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Curriculum Units by Fellows of the National Initiative
2018 Volume IV: Big Numbers, Small Numbers

Exploring the Metric System and EM Spectrum Through Base Ten Numeration

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> "Nature uses only the longest threads to weave her patterns, so each small piece of her fabric reveals the organization of the entire tapestry" - Richard Feynman

Introduction

The scale of the known universe is truly immense, from quantum level interactions that affect molecular affinities to collisions of whole galaxies. In fact, even the nucleus of an atom, which is tiny in comparison to the whole atom, is made of smaller parts, called nucleons, and even the nucleons (protons, neutrons) are made up of smaller parts. Our humble existence is predicated on these interactions. No other field of science, besides physics, truly seeks to understand this broad scope of knowledge. As a result, many students struggle with contextualizing content and the related quantities associated with cosmic energy outputs and extreme distances in physics. Scientists have utilized the base ten numeral system for hundreds of years to accurately represent quantities; from the distances between particles in an atom to the energy output of a quasar. This rather sophisticated but very powerful and simple to use, mathematical tool is often not discussed sufficiently in the classroom, which creates deficits in areas of scientific notation, unit representation, exponential arithmetic and significant digits.

The electromagnetic spectrum provides a practical context to integrate the concept of scale, and refine skills of scientific notation, due to the inherent inverse relationships between wavelength and frequency. The District of Columbia has adopted the Common Core standards which requires high school students to use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media (HS-PS4-1). Students continually struggle with articulating these mathematical relationships, and these difficulties serve as the impetus for this unit. After three years in DCPS, I am keenly aware of the importance of truly understanding quantities and groups of quantities with respect to the base unit, implementation of scientific notation, and explorations of scale. All three of these basic topics are fundamentally important to all STEM fields. This 3-4 week unit seeks to enhance understanding of the relation between mathematics and physics through the continual implementation of scientific notation throughout the year, with a concentrated emphasis on SI units systems. In addition, students will develop a sense of scale through base ten numeration with the electromagnetic spectrum. The size of each relevant wavelength will be compared to metric distances to provide a familiar frame of reference. The culminating work will occur during the months of September to November as well as March; students will conduct several inquiry investigations (i.e., calculating wave speed in various media and developing EM spectrum mind maps that depict relationships based in data). It is my hope that this unit will motivate students to think critically about their physical environment as well as refine their number sense to provide valuable insight in comparing like quantities in a base ten system.

Demographics

Next year I will begin a new chapter in my teaching career at Woodrow Wilson High School (WWHS) and serve as the 11th grade academy Physics and Environmental Science teacher. WWHS is a relatively high-performing school in Washington, DC that consists of approximately 1,800 students. The diverse student body presents nuance challenges for instructional delivery because of persistent achievement gaps within the school. Students have historically tested below grade level in mathematics, with only 22% of students meeting academic expectations. The socioeconomic issues associated with urban schools are still present (i.e., in seat attendance, classroom behavior, etc.) but to a lesser extent than other schools. The two feeder schools for WWHS are Deal Middle School and Hardy Middle School which represent two different socioeconomic populations in DC. The physics department has struggled to remain stable due to high turnover of teachers and administrators. After three years of teaching in the District of Columbia Public School (DCPS), I have learned that students respond best to a positive, dynamic classroom, with hands-on activities. The more the student understands the content’s relevance to their dealings with the world, the more likely they are to gain a greater depth of knowledge. This unit will challenge students’ notion of scale by introducing base ten numeration with the electromagnetic spectrum and creating visual representations of mathematical relationships through the analysis and application of the metric system.

Content Objectives

This 3 – 4 week unit is designed to elevate high school students’ conceptual understanding of several interdisciplinary concepts (i.e., scale, units, and scientific notation) by solving problem sets, conducting inquiry activities using the metric system, and comparing datasets related to wavelength, frequency, and energy. The unit will begin during the first advisory period (September – November), and then will continue in the third period (April-March). Topics will follow NGSS standards (see Appendix) as well as DCPS scope and sequence. The unit is divided into three sections (i.e., base ten implementation/ unit interpretation and comparing quantities across a spectrum of the powers of ten). The unit has several objectives to assess student growth including:

1. Base ten arithmetic and unit interpretation.

2. Measurement precision, accuracy, and error.
3. Comparing quantities using scientific notation across the EM spectrum.

Unit Content

Place Value with the Decimal Numeral System

The universally adopted decimal numeral system powerfully, but nearly invisibly, facilitates our capacity to compute large and small numbers. Prior to this invention, early civilizations used unique symbols to denote each order of magnitude, causing rapid proliferation of symbols to represent large quantities. The Roman numeral system adopted a slightly more elegant system by subtracting or adding numerals depending on the position of the numeral of greater value. Seven symbols (i.e., I, V, X, L, C, D, and M) represented thousands of numbers based on this addition and subtraction convention.[5] However, the advent of the decimal numeral positional notation system enabled sequences of the ten Hindu-Arabic symbols (i.e., 0, 1, 2, 3, 4, 5, 6, 7, 8, 9) for the digits to represent quantities relative to their position within the sequence of digits, with each place representing a different power of ten. The decimal number system facilitated a scientific and commercial renaissance in the West due to the efficacy of representing large numbers with fewer symbols and digits. [2] This modern positional notation is often taken for granted in secondary science courses, inhibiting rich opportunities to make meaningful connections with measurement, scientific notation, relative error, and scale comparisons. One can compare how the number 643 would be represented in the various numeral systems and easily observe the compactness of the decimal system with respect to its predecessors.

Numeral Systems	
Roman Numeral	Decimal Numeral
DCXLIII	643

Table 1. Depicts how three numeral systems (i.e., Roman, and Decimal) represent the number 643.

We should recall the elaborate structure that gives meaning to the compact form 643. It depends on several conventions that exploit algebra in clever ways.

Ex: 643

Expanded Form $= 600 + 40 + 3$

Second Expanded Form $= (6 \times 100) + (4 \times 10) + (3 \times 1)$

Third Expanded Form $= (6 \times 10 \times 10) + (4 \times 10) + (3 \times 1)$

Polynomial Form $= (6 \times 10^2) + (4 \times 10^1) + (3 \times 10^0)$

The three-digit number 643 consists of various place value pieces, in this case, 600, 40 and 3. Thus, hundreds, tens, and ones are implicitly signified by the integers in the corresponding sequence. The number can be decomposed as a sum of in its place value pieces of 600, 40, and 3. Furthermore, each piece can be expressed as multiples of the base ten units 1, 10 and 100, as shown in the second and third expanded form.

These forms clearly denote the order of magnitude with respect to the decimal numerals system's, positional notation. It should be noted that the place value that 6 represents (hundreds) is a quantity ten times as large as the place value 4 represents (tens). Likewise, 4 represents a place value ten time as large as the place value that 3 represents (ones). These terms can be written as a base ten polynomial as shown above. This process of decomposing a number in expanded forms further reinforces the nuances of the decimal numeral system, allowing students to grapple with orders of magnitude as it relates to the position of numbers in a sequence.

Exercise 1. For the number 643, which digit represents the largest value of the number and why?

As stated above, the number 643 consists of three base-ten place pieces (i.e, 600, 40, and 3). Of the three, the 6 represents the largest portion of the value since it is positioned furthest to the left in the hundreds place. The second larger number would be 4 represented four groups of tens and so forth. We note that 600 is more than 10 times (in fact, 15 times) larger than 40. These size relationships are vitally important for students to recognize when performing arithmetic operations or interpreting data.

Exercise 2. Calculate the relative value for each digit for the number 643?

643

= 600 + 40 + 3

= 600/643 40/643 3/643

= 0.933 or over 93% 0.062 or about 6% 0.004 or under ½ %

To further articulate the significance of the positional notation system students should calculate the relative value of digits with respect to a given number. If we reexamine the number 643 and again apply the associative property of addition we are left with the three corresponding place value pieces of 600, 40 and 3. To determine the overall percentage each piece represents one must divide by the total value which for this example is 643. The calculations illustrate that 93% of the number's total value is represented by 6. This exercise also highlights the relevance of rounding and significant digits. Based on our computations, 3 represents less than 0.5% of the number, a negligible difference in the overall value of the number. Understanding these fundamental concepts of place value will be essential with respect to understanding the metric scale, scientific notation, and scale.

Measurements and Units: Using Base Ten Numeration

The metric system serves as an ideal model for describing a base-ten unit system in a scientific context since most scientific fields are predicated on the ability to quantify information with SI units. The French were the first to develop standards for length (metre) and weight (kilogram) and adopt the decimal system which later developed into the metric system we recognize today. In 1960 the Systeme Inernational d'Unites was adopted. It established six bases units of measure: the meter, kilogram, second, ampere, degree Kelvin, and candela; along with sixteen corresponding derived units. [4] The system utilizes prefixes to denote the order of magnitude with respect to the base unit. For example, a kilogram represents 1000 grams or 10^3 grams (see Table 2).

The metric system was developed to be compatible with the decimal numeral system

Prefixes in the Metric System

kilo	*hecto-*	*deca-*	meter gram liter	*deci-*	*centi-*	*milli-*
1,000 times **larger** than base unit	100 times **larger** than base unit	10 times **larger** than base unit	base units	10 times **smaller** than base unit	100 times **smaller** than base unit	1,000 times **smaller** than base unit
10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^$^{-3}$
10×10×10	10×10	10	1	1/10	1/(10×10)	1/(10×10×10)

Table 2. Illustrates the function of the derived prefixes to denote the order of magnitude for each base unit

Unit Conversion with the metric system

Exercise 3. Desmond brought five weights to the weight room each of which are 10kg. How many grams did Desmond bring to the weight room?

1 weight (wt) = 10 kilograms (kg)

5weights × 10kg = 50kg (total weight)

1000g = 1kg

50kg × 1000g/kg = 50,000g (total weight)

Or

50kg = 50(1000g) = 50,000g (total weight)

The prefixes kilo indicates three orders of magnitude from the base unit gram; thus, the proportional relationship of a kilogram to a gram is provided as 1000:1. If a kilogram equates to a thousand grams, and we have a total of fifty kilograms (five weights each with a weight of 10 kg); we can calculate the total weight in grams by multiplying 50kg by 1000g/kg. The total amount of weight moved to the weight room is equal to 50,000 grams. This relatively simple exercise will familiarize students with base ten notation with regards to the various SI units that will be introduced throughout the course of the year.

Exercise 4. The length of a sports car is 4.80 meters. Convert the length to millimeters and decameters.

1 millimeter = 1/1000 meter

1 millimeter = 1/ (10 × 10 × 10) meter or 1 meter = (10^3) millimeters

4.80 meters = 4.80 × (1000 millimeters)

= 4800 millimeters

The prefix milli represents 1/1000 of a base unit thus 1-meter equals 1000 millimeters. Since the length of the sports car is 4.80 meters, converting the length from meters to millimeters would increase the numerical value by three orders of magnitude, resulting in 4800 millimeters.

1 decameter = 10 meters   or 1 meter = (10^{-1}) decameters

4.80 meters = 4.80 / ((1/10) decameters)

= (4.80/10) decameters

= 0.48 decameters

Conversely, converting the sports car's length from meters to decameters would reduce the value by one order of magnitude, resulting in 0.48 decameters. A decameter consists of 10 meters, and since the car only measures 4.8 m thus we divide 4.8 m by 10 m.

Exercise 5. The speed of light in outer space is approximately 300,000,000 meters per second; however, the speed of sound in air is approximately 1,000,000 times slower. What is the approximate speed of sound in air?

3×10^8 m/s (speed of light)

$\div 1 \times 10^6$

3×10^2 m/s (speed of sound)

300,000,000 m/s

÷ 1,000,000

----

300 m/s

One could approach this problem by utilizing standard form or by applying scientific notation. The speed of light is approximated at 300,000,000 m/s or 3×10^8 m/s, and it is estimated to be 1,000,000 or 1×10^6 faster than the speed of sound. To approximate the speed of sound we must divide 300 million m/s by 1 million, resulting in approximately 300 m/s or 3×10^2 m/s. It should be noted that the speed of sound is not fixed and is dependent upon the medium in which sound is traveling through. Although the speed of light remains relatively constant in the vacuum of space once it interacts with Earth's atmosphere, this too will affect the overall speed of light. This problem is assuming ideal conditions to simplify the arithmetic calculations. Alternatively, students could calculate the speed of sound travelling through liquid or a solid and compare it to the speed of light. In addition, students could estimate the relative distance of lighting using the relative speed of sound to light ratio. During a thunderstorm, one first observes the flash of lightning followed by thunder. Since sound waves travel roughly at 300 m/s in air at sea level one could estimate the relative distance of the lighting by counting by one one-thousand, two-one-thousand and so forth until the sound wave was heard. At three seconds the sound wave would have traveled roughly 900 m or approximately 1 km. This extension activity would provide a real-world application to estimation with respect to wave speed. This method is accurate to within ten percent due to the uncertainty associated with the speed of sound, but students will assess if this is acceptable in the framework of estimating distance.

These relatively simplistic base-ten problems highlight major deficits and misconceptions that my students have in working with the metric system, namely the arithmetic operations involved with powers and exponents. Throughout the year students will refine and familiarize their understanding of base-ten operations with problem sets and inquiry labs. Students often fail to recognize the inherent patterns within the metric system with respect to orders of magnitude and thus find it difficult to explain concepts involving large and small quantities.

Measurements: Precision and Accuracy with Scientific Notation

The *cubit*, used by ancient civilizations along the Nile delta, is the earliest known unit to measure length. A cubit consisted of the length of a forearm from elbow to the tip of the middle finger.[2] The *grain* was established to measure the mass of objects, often in conjunction with precious metals to determine overall value in commerce. The standards for cubit and grain varied regionally, creating ambiguity among empires. However, with gradual refinement and the incorporation of base-ten numerical systems, the scientific community adopted the SI system as the standard units of measure in 1960.[4] The iterative refinement of place value has allowed science to communicate quantitative measurements more effectively. The adoption of the SI systems established a total of seven base quantities including; length, mass, time, electric current, temperature, amount of substance, and luminous intensity (Table 3).

SI Base Units

Base Quantity	*Name*	*Symbol*
Length	meter	m
Mass	kilogram	kg
Time	second	s
Electric current	ampere	A
Temperature	kelvin	K
Amount of substance	mole	mol
Luminous intensity	candela	cd

Table 3. Lists seven of the base quantities established in 1960 from the international system of units.

The act of measuring is one of the fundamental skills in any scientific discipline, as the nature of science relies upon the ability to test ideas quantitatively. The accuracy and precision of measurements depends on the available scientific equipment and thoroughness of the investigator. It is imperative to know the limits of one’s ability to report quantities when using scientific notation and significant digits.

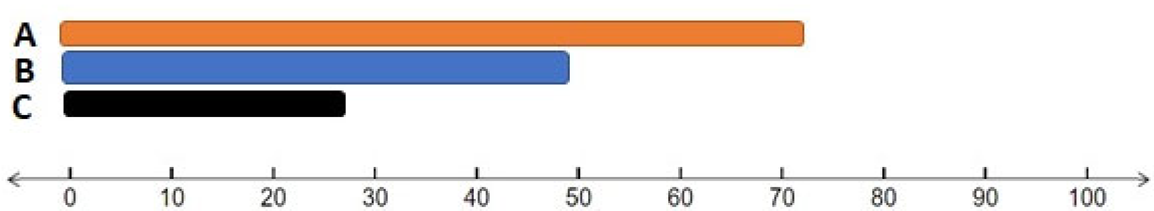

Figure 1. Illustrates the possible instrumental error of using a 100 cm ruler when quantifying the length of objects A, B, and C.

Exercise 6. What is the length of the objects in Figure 1? Justify your answer.

Sample Solution: Reading off the lengths from the ruler below the colored bars, we can verify that

Object A ≈ 70 cm

Object B = 50 cm

Object C ≈ 30 cm

Instrument error, relative error

The 100 cm ruler is subdivided into ten equal intervals of 10 cm, limiting the overall precision of measurements to the nearest tens. The objects in question do not all align to the subdivision of units and thus must be rounded to approximate the lengths leading to an error up to ± 5cm. If the true length of object A was 73.0 cm then the relative error would equate to 4.1% (see below).

Object A = 73.0 cm (Actual value)

Object A = 70 cm (Measured)

73.0 cm – 70 cm = 3.0 cm

3 cm / 73 cm = 0.04109

(0.04109)(100) ≈ 4%

To determine the relative error, one must divide the absolute error, the difference between the actual value and the measured value, by the actual value. If the actual value of object A equates to 73 cm and the measured value rounded to the nearest tens at 70 cm then our absolute error would equate to 3 cm. The absolute error divided by the actual value results in the relative error between the two quantities.

27.0 cm – 30 cm = 3.0 cm

3 cm / 27 cm = 0.11111

(0.11111)(100) ≈ 11%

Similarly, we can perform a similar computation with respect to length C and determine the relative error associated with this measurement. The relative error associated with 30cm is significantly larger since 3cm represents a larger proportion of the overall value, as such the relative error associated with rounding equates to 11%.

Significant figures and Estimation

The number of digits reported in a measurement reflects the precision or accuracy with which the quantity in question is known. Students are often tempted to exceed the limit of instrumental precision by estimating within the labeled subdivision. For example, one might expect 28.0 cm for object C and 73.0 cm for object A, while uncertainty lies within ± 5cm. These values exceed the limit of precision by two orders of magnitude and should be discussed openly. If the intervals of the 100 cm ruler were subdivided into 100 equal intervals of 1 cm, then reporting 28 cm for object C would be appropriate. However, the limits of our scientific instrument are to the nearest tens, and because of this, all measurements should be rounded to the nearest tens. Object C is greater than 20 cm and slightly less than 30 cm, as a result the object would be approximately 30 cm in length. Similarly, object A is slightly greater than 70 cm and significantly less than 80 cm, rounding to 70 cm in length. These practices become more important when measuring quantities with greater precision.

Scientific Notation

The origin of scientific notation began with Archimedes, a prolific mathematician and inventor from Greece who, among many other works, attempted to estimate the number of grains of sand on all the beaches of the whole world. [2] However, our modern system of scientific notation is largely attributed to Rene Descartes, who was the first mathematician to use the Hindu-Arabic numerals as exponents. The standard form of scientific notation is depicted below.

$m \times 10^n$

where *m* represents a decimal number between 1 and 10; *m* is called the *coefficient*. This coefficient exhibits the overall precision of the quantity. It should only contain digits that are confidently known; these are called *significant digits*. The second component of scientific notation, *n,* denotes the number of digits or orders of magnitude of base ten. It is important to remember the *m* must be a decimal number between 1 and 10 for scientific notation. Scientists regularly employ scientific notation, since it clearly indicates the key features that they want to know about a number: it size (with the exponent *n*), and its accuracy (with the coefficient, and specifically, with the number of decimal places in the coefficient).

If we refer to our previous discussion of the various lengths of objects we would note that all measurements are only one order of magnitude of 10 with very limited precision, resulting in the following measurements, written in scientific notation. It is important to note that zeros which are reported with the coefficient in scientific notation are considered significant; meaning that the degree of certainty is at least two orders of magnitude. This is equivalent to estimating between the subdivisions of the ruler, which was previously discussed.

1. 7×10^1 cm $\neq$ 7.0×10^1 cm
2. 5×10^1 cm $\neq$ 5.0×10^1 cm
3. 3×10^1 cm $\neq$ 3.0×10^1 cm

Exercise 7. Compare the mass of the Jupiter with the mass of the earth.

1.89×10^{27} kg = Jupiter or 1,890,000,000,000,000,000,000,000,000 kg

5.97×10^{24} kg = Earth 5,970,000,000,000,000,000,000,000 kg

Sample solution:

Step 1: Divide the coefficients

$1.89 \div 5.97 = 0.317$

Step 2: Subtract exponent in the denominator from the exponent in the numerator

$(10^{27}) / (10^{24}) = 10^{27-24} = 10^{3}$

Step 3: Combine the resulting coefficient with the new power of ten

0.317×10^{3}

3.17×10^{2} or 317 times the size of the earth

This scientific notation problem can be broken into three steps. First, divide the coefficients of Jupiter and Earth, as shown above. Next, subtract their exponents through the application of the quotient rule, $(10^{27})/(10^{24}) = 10^{27-24} = 10^{3}$. Finally combine the results, to get 0.317×10^{3} and then convert this to scientific notation. To do this, move the decimal point to produce a coefficient between 1 and 10 (so 0.317 -> 3.17), and change the exponent to compensate (so 3 -> 2). Thus, the result is purely 3.17×10^{2} or 317 times the mass of earth since the units are canceled when dividing. The initial measurements indicate an accuracy of three significant digits, which should also be reflected in our result.

Analyzing Scale along the Electromagnetic Spectrum

The electromagnetic spectrum spans nearly twenty-four orders of magnitude, from high energy gamma rays with a wavelength less than a nanometer to mile-long radio waves carrying signals for communication. Understanding base-ten and scientific notation is imperative prior to analysis.

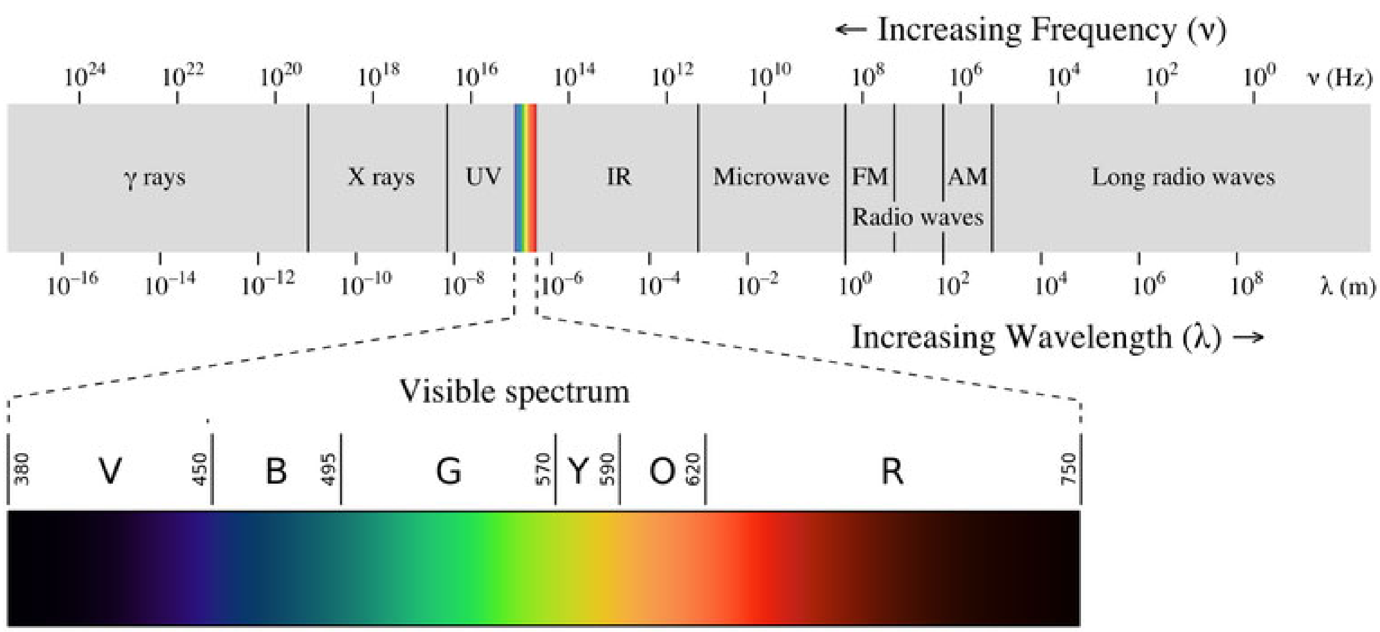

Figure 2. Depicts the electromagnetic (EM) spectrum with respect to relative frequency and wavelength. The classification of each EM wave spans varying degrees of magnitude with the visible spectrum spanning less one order of magnitude and radio waves spanning 8.

The electromagnetic spectrum (EM) is arranged along a logarithmic spectrum which allows for opportunity to discuss the difference between geometric means (GM) and arithmetic means (AM) in relation to types of wave.

The geometric mean between two numbers, *x* and *y*, is equal to the square root of the product, $\sqrt{xy}$. Given that the wavelengths along the EM spectrum are already expressed in scientific notation the geometric mean would require the multiplication of the minimum and maximum range. Since the square root is inversely related to exponential functions, the sum of the powers of ten is divided by two to get the geometric mean, as shown above. The logarithmic scale of the EM spectrum effectively transforms the geometric mean into the arithmetic mean. The geometric mean can be thought of as the "multiplicative average" whereby GM(x, y)/y = x/GM(x.y), just as AM(x,y) – y = x – AM(x,y). The relevance of the geometric mean may be beyond the scope of the classroom but could be included if opportunity should arise.

Exercise 8. Broadly compare the wavelengths of the visible spectrum to radio waves using the logarithmic scale. What inferences can be made from the range in which both EM waves occupy? Justify your answer.

From the electromagnetic spectrum diagram we read off the following ranges:

Visible Light: 4×10^{-7} (400 nm) to 7×10^{-7} (700 nm)

Radio Wave: 1×10^{0} (1 m) to 1×10^{5} (100 km)

According to the logarithmic scale from the EM spectrum radio waves are 1×10^{7} to 1×10^{15} m. Taking 5×10^{-7} as the typical light wave, 1 meter is 2×10^{6} times as long, and 10^{5} meters is 2×10^{11} as long. The EM waves that encompass the EM spectrum span varying ranges and thus varying orders of magnitude. Radio waves in the broad sense include FM, AM, and long radio waves which range in five orders of magnitude in wavelength, from 1 m to 100 km. Conversely, visible light is restricted to less than ¼ order of magnitude in wavelength, from 400 nm to 700 nm.

Wavelength, frequency, and energy are closely linked. Radio waves carry communication information over varying distances and mediums, thus occupying a wide range within the electromagnetic spectrum. However, visible light requires a specific energy bandwidth whereby some frequencies are reflected and absorbed from objects. In addition, these frequencies must carry energy high enough to interact with organic matter, but low enough not to destroy it, as our rods and cones would be destroyed. Providing context to the relative application for each EM wave will allow students to recognize patterns among wavelength, frequency, and energy. For this exercise however we will only be comparing EM waves with regards to wavelength size.

Exercise 8. Compare the frequencies of X rays to microwaves using scientific notation. What conclusions can be drawn? Justify your answer.

Again reading from the spectrum diagram, we find the following ranges:

X-Ray: (1×10^{-8}) m to (1×10^{-11}) m

Microwaves: (1×10^{0}) m to (1×10^{-3}) m

Calculating Geometric Means

X-Ray: (1×10^{-8}) to (1×10^{-11})

$\sqrt{(1 \times 10^{-8})(1 \times 10^{-11})}$

$\sqrt{1 \times 10^{(-8)+(-11)}}$

$\sqrt{(1 \times 10^{-19})}$

$\approx 1 \times 10^{-9.5}$

Microwave: (1×10^{0}) to (1×10^{-3})

$\sqrt{(1 \times 10^{0})(1 \times 10^{-3})}$

$\sqrt{1 \times 10^{(0)+(-3)}}$

$\sqrt{(1 \times 10^{-3})}$

$\approx 1 \times 10^{-1.5}$

Comparing EM Wavelengths with Scientific Notation

X-Ray: $(1 \times 10^{-9.5})$ m

Microwaves: $(1 \times 10^{-1.5})$ m

$(1 \times 10^{-9.5})$ m

$\div(1 \times 10^{-1.5})$ m

Step 1: Divide the coefficients

$1 \div 1 = 1$

Step 2: Subtract exponent in the denominator from the exponent in the numerator

$(10^{-9.5})/(10^{-1.5}) = 10^{-9.5-(-1.5)} = 10^{-7}$

Step 3: Combine the resulting coefficient with the new power of ten

1×10^{-7}

To estimate the relative difference in wavelength between X-rays and microwaves we must apply the geometric means between the ranges of the two EM waves as estimates (see above). As previously discussed, the geometric mean can be thought of as the multiplicative average between a set of numbers. However, since the EM spectrum falls along a logarithmic scale the geometric mean is essentially the arithmetic mean with regards to the exponents. This problem is relatively complex due to the extreme nature of the order of magnitudes and our understanding of geometric means. However, if we reduce this calculation into two parts, decimal digit division and subtracting the exponents by applying the quotient rule, it becomes more manageable for students. In the first step we divide the decimal digits of 1 m by 1 m, resulting in 1. Next, we apply the quotient rule to determine the overall magnitude difference of 10^{-7}. Combining both calculations results in the value of 1×10^{-7} , thus an x-ray is on average seven factors of 10 smaller than a microwave. In this case, since the two types both span 3 orders of frequency/wavelength magnitude, if you compare longest to longest, you will get a factor of 10^{7} , and the same with shortest to shortest.

Arithmetic Awareness

This unit is structured to enrich physics content by elucidating patterns inherent within the decimal numeral system. In other words, physics content will remain the driving force throughout the year, but intentional activities will be strategically placed to blend mathematics into the foreground whereby students will critically examine arithmetic operations. A major deficit with many students is the lack of arithmetic awareness, which comes from not asking a simple question: "Why?" At the beginning of the year we will analyze the significance and usefulness of decomposing a number into its place value constituents, as discussed above, and will begin discussing the importance of units. It is my hope that this will provide greater context to the problem sets and labs performed throughout the year. The measurement lab will be performed early in the year to address misconceptions associated with precision, significant digits, and relative error. Understanding the fundamentals of base-ten will become vital with the introduction SI units and the metric system. To promote my students' grasp of these ideas, a considerable amount of time will be spent converting units with open discussions in small groups. Ultimately, the goal is to create a classroom culture that fosters insight about arithmetic operations. By the third advisory, students will be asked to apply these principles to the EM spectrum and critically analyze the various EM waves in terms of a logarithmic scale.

Teaching Strategies

Station Rotations and Small Groups

Station rotation facilitates the engagements of students by rotating students through several concurrent activities throughout the class period or week, depending on the model. This instructional strategy allows students multiple opportunities to refine conceptual understanding and mastery by participating in activities that target various modes of learning (e.g., kinesthetic, auditory, visual). Students will spend approximately 20 minutes working independently or in groups on activities. As a class, students will share findings, observations, and misconceptions that persist. These open discussions will provide a framework for peer evaluation and allow students to guide topics.

The unit will be interspersed between the first advisory (September-November) and third advisory (April-March) with cooperative learning in heterogenous groups. Stations will consist of students at various mastery levels to foster leadership roles within the classroom (i.e, timekeeper, recorder, reporter, analyst). Students will apply, and practice concepts introduced during the first half of class. The number of stations may vary based on the number of students and classroom dynamics, however this pedagogical model is designed to foster independence, communication, and accountability among the students. In addition, student rotations will enable differentiation among groups to ensure that content mastery is achievable.

Inquiry Activities and Lab

As a science, physics offers a plethora of opportunities for students to apply mathematical concepts and refine arithmetic skills when describing physical phenomena. Student engagement and content mastery is dramatically increased with hands-on activities, as many of my students are visual and kinesthetic learners. This unit will seek to strengthen students' content mastery of base-ten arithmetic and the nuances associated with measurement error and units. Students will be asked to participate in several measurement activities to elicit misconceptions associated with comparing quantities in scientific notation as well as examining instrumental error. To grapple with logarithmic scales, students will create a base ten scale with a minimum of six orders of magnitude for examining the size of objects, where ultimately, they will be able to infer patterns in the EM spectrum. Ultimately, the inquiry activities and hands-on labs are aimed to foster memorable learning experiences. In addition, the labs will allow a closer examination of the arithmetic operations that are essential to many topics within physics (scientific notation, relative error).

Pre-Assessment and Post-Assessment

A pre-assessment will be administered prior to the unit to establish baseline data of students' capabilities and content knowledge. Questions will consist of a selection from the district's standardized test (PARCC) as well as foundational arithmetic base-ten questions related to measurement. Students will be asked to interpret graphs and data as well as estimate large and small quantities using scientific notation. Data will be compiled from last year's PARCC assessment to determine overall academic performance. Students will be assigned problem sets that will gradually challenge their skill level, and will be arranged in heterogenous groups. Each problem set will consist of five to six questions that will begin with simple arithmetic questions and end with problems that require a multi-step solution. These problem sets with serve as informal assessments and facilitate discussion for station rotations. Students should be able to articulate their attempt at a problem and will be required to record every step. Our district focuses on data-driven results and it is my hope that these assessment metrics will demonstrate the overall effectiveness of this unit.

Classroom Activities

Creating a Base-Ten Scale

Students will create a customized base-ten scale using their age (i.e., days, months, or years) as a base unit and make comparisons along six orders of magnitude. For example, a student that is 16-year-old would have a base unit of 16 and would provide three temporal examples of increasing magnitude (160, 1,600 and 16,000 years) and three of decreasing magnitude (1.6, 0.16 and .016 years).[3] This inquiry-based activity will allow students to compare quantities using scientific notation and refine their base-ten arithmetic. In addition, this activity will provide a temporal context that will enable students to grasp orders of magnitude. The base-ten scale will be peer evaluated using a rubric with a gallery walk.

Measuring lengths with variance / Decomposition of a meter

The ability to accurately measure and report precise quantities is a skill that is fundamental to all disciplines of science, especially physics. Students will participate in a series of measurement labs that will examine base-ten in relation to the metric system and practice measuring objects of various lengths. Groups will be given an assortment of objects with rulers at different levels of precisions and accurately report lengths in scientific notation. Each group will have the same objects with different instruments of precision; some may have a ruler with inches others, a ruler with centimeters or millimeters. Students will share their results and discuss possible errors that may have arisen due to instrumental error. In addition, students will perform unit conversions to determine how accurate each group's measurement was, relative to their own. These inquiry investigations will elicit discussions on the importance of measuring in relation to relative error and rounding.

Comparing orders of magnitude in the electromagnetic spectrum

During the third advisory students will compare EM waves along the logarithmic scale, investigating patterns in wavelength, frequency, and energy. Comparisons will be made broadly, examining the overall range with each type of EM wave. In addition, students will take the geometric mean of the upper and lower limits of wavelength for each type of EM wave and compare different parts of the spectrum to more precisely determine differences in the order of magnitude of wavelengths for these different types of radiation. Inferences will be made about energetics for each EM wave and their application.

Appendix

Standards

The unit will incorporate standards from the Next Generation Science Standards (NGSS) as well as common core high school mathematics.[1] During the first advisory students will grapple with SI units with regards to kinematics and use units to guide in solving for multi-step conversion problem sets (CCSS.MATH.CONTENT.HSN.Q.A1). Students will then explore base-ten numeration utilizing scientific notation by creating a logarithmic scale with their age as a base unit. Student will calculate how many mega seconds old they are when they reach a billion, to further provide context about scale. The culminating activity will have students analyze the EM spectrum to elicit patterns with regards to wavelength and frequency (HS-PS4-1).1 A fundamental understanding of exponents will be paramount to recognizing patterns within the EM spectrum as such a considerable amount of time will focus on arithmetic operations involving exponents, in the form of scientific notation problem sets.

Bibliography

1. NGSS Lead States (2013). Next Generation Science Standards: For States, By States. High School Physical Sciences
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3. Boeke, K. (1957). *Cosmic view: The universe in 40 jumps*. New York: J. Day.
4. Gupta, S. V. (2010) *Units of measurement: past, present and future: international system of units*. Heidelberg: Springer.
5. Lam, Yong., and Tian Se. Ang. (2004) *Fleeting Footsteps: Tracing the Conception of Arithmetic and Algebra in Ancient Chine.* River Edge. World Scientific.