strategy I will incorporate in my lesson is cooperative learning. My classroom is set up with tables instead of desk for group activities and discussions. When I set up a cooperative group, I can arrange the groups so that there is a mix of high, medium, and low ability students. Low level readers are placed with a higher-level reader. Strategically grouping students is necessary to enhance the learning process of each individual student. Students that need frequent redirection, need one-on-one guidance, or have behavior issues are given preferential seating in a group I will give extra support to. When grouping for intervention I prefer to group my students according their skill level, so that I can give extra guidance and support to the students' with basic math skills (this sometimes means two groups that need extra support). IEP modifications for students with special needs are used when appropriate. These include extra time for completion of assignments, assistance with reading, shortened assignments, completed samples and examples, simplified instruction, frequent and immediate feedback, redirection, and an abundance of positive reinforcement. My students need plenty of praise and encouragement on a daily basis.

Use of technology is an important strategy in my classroom. Technology is everywhere in almost every part of our daily lives and community. It affects how we live, work, play, and most importantly learn, so it only makes sense that we incorporate technology into our daily lessons. Combining new technology with traditional classroom instruction is one example of how the introduction of new technology can enhance the learning experience and create new opportunities for our students. When technology is readily available and used as an instructional

tool, students have better understanding of concepts learned. I noticed that when I taught the same material

for a while my students found it less than exciting so technology was my key in engaging my students. A quick internet search helped me identify ways to supplement my lessons with interesting new material. I made it a habit of searching before I begin a new unit so I would be prepared and have a smooth transition from my instruction into a supplemental lesson using technology. I found plenty of supplemental lesson using video clips, tutorials and more, that brought my lessons to life. So, integrating **technology in the classroom** offers the opportunity to increase student interest and teach valuable skills, plus get them engaged. I try to make lessons engaging as much as possible so students are motivated in learning each new concept. Hence, implementing technology into my math lessons is essential in enhancing my instructional delivery and supporting my students' visual learning style.

Classroom Activities

Guided Instruction and Modeling

When I start a lesson I usually have students be aware of the lesson objective and the expected learning outcome. Once students understand the expectations of the lesson I will begin my guided instruction and modeling. It is important that I provide examples of how to use the place value chart and number line to help them understand whole numbers and decimal fractions. I incorporate modeling and reviewing in our daily lesson to support long term memory of the concept taught. My students need plenty of visual instruction for them to comprehend.

I plan to make the lessons interactive and implement time for group and partner discussions to build understanding and vocabulary. The more interested and engaged students are, and the more interactive each learning session is, the more students will enjoy, learn from and retain information from the lesson.

To further engage students in learning concepts I plan to use math videos as a supplemental learning tool. Video provides a means of interactive instruction and is a very flexible medium. Having the ability to stop, start and rewind which is absolutely imperative for me during my instruction. It provides the option to stop each video and challenge students to predict the outcome of a demonstration, and elaborate on the math procedure. I also have the option to rewind a section of the video to review a segment to ensure that my students understand the key concept. Using videos is a good strategy but I know it's important to select the right video for each lesson to align with your instructional outcome. I have found several good videos on base 10 model, place value, using number lines, adding and subtracting (whole numbers, decimals and money), and estimating. Also, our school has bought math programs to support our students which include; Reflex Math, Adapted Minds, Envision (includes Virtualnerd.com), STAR Math (Accelerated Math), and Gizmo. I plan to use these programs to supplement my instruction, as independent practice or as an intervention tool. Another site I occasionally use to supplement my instruction is mathplayground.com, which has a few good instructional videos. I emphasize to my students that several of these sites are free websites and they are allowed to use them as tutorials.

In this unit I will be instructing five lessons in the strand of number system:

- Place Value – whole numbers, place value, and decimals to the hundredth. To develop further understanding of place value concepts, my activities will involve concrete models (place value mats, place value chart, and number lines), and practice using place value language orally; and illustrations;

and will develop connections to real world situations with numbers at school, home, and community to make learning meaningful.

- Procedural computation of addition and subtraction – using the strategy for adding and subtracting using expanded base 10 form. In this lesson I will be using guided instruction and modeling. Students will use a whiteboard to practice as they learn the concept. Think pair share strategy will be implemented as a discussion tool during the instruction.
- Decimal fractions – When I am instructing this lesson I will be using place value and the number line to support student's conceptual understanding of decimal fractions. This lesson is an important math concept for students to learn; for them understand the connection of decimal and fraction. In this lesson I will be implementing more video clips and power points for visual instruction. Interactive notebook strategy will also be utilized to reinforcement understanding. Hands on activities and grouping will be incorporated to engage students as they learn the concept of decimal fractions.
- Connecting Decimals to money – Using real world context to help students understand the concept of money. In this lesson we will use real world situations such as shopping, selling goods, and household budget.
- Estimation and basic error in connection to the concept of money –Using real world situations with money, students will have a better understanding of the amount of error when rounding to the nearest whole number and to the nearest tenth. This is for numbers between 1 and 10. In general, it would be to one digit, or to 2 digits.

Math Interactive Notebook

Using math interactive notebooks and math journals in the classroom helps my students learn math vocabulary and is a great way to reinforce math problem solving skills. With this activity I have students partner up to work together in completing their vocabulary entries in their math interactive notebook. Each entry usually takes 45 minutes to complete since we make each template from scratch. Have students write the title: The Basics of Place Value. Students are partnered to discuss and share ideas as they complete their task. In their entries they will show a place value chart which flips up to reveal the values of each place. Next, students will explain the meaning of place value versus the value of a whole number. Then students will explain the meaning of standard form, expanded form, and word form. The math interactive journal is used as a supplemental teaching tool throughout the unit to reinforce concepts being taught. Alongside the interactive journal activity the think pair share strategy will be implemented to enhance understanding and usage of math vocabulary. The interactive journal is a great way to have students engaged in their learning by completing anchor charts to enhance their understanding using visual props such as number lines, place value chart, vocabulary foldables, step by step procedures in solving problems, and other important fundamental components of math. This strategy will be utilized throughout the year because our students are visual and hands on learners. The journal helps students with homework and class assignments, which is part of their independent practice.

Independent Practice

Allowing students to practice what you taught them is probably the most important component of the lesson.

Students are expected to remember what to do or how to do something, so they must have the opportunity to practice what they learned. Without a good deal of independent practice, my students may be able to remember for several hours how to do the concept they learned or they may even remember it next day, but chances are they will forget by the next week. So providing independent practice right after each lesson is important. Also, allowing students to work out problems or answer questions on their own improves the

chances that the information will be transferred into long-term memory. There are several ways I use to incorporate independent practice in the classroom; practice on individual whiteboards, group activities, peer tutoring, presentations, homework sheets, independent practice sheets, and math puzzles. I try to implement at least two of these strategies each week to give my students an opportunity to practice what they learned. When my students are completing class assignments, I use my formative assessment strategy of observation and monitoring to check on their progress. If a student is struggling, I will give guided support to help that student complete their assignment.

Relevant Project Ideas

I have several ideas for a project that students will be completing once the unit has been taught and the following is my list of ideas:

- A project board of how they connect their learning into their own real world situation.
- A shopping spree of x amount of dollars and how they use the concepts to connect to their spending.
- Role Play – of how they budget using their income and paying bills. How do the concepts you learned connect with this situation?
- In connection of content relevancy is to find out the profit vs. hours in making the jewelry. With this project students will be able to determine how much profit they would make when they sell their jewelry. Students will keep track of how many hours is spent on making their jewelry with their parents. The data will be recorded weekly and they will also keep track of their sales. Once we have compiled the data for at least two months we can start figuring out if the family is actually making a profit for their sales. I believe my students would enjoy this project because it has a home connection and they could include their parents in developing their data analysis. The parents would also be interested in the results.

Resources

(1)Howe, Roger and Reiter, Harold, “The Five Stages of Place Value”

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Spangler, David B. “Math for Real Kids.” Problems, Applications, and Activities for Grades 5-8, Addison-Wesley Educational Publishers, Inc. 2007

O’Connell, Jane and Flowers, Linda, “Math Games and Activities” Middle School, Frank Schaffer Publication, Inc. 1993

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Fishman, J.A. (1996) Stabilizing Indigenous Languages (pp. 80-91). Flagstaff, Az. Northern Arizona University

Websites

www.virtualnerd.com

https://www.mathplayground.com

https://www.teacherspayteachers.com

www.commoncoresheets.com

https://www.mathworksheetsland.com

Appendix

Implementing District Standards-Common Core State Standards and Dine' Content Standards

The Common Core State Standard (CCSS) is a research-based set of learning progressions detailing how students' mathematical knowledge, skill, and understanding develop over time. The standards supports the development of knowledge and skills students need to prepare for mathematics in college, career, and in life. These standards define what students should understand and be able to do at each grade level, but at our school, most students are usually not at grade level. The standards encourage students to solve real-world problems using the components of mathematical practices.

The middle school math teachers at our school had the opportunity to participate in a Professional Development summer session. In this session we aligned the math standards from Kindergarten to 8th grade. In the number system strand we noticed that this particular strand aligned from Kindergarten to 8th . Number system starts at the Kindergarten level where they learn to count in sequence and compare numbers. In 1st grade students begin to learn the base 10 concept. In 2nd grade students recognize that the digits in each place represent amounts of thousands, hundreds, tens and ones. In 3rd grade students use their knowledge of place value to round whole numbers to the nearest 10 or 100s. They fluently add and subtract within 1000 using strategies and algorithms based on place value. In 4th grade students use place value to round multi-digit whole numbers to any place. They fluently add and subtract multi-digit whole numbers using the standard algorithm. In 5th grade students recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to the right and 1/10 of what it represents in the place to its left. In 6th grade students extend their previous understandings of number and the ordering of numbers to the full system of rational numbers, which includes negative rational numbers, and in particular negative integers. Therefore, my unit is created to review concepts starting from Kindergarten level because my students have not mastered these standards. I am using this unit to help my students develop a concrete understanding of the number system.

Dine' Content Standard-Student Objective: I will utilize the Navajo language and culture, I will listen, communicate, observe, and understand appropriately. In this unit students will learn to use their Navajo

language for money (e.g. penny, nickel, dime, quarter, dollar and other money amounts.)

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