Estimating Big Numbers: Do You Really Understand Them?

Curriculum Unit 18.04.07, published September 2018
by Lynnette Shouse

Introduction

Numbers are all around us every day. They are a crucial part of our everyday life. Mathematics is now so critical to understanding modern society that some have labeled it the "new civil right."(1) We use numbers without really thinking about them. This unit will focus on developing deeper number sense, interpreting place value and order of magnitude, and how all of these support making reasonable estimations.

Here are some questions I thought about as I prepared my unit. Can students really understand the differences between really big numbers, and between really small numbers? How deep is the knowledge required of our students in determining relative size of numbers based upon the position of the digits? Do students grasp the concept that our place value system is based upon the number 10? Do they know what a thousand, ten thousand, hundred thousand, or even a million items would really look like? As John Allen Paulos said in his book *Innumeracy*, "Knowing that it takes only about eleven and a half days for a million seconds to tick away, whereas almost thirty-two years are required for a billion seconds to pass gives one a better grasp of the relative magnitude of these two common numbers."(2)

This unit will focus on clarification of place value and order of magnitude, and likewise improving student's estimation skills. By reviewing what estimation is and how we can create a reasonable guess, my hope is that students become more comfortable working with and manipulating larger numbers such as ten thousand, hundred thousand and continuing into the billions. In keeping with our theme I will use three key texts: *Great Estimations* and *Greater Estimations* by Bruce Goldstone and *Millions, Billions, and Trillions, Understanding Big Numbers* by David Adler. I will also be using a video to introduce and help to visualize the idea of order of magnitude, called *The Powers of 10*.

Rationale

Grissom Elementary is a high-performing school with diverse ethnicities represented. It has students in pre-kindergarten through sixth grade. Approximately half of the student body is made up of Hispanic and African-

American students. The other half is a combination of Caucasian, Asian, and Native Americans. Many students attend Grissom based upon school choice, where parents apply to attend this site within the larger Tulsa Public Schools system. The school was founded in 1969 and is named after astronaut Virgil “Gus” Grissom. His name implies exploration and service which is a major focus of student life. The school is a demonstration school for the Oklahoma A+ Initiative, which integrates the arts into all areas of the curriculum.

My students have been identified as gifted based upon their classroom performance and/or criteria established by the district to signify advanced potential. My fourth grade students have been with me as their gifted teacher for the past four to five years. My students come from diverse ethnic backgrounds and have academic abilities that are advanced when compared to other students in their age/grade levels. Sometimes they seem trapped by their giftedness, and they feel that they have to know the right answers at the beginning of a study. They can be afraid of being wrong and of letting their peers see that they do not always know the correct answer. They are not comfortable challenging their own thinking on a topic. Most are used to being “smart,” which in their minds translates to: if I don’t know the answer then it is not worth studying. They can sometimes have the fixed mindset that I’m smart and I want to make sure that I get the right answer. If they are questioned about their thinking, many times, they cannot explain how they arrived at their conclusion. They say, “I don’t know how I know, I just know.” I will use this response as a pretext for a class discussion about how to solve a problem.

I want to use a visual concept of the skill, starting with a hands-on demonstration, then match that to a pictorial representation, and only later proceed to the algorithm. I hope this will assist them in breaking out of that fixed mindset and move towards a more growth mindset approach that says, “The more that you challenge your mind to learn, the more that your brain cells grow.”(3)

I want my students to use their reasoning and thinking abilities to work through problems and discuss with others how they are solving the problems, and work in small groups of peers. Not only is the review of these skills necessary to go deeper, it will allow me as the instructor to use relevant problems that will allow my students to apply problem-solving skills. I also want them to feel comfortable not knowing the answer immediately and be willing to talk about their misunderstandings in small group discussions. I want the students to … “learn to reason and to *justify* their solutions to learn that mathematics is about making sense.”(4)

Content Objectives

This unit will take approximately three weeks to teach. I will present it during the first quarter, and it will share time with other classwork. The first week will focus on the concept of place value and how the base ten system is an efficient way to represent whole numbers. As a pre-assessment activity, I will give students a target number, 467, and having them think of all the ways they can represent that number. They may create math equations using various operations or properties (e.g. 467 x 1), they may decompose the number into smaller addends (400 + 60 + 7), or possibly even create a graph to show their understanding of that target number. Additional work will center on the Oklahoma fourth grade standard (4.N.2.7), “Comparing and ordering whole numbers using place value, a number line, and models such as grids or base 10 blocks.” I do not anticipate much misconception during this initial week as it should be review of third grade topics. Confusion may arise if I use a larger number moving into the thousands and ask them to repeat the task. This will not be a review of previous work. I will stretch their thinking by helping them grasp the concept that the largest place where the numbers differ, determines which is greater. I will ask them to compare numbers with 4, 5, and 6 digits.

The second week will be focused on the texts, *Great Estimations* and *Greater Estimations,* as students move towards using very large whole numbers and plotting these big numbers on a number line. An anchor activity will be to have jelly beans in cups of various sizes (1/4 cup, ½ cup, 1 cup, and 2 ½ cups), have students estimate the number of jelly beans in each cup starting with the smallest cup first. Then use their knowledge of the base ten system to estimate how many jelly beans will be in a cup that is twice as big, four times as big, or ten times as big. Students will count the actual number of jelly beans in each cup. I will use class discussion to compare and contrast estimations and actual counts. Similar scenarios will be used in classroom demonstrations and the estimates will assuredly become more precise and accurate. Activities like this will address Oklahoma math standards in both fourth (4.N.1.5.) and fifth grade (5.N.1.4.), "Solving multi-step real-world problems requiring the use of different operations with whole numbers." Students will be writing and illustrating in their math notebook daily to cement their thinking and reasoning about the relationships between our number system and effective estimations. The writing in their notebooks may be numerical, pictorial or narrative forms of their learning. The use of writing or drawing is encouraging students to see the mathematical ideas. As Jo Boaler states in *What's Math Got to Do With It,* "Mathematicians draw all the time."(5) My use of the notebooks will address the fourth and fifth grades standard for language arts of listening to others' ideas and asking and answering questions to clarify meaning. By writing it down, sketching it out, or creating a similar math problem, they are participating in the productive struggle of mathematics.

The third week will focus on the connection between estimating and determining placement of large multi-digit numbers accurately on a linear number line. The primary focus text will be *Millions, Billions and Trillions, Understanding Big Numbers* by David Adler. A number line around the school that will depict the orders of magnitude by showing the relative space between numbers will be their culminating project. This number line will begin at my classroom door and the initial tick marks will be determined by using a paper image of base ten blocks cut apart and placed end to end, thus establishing the mark for ten and the mark for one hundred. Students will use these markings to then estimate where numbers such as one thousand, ten thousand, and possibly one hundred thousand will be located on the number line. As we continue through the unit, students will be adding to the length of the number line by establishing the possible position of one million and one billion. Students will also be placing individually researched quantities, such as the lifetime earnings of differing vocations or how many seconds old they are, correctly on this number line to show their use of integers in real-world situations. Additional number lines will be added to our classroom where one represents one million and the other will represent one billion. Start by subdividing into ten equal subintervals, and discussing what the tick marks would represent. Then subdivide one of the subintervals into 10 equal smaller intervals, and discuss what the tick marks there would represent. I will propose some meaningful (populations, expenditures) multi-digit numbers that will fit in the interval you have just discussed, and ask my students to locate it. Hopefully, they will see that after the 3rd digit, or maybe the 4th, the intervals have gotten too short for them to distinguish further digits. These additional number line representations will allow students to see that the first few digits determine almost all of the location no matter the length of the number line. These activities will reflect the Oklahoma standards from fourth (4.N.2.7) and sixth grade (6.N.1.1) that require them to use a number line to represent integers and recognize the concept of magnitude. The order of magnitude will definitely be a new way to discuss numbers. I anticipate some misunderstandings around this topic and so the classroom activities will address the possible misconceptions. Students have not had previous exposure to order of magnitude and thus will have a gap in their grasp of the scale of numbers.

Even though fourth graders should have already mastered place value and how to estimate for small numbers, this unit will focus on creating a deeper conceptual knowledge of both skills. Also a significant emphasis will be placed upon the concept of the order of magnitude. To that end the video *The Powers of* 10 will expose them to much larger numbers than they have been used to, and will be the initial introduction to using exponents for the base ten units, which will appear again when we are writing larger numbers in expanded form. Class discussion will center on then using information from the video to write large numbers using scientific notation. (E.g. 1,000 can be written as 1×10^3.) A strong robust foundation in number sense will be required for them to continue to learn mathematics at a more conceptual level. This unit is designed to have students take a deep "dive" into using the above mentioned skills and fully develop their number sense and ability to work with numbers in a strategic and flexible manner as well as to make a direct connection to their daily lives.

Unit content

Number sense

"Number sense is the foundational building block for all strands of mathematics."(6) Number sense is not just one thing, and a sense of very large numbers is a kind of number sense that is not much cultivated. Students who struggle to solve problems, find relationships or patterns, or even grapple with data usually lack number sense. This term can mean many things but the demonstrations of number sense include: a student can visualize what a number is, can compare quantities and determine what a large amount is and what it is not, and can use numbers flexibly to perform operations strategically. Students need to work with numbers in a multitude of ways to grasp how our number system works. Readily recognizing how to group or cluster items is but one component of a robust number sense. This understanding leads to using the number system in a systematic fashion.

Base ten system

Our number system (base ten system) is a positional system. Given that a digit can represent different values based upon its position is an elegant and organized way to keep track of amounts. This system also shows students that each digit in a number expresses a power of ten. A digit stands for a number ten times as large as it would, if it were one place to the right, and only one tenth as large as it would, if it were one place to the left. This process does not end no matter how far away from the decimal point you move in either direction. Relative place value can also be applied to pairs of places that are not next to each other. For example, digits separated by two spaces represent quantities that differ by a factor of one hundred. For example, in the number 37, 364 each digit increases by a factor of ten as you move to the left. In the example 4 represents 4 x 1; and 6 represents 6 x 10; and the first 3 represents 3 x 100 and the 7 represents 7 x 1,000 and the second 3 represents 3 x 10,000. Examples of the five stages of place value follow:

37,364 = 30,000 + 7,000 + 300 + 60 + 4

= 3 x 10,000 + 7 x 1,000 + 3 x 100 + 6 x 10 + 4 x 1

= 3 x (10 x 10 x 10 x 10) + 7 x (10 x 10 x 10) + 3 x (10 x 10) + 6 x 10 + 4 x 1

$= 3 \text{ x } 10 + 7 \text{ x } 10^3 + 1 \text{ x } 10^2 + 3 \text{ x } 10^1 + 4 \text{ x } 1^o$

The second equation is called expanded form. Students are familiar with this type of notation. It shows that a base ten number is to be understood as a sum of pieces. We call them "place value pieces." The second equation shows the multiplicative nature of the place value pieces by showing that each place value is a digit times a base ten unit (i.e., 1, 10, 100, 1000, etc.). The third equation show the multiplicative structure of the base ten units: how 1000 is ten times ten times ten, 100 is ten times ten, or a power of ten. The third equation is the one that is missing from our elementary curriculum and it demonstrates the use of exponents and powers to denote magnitude of the number. This last expression can also be termed the polynomial form. This information and understanding is key to being successful in later algebraic problems and scientific notation. The last expression will also be connected to the *Powers of* 10 video. When students understand this conceptual structure of our base ten system, they can then use strategies to assist them in breaking apart (decomposing) or putting together (composing) numbers effectively. They are building their number sense.

Order of magnitude

The orders of magnitude of numbers are used to make approximate comparison. Order of magnitude can be defined in a whole number as one less than the number of digits required to write it. Thus, the order of magnitude of 146 is two. The mathematical concept relates to place value and the relative size of a number. It is specifically important when working with very large numbers that require the use of many places in the place value system. The importance of this concept provides a sense of scale. Students will be exposed to the sense of relative size within our universe through the video *Powers of Ten*. Students are not familiar with the language and terminology used in orders of magnitude in elementary school. However, this gap in their understanding leads to many misconceptions when they approach middle and high school mathematics. The lack of targeted instruction in this area will be addressed in this unit.

Estimation

In our seminar we worked with the idea that if you can determine the order of magnitude of a number, you know something important about it. This statement connects the place value connection to making good estimations. Estimating is more than making a guess, it is using strategies to come up with a reasonable number. In other words, an estimate should be close to the actual number. As Lawrence Weinstein and John A. Adams stated in their book, *Guesstimation,* "Once you have estimated the answer to a problem, the answer will fall into one of the three "Goldilocks" categories: 1.Too big; 2. Too small; 3. Just right."(7) Students need to learn that estimating is a skill that can improve with additional practice. They should learn to use techniques that will assist them in being close to the real number. To start with, "close to the actual number" would mean that they are close to the correct order of magnitude. In some cases, they may also determine the first digit. Students should estimate the area of a twenty foot by twenty foot room to be under a thousand square feet, not several thousands. That answer is in the correct order of magnitude. Three strategies, described in the Goldstone books, to make better estimates, which will be described in more detail, are eye training, clump counting and box and count. Each of these strategies provide students with concrete ways to improve their estimating skills and will be utilized in the course of this unit.

In addition to using mathematical vocabulary and processes, students will be using writing strategies to "concretize their thinking."(8) Students will use note-booking to keep track of their thinking, wonderings, and realizations as we maneuver through this unit. It will allow them to keep a record of their classroom activities, pictorial representations, and strategies used for future reference. The notebooks will also allow me as the instructor to assess in their responses their thinking patterns or misconceptions. It will allow us, as a learning cadre, to create math problems and to creatively solve those using strategies that will be discussed within their small groups and by the whole class. As Joan Countryman states, “Writing strengthens a student’s experience with a new concept. They fix on the page connections and relationships between what they already know and what they are meeting for the first time.”(9) Using note-booking as a way to capture thinking will also allow these students to see the connections between language arts structures and math learning.

Text Selection

Even though we will be working on math concepts, I have selected three main texts to use during this unit. These texts are similar to each other in structure, and the mathematical theories in each are pedagogically sound. The bold graphics and illustrations will appeal to the visual learners and each text has opportunities for students to engage with it interactively.

The first text is *Great Estimations* by Bruce Goldstone. The text introduces estimating in a fun way with colorful found objects illustrations. Students usually follow the “Goldilocks pattern” when estimating: either too small, too large or just right. However, they struggle when trying to determine if their estimation is reasonable. This picture could be used to introduce how to cluster items to estimate the total number of bears in this picture. If you cluster all the orange bears together they represent ten bears. Students can then use this number to estimate the total number of bears, regardless of color, in the figure 1.

Figure 1

The text starts out using groupings of items arranged in a myriad of ways to work on the technique known as eye training. This technique trains your eye to look for groupings of smaller numbers and then adding those small groupings together to arrive at a reasonable estimate. Another technique used to facilitate estimating reasonably is termed clump counting. It requires students to count a specific number of items in the picture, like 10, and then notice the amount of space required for that clump. Then use that clump to estimate the total number by multiplying by ten or by one hundred to arrive at an estimate. This strategy reinforces the powers of ten to support a student in estimating appropriately. Clump counting allows students to choose numbers that are easy to work with to find a close estimate. The final strategy introduced is box and count. If the picture has lots of things spread out then students can draw, or imagine, boxes overlaid on the image to divide it into manageable pieces. Once the boxes are in place, each individual box is given an estimated number and then the total number is estimated by multiplying the small estimate by the number of boxes overlaid on the picture. My example is if I draw 100 boxes on a graphic, then estimate the number of items in one box, I then multiply the single box estimate by one hundred to arrive at an estimate for the total number of things pictured. (Or written mathematically: total = $100 \times X$, where X is the estimate from one box).

The second book I will be using is *Greater Estimations* by Bruce Goldstone. *Greater Estimations* uses the same three techniques previously described, but it also moves into estimating volumes and measurements. The most significant differences in the two texts are the variance in numbers. The first text uses numbers reaching into the hundreds of thousands. This text focuses on really big numbers, millions and billions. An example of a picture representing possible blades of grass which may be estimated in the billions is Figure 2.

Figure 2

Students would need to isolate a small portion of the picture using the box and count method, estimate the number of blades in that portion and then use information about the size of the picture to estimate a total number of blades of grass. This requires working with very large numbers and the use of area to calculate the approximate number of blades of grass. Both texts reinforce the five stages of place value by encouraging students to also use exponents when working with really big numbers. The author's letter at the end of this text highlights an important point for students--that estimating is an everyday activity. Estimating has implications vocationally, politically and socially. It affects how parents prepare meals, how planners set up public events, and how governments allocate funds. This direct connection will enable students to see the correlation between math and real world application.

The last text will be *Millions, Billions, and Trillions, Understanding Big Numbers* by David Adler. The illustrations make these numbers accessible for students. The problems presented show how these large numbers are used in our everyday life. An example of how a picture could be used to assist students in developing their understanding of these very large numbers is provided beneath figure 3.

Figure 3

This picture is a herd of sheep. Students can use their estimating skills to find out how many are in this picture; however, if the question then becomes how many sheep are located in the United States, then students must do some further calculations. This would require some research and then a knowledge of using multiplication using the orders of magnitude to provide an approximation of the total number of sheep in the United States. An extension of this problem would be to calculate how many sheep would be in a square mile or how many per acre? This example would provide a chance to discuss how many significant digits should we keep in a large number? Do we need to keep all of them or would rounding to the two or three significant digits be a reasonable estimation? This discussion will also be informed by the previous number line activities.

The text is presented in a logical sequence to assist students in arriving at the understanding that each of these numbers is based on the powers of ten concept. The examples of where these large numbers are useful also provide chances for students to delve deeper into realization of how immense these numbers are. A misconception I believe students will have is how these very large numbers make a direct connection to their daily life. So we would proceed to work in small groups to answer the following questions: How many minutes old are you? How many million seconds old are you? How many billion? How many billion seconds old are your parents? I believe their first estimates would be significantly too small. This would then lead to that "productive struggle" when students would need to reason through the steps to arrive at a reasonable approximation of their ages in minutes and seconds. Students will also talk about how many people are in the school, and in Tulsa, and in Oklahoma, and in the US, and compare these numbers, and perhaps also compare US or Oklahoma population to the whole world.

Teaching Strategies

Pre-launch

Before a new unit takes place, I like to use a strategy that is, in essence, a pre-assessment. I call it pre-launch with my students. (My school mascot is the Grissom Stars and we are named after Virgil “Gus” Grissom—consequently the space vernacular.) Students are presented with an initial set of tasks to determine their background knowledge and use of strategies. I have them cut out and paste the tasks presented in their notebook and then they proceed to work out the problem(s) individually using whatever strategy they think is appropriate to arrive at an answer. This pre-launch allows me to set the stage for our upcoming learning and to also assess their familiarity with or misconceptions about our study. Many times gifted students have mastered some content that is above their age/grade level peers, but they may have some misconceptions that need to be addressed, or a student may need more depth of exposure than provided by my original plan. This first day procedure gives me a chance to ensure that my unit is targeted to meet their academic needs and provides the structure for us as a learning cadre to pique interest in the topic. Students will also re-visit these same tasks later in the unit to use new-found strategies in solving the problems. A pre-launch is like the pre-launch in the space program, it assesses our course and ensures that the entire group of learners is on board ready to launch the unit.

Demonstrations

Many times demonstrations are relegated to science classrooms, but math lends itself to using hands-on manipulatives as well. During the course of this unit, both students and I will be engaging in using supplies from the kitchen to work on our estimation skills. I will start each segment, anchored to a picture book, with a demonstration using household food. These demonstrations will also help make the direct connection between these very large numbers and everyday life. It will also help them internalize that estimations need to be close to a real number and that choosing a range of appropriate answers is also necessary. (E.g. should this be to the nearest one hundred or the nearest one thousand?) I will discuss with students the number of significant digits required in their estimations and what order of magnitude is their estimate. The overall message should be that the order of magnitude is good, the first digit is great, and two correct leading digits is terrific (and perhaps lucky). This can be reinforced by the work with locating numbers on number lines, and probably also with the work with measuring cups.

Small and whole group discussions

Whoever is doing the most talking is doing the most learning. Discussions amongst students is a strong indicator of their level of understanding. When students are sharing with their peers or discussing in whole group how they arrived at an answer, all of the learners in the room, including the teacher, benefit from hearing. As Jo Boaler says, “Children should also be encouraged to ask questions of themselves and others.”(10) Students’ talks must be centered on explaining how they arrived at their answer. Small group and subsequently whole group discussions will allow all learners to “see” mathematical methods used and hear the reason backing up their use. This will give me an opportunity to hear how students are engaging with the content, see their strategy usage, untangle their misconceptions, or probe to go deeper into making connections to other concepts. Some specific prompts that I may use are:

- How did you think about the problem?

- Why did you do it that way?
- Can you think of another way to do the problem?
- How do the two ways relate?
- What could you change about the problem to make it simpler or easier?

Having students talk throughout the course of the unit will deepen their knowledge and strengthen their number sense. This will contribute to their abilities moving into other areas of math instruction.

Note-booking

The format that I will use will be an interactive format. Students will use their notebook each day to keep track of our pre-launch activities, work on demonstrations, as well as the problem-solving component of this unit. Students will be encouraged to represent all of these activities in ways that make sense to them. They can use drawings, problems, words, or any combination of the above items. This allows them to refer back to their own notes throughout the unit. It can also help them discover patterns in our activities and connections between the larger themes. The interactive portion will be where students will save space at the bottom of each page for me (approximately one pinky length measuring up from bottom of the page) to jot down questions, ideas or notes to them on a weekly basis. I pick up the notebooks a few times per week to look through their notations. I use this discovery time to assist me in planning for our next steps, and to ensure that I am not leaving any students behind in their understandings. Note-booking is also a simple way to share with parents the path of learning that their child is on at the present time. It is a built in record of their interaction with the content and a way to teach students to take notes for themselves. Taking notes is listed as a key strategy for positively impacting student learning in *Building Academic Vocabulary*. I can also use the note-booking to have direct instruction in note-taking and organization of those notes.

Classroom activities

Pre-launch

Each student will receive two index cards, each having a number on it, and each group will sort its cards from least to greatest. The numbers will vary in size from hundreds to billions. As a follow up activity students will be given a target number to represent in their notebooks. They may create pictures, computational problems, word problems, decompose or compose it, or graphically represent it. I will have a variety of card sets, some of which are obvious to sort, say order of magnitude or largest digit, and others of which have some pairs that have the same magnitude, and only differ in some digit three or four places from the leading digit.

Estimating demonstrations

Our first one will involve putting jelly beans in cups of graduated sizes. (1/4 cup, ½ cup, 1 cup, and 2 ½ cups) This will give students a chance to put their estimation skills to work. It will also help them see the multiplicative power in our base ten system by noticing that if 10 jelly beans fit into the ¼ cup then 20 might fit into the ½ cup measure, and 40 jelly beans might fit into the cup. Also, the leading digit will probably just scale up like that, but the second digit may not. Discussion as to why this happens will be instructive. Students will check how closely the number scale is to the exact counting of jelly beans in each measuring cup. The process of counting actual items in each measuring cup will be repeated to see if exactly the same number of items go in each time it is filled. The second demonstration, anchored to the *Greater Estimations* book, will involve using grains of rice in the same cups. I want students to realize that more grains of rice will fit into a ¼ cup than jelly beans due to the size of the item. Scale of items is paramount when determining a reasonable estimate. Students will estimate how many grains of rice will fit into a jelly bean. We will use the same idea of estimating how many grains of rice will fit into the same cups as the jelly beans. In this demonstration though I will first give each table group just 1 jelly bean and 1 grain of rice, and will have them pre-estimate how many will fit into the cups and noting an estimate in their notebook. Then we will actually count out the number of grains of rice in the ¼ cup and make estimates for the other sizes. The final use of the cups will be to estimate the number of granules of sugar. Again, the logic should be that more granules of sugar will fit into the same cups than either the rice or the jelly beans. Once again students will use 1 jelly bean, 1 grain of rice and 1 sugar granule to make an initial estimate. They will also estimate how many granules of sugar will fit into a grain of rice, or in a jelly bean. Then the procedure will repeat. How many granules do you think will fit into the ¼ cup? Each group will explain their estimate so that all receive the advantage of their peers' thinking process. The granules of sugar will reach into the millions and billions since approximately ¼ cup of sugar = one million, as stated in Adler's book, *Millions, Billions and Trillions.*

I will use an activity from *Great Estimations* using cereal O's to estimate the total number of O's in the box. First, students count one hundred O's and cluster the O's into a circle. Next students count one thousand O's and put these into circle. I can use this as a discussion on the order of magnitude of each amount represented. I will have them notice that the larger circle has a diameter only 3 to 4 times the diameter of the smaller one, and discuss why that might be true. Students will write one hundred and one thousand using both the fourth step in place value [100 = 1 ´ (10 ´ 10); 1000 = 1 ´ (10 ´ 10 ´ 10)] and using exponents. ($100= 10^2$; $1000 = 10^3$) The two circles are then used to estimate the total number of O's in the entire box of cereal. Once the entire box is emptied those O's will be shaped into a circle; this demonstration is using the eye training strategy to notice the relative size of each circle of O's to estimate a larger amount.

Students will use differences in weight to think about the concept that larger weights take up more space. I will have one pound, five pound, and ten pound bags of dog food for them to pick up. I will lead a discussion about the size of the bags in relation to the weight of the bag. I will then project the picture of page 25 in *Greater Estimations.* I will point out that the linear dimensions of the 10 pound bag are nowhere near 10 times the dimensions of the 1 pound bag. We might discuss how much dog food could fit in a bag with dimensions 10 times the one pound bag. I will use this discussion to make an analogy with the thousands cube in the base ten blocks set. Benchmarks of the Brussels griffin dog weighing about ten pounds and the Bernese mountain dog weighing one hundred pounds should help student small groups generate an estimate of weight for other dogs pictured. Again, the ratio of linear dimensions will be much less than 10. Two examples that are similar in size would be the Yorkshire terrier and the German shepherd. The relative size of the terrier and shepherd are similar to the griffin and mountain dog. This demonstration will lead to a whole class discussion where each small group will give two-three examples of other dog breeds chosen and the estimate of weight for each. The groups will be able to share their thinking and justification about how the estimate was derived.

Interactive math notebook

The interactive portion will be where students will save space at the bottom of each page for me (approximately one pinky length measuring up from bottom of the page) to jot down questions, ideas or notes to them on a weekly basis. By the end of the week, students will respond the following prompts in narrative writing to show their learning: What did I learn? And how can I show this learning? I can also use the note-booking to have direct instruction in note-taking and organization of those notes.

Number line

Students will create a large-scale number line in the hallway in our school. It will begin at my door with the zero mark, ten spaces will be added by using a paper tens bar cut apart and glued onto this line end to end. The next mark will be where one hundred would come based upon the paper one hundred grid being cut into strips of tens and also glued in a linear fashion on the number line. Students will then use their estimating strategies to determine where they believe one thousand would be located. Students will use ten, one hundred grids cut apart to measure and check. This process will continue until we have determined where, at minimum, one million might be located if we could continue the line out into the neighborhood. As an additional component, I want students to use some calculations to add in reference points on their number line. A few possibilities are:

How many times has your heart beat from birth to your last birthday? Or in all?

How many minutes old are you?

How many seconds old are you?

How much will you earn over a lifetime if you choose *x* profession? (Professional athlete, CEO, teacher, small business owner, lawyer, dentist, minimum wage worker) I will use this question to discuss maximum and minimum earnings to see if the difference is less than one order of magnitude. This is just an exposure of this concept, I will not be assessing this topic.

Appendix

Implementing District Standards

Oklahoma Academic Standards—Mathematics

4.N.1.5 Solve multi-step real-world and mathematical problems requiring the use of addition, subtraction, and multiplication of multi-digit whole numbers. Use various strategies, including the relationship between operations, the use of appropriate technology, and the context of the problem to assess the reasonableness of results.

4.N.2.7 Compare and order decimals and whole numbers using place value, a number line and models such as grids and base 10 blocks.

5.N.1.4 Solve real-world and mathematical problems requiring addition, subtraction, multiplication, and division of multi-digit whole numbers. Use various strategies, including the inverse relationships between operations, the use of technology, and the context of the problem to assess the reasonableness of results.

6.N.1.1 Represent integers with counters and on a number line and rational numbers on a number line, recognizing the concepts of opposites, direction, and magnitude; use integers and rational numbers in real-world and mathematical situations, explaining the meaning of 0 in each situation.

Oklahoma Academic Standards—Language Arts

Standard 1: Speaking and Listening: Students will speak and listen effectively in a variety of situations including, but not limited to, responses to reading and writing.

5.1.R.2 Students will ask and answer questions to seek help, get information, or clarify about information presented orally through text or other media to confirm understanding.

5.1.R.3 Students will engage in collaborative discussions about appropriate topics and texts, expressing their own ideas clearly while building on the ideas of others in pairs, diverse groups, and whole class settings.

Notes

---

1. Jo Boaler, "Introduction," *What's Math Go to Do With It?*
2. John Allen Paulos, "Examples and Principles," 12.
3. Carol Dweck and Lisa Blackwell, "You Can Grow Your Intelligence,"
4. Jo Boaler, "What's Going Wrong in Classrooms," *What's Math Got to Do With It?*
5. Ibid. 108.
6. Jessica F. Shumway, "Number Sense, What Does It Mean?" *Number Sense Routines. 8.*
7. Lawrence Weinstein and John A. Adams, "guesstimations," *How to Solve Problems. 2.*
8. Thomas Rowan and Barbara Bourne, "Evaluation," *Thinking Like Mathematicans,*
9. Joan Countryman, *Writing to Learn Mathematics. 10.*
10. Jo Boaler, "Giving Children the Best Mathematical Start," *What's Math Got to Do With It?*

Annotated bibliography

Adler, David. *Millions, Billions, and Trillions, Understanding Big Numbers*. New York:

Holiday House, 2013. A picture book about very big numbers with vibrant illustrations.

Boaler, Jo. *What's Math Got to Do With It?* New York: Penguin Books, 2015. Dr. Boaler discusses issues with mathematics instruction and offers ideas for both parents and teachers.

Burns, Marilyn. *About Teaching Mathematics, a K-8 Resource.* Sausalito, California: Math   Solutions Publications, 2000.

Casa, Tutita M., Kyle Evans, Janine M. Firmender and Madelyn W. Colonnese. "Why Should Students Write in Math Class?" *Educational Leadership* 74, no. 5 (Feb. 2017).

www.ascd.org/publications/educational-leadership/feb1y/vol74/num05/why-should-students-write-in-math-class€.aspx. I used this article to find out more on writing in math class.

Dweck, Carol. *Mindsets in the Classroom.* New York: Ballantine Books, 2006. This book discusses the fixed and growth mindsets model and provides activities for implementation in the classroom.

Eames Office. “Powers of Ten.” Accessed on July 11, 2018.

https://www.youtube.com/watch?v=0fKBhvDjuy0. This video shows the orders of magnitude from a picnic in Chicago to the outer bounds of the universe.

Goldstone, Bruce. *Great Estimations.* New York: Henry Holt and Company*, 2006.* Bright colorful pictures depicting many collections used to train the eye to make better estimates.

Goldstone, Bruce. *Greater Estimations.* New York: Henry Holt and Company, 2008. A picture book to work with larger numbers, ranging from thousands to billions.

Marzano, Robert J. *Building Academic Vocabulary Teacher’s Manual.* New York: ASCD Press, 2001. The companion to Dr. Marzano’s book on the most useful strategies to improve academic engagement.

Paulos, John Allen. *Innumeracy.* New York: Hill and Wang, 2001. The author describes how many people do not understand math literacy.

Rowan, Thomas and Barbara Bourne. *Thinking Like Mathematicians, Putting the NCTM Standards into Practice.* Portsmouth, New Hampshire*:* Heinemann, 2001. Provides an overview of the NCTM standards for math and how to use every day in the classroom.

Shumway, Jessica F. *Number Sense Routines, Building Numerical Literacy in Grades K-3.*

Portland, Maine: Stenhouse Publishers, 2011. This book provides foundational Mathematics practices to ensure that students are successful in math.

Sousa, David A. *How the Brain Learns Mathematics.* Thousand Oaks, California: Corwin,

2015. Dr. Sousa discusses the implications of brain research on math instruction for students in elementary all the way through high school. Book has great resources.

Vasileiadis, George. “Powers of Ten Ultimate Zoom.” Accessed on June 20, 2018.

https://www.youtube.com/watch?v=F16qnZzVRAU. This video also shows the orders of magnitude.

Wise Wanderer. “ Mind Blowing!...The Earth Compared to the Rest of the Universe.”

Accessed on June 20, 2018. https://youtube.como/watch?v=1Eh5BpSnBBw. This video shows the relative size of the planets and stars throughout our universe.

Weinstein, Lawrence and John A. Adams. *Guesstimation.* Princeton, New Jersey: Princeton University Press, 2008. An intriguing little book filled with opportunities to guesstimate answers to real world problems.
