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Decimal Place Value: A Culturally Responsive Approach

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Introduction

They were unmotivated, effort-deficient, and badly behaved this past year. I was unexpectedly met with a group of 5th grade demoralized students who had given up on math and themselves. It showed up in every aspect it possibly could, but mostly in their words to me about themselves. It hurt to hear them refer to themselves as the "dumb class." They often believed every negative thing they heard about themselves and ignored anything positive. In all my ten years of teaching (in another district), I had never encountered such negativity. I didn't know what to do, but I knew giving up was not an option. So here I am, developing this unit.

"I know I can-- be what I want to be. If I work hard at it, I'll be where I want to be." These lyrics are from a song titled "I Can," by a rap artist that goes by the name Nas. It was released in 2002 but remains relevant for students today. While children continue to face adversity in urban communities, the song is empowering. My students need to know that I believe in them no matter the circumstance. I want to inspire them to have a strong conviction in themselves.

I intend to put things in place to empower my scholars. To encourage and inspire them to believe in themselves, this unit will be culturally responsive. Zaretta P. Hammond recommends three ways to do this.[1] She says to "Gamify," "Make it Social," and "Storify" lessons to make them more culturally responsive. While being culturally responsive does not solely mean using racial pride to motivate students, I will focus on positive self-identity, purpose, and hope by connecting math to the past, present, and future of my students' race and culture. Given their lack of interest in math, it is my hope that making these connections will increase their motivation, effort, and positive behavior which will then enable increased math development. This will also help foster stronger relationships with my scholars, which essentially makes teaching so amazing!

Social connections such as friendships, teacher-student relationships and closeness to the community, are known to be closely related to well-being and personal happiness. A lack of social connection can often put people at risk for bad habits. In this case, my scholars struggle in math and have some poor academic habits. They see math as irrelevant and too difficult. It is my firm belief that students need to see that math is important in their day-to-day lives and experiences, or they become disengaged and uninterested, as is the case. In "Mathematical Mindsets," Jo Boaler says, "Over the years, school mathematics has become more and

more disconnected from the mathematics that mathematicians use and the mathematics of life.[2] Students spend thousands of hours in classrooms learning sets of procedures and rules that they will never use, in their lives or in their work." It is my goal with this unit for students to see and do the math in their world.

I want to help my students achieve a wholesome and more conceptual understanding of place value. Place value includes not only the position and the value of digits but also the decomposition of numbers and a number's relationship to other numbers in the base ten system. Place value is the foundation of basic mathematics and, if not mastered in the elementary grades, often makes working with operations more difficult as well as learning higher levels of mathematics much more challenging. By focusing on decimal place value in my students' world by connecting to the past, present, and future, I hope to develop and increase a positive self-identity, purpose, and hope in all my scholars.

In ""Multiplication is for White People:" Raising Expectations for Other People's Children," Lisa Delpit proposes that one reason "why African American students are not achieving at levels commensurate with their ability," is due to the curricular content.[3] "If the curriculum we use to teach our children does not connect in positive ways to the culture young people bring to school, it is doomed to failure." Additionally, in "What is Mathematics, Really?," Reuben Hersh says, "If mathematics is conceived apart from mathematical life, of course it seems – dead."[4] I want my scholars alive and ready for the world socially and mathematically!

Demographics

I teach 4th and 5th students math and social studies in the urban setting of the Pittsburgh Public School District. My building is the largest K-5 Elementary School in the city with approximately 500 students. The community is predominantly African American (approximately 95%) and the school percentage is similar. As a neighborhood and community school, only students living in the community can attend. The school provides many needs including dental, eye, mental and other health screenings. The school has partnerships with various outside organizations that provide services including, but not limited to: mental health, mentorship, gardening, ESTEAM (Entrepreneurship, Science, Technology, Engineering, Art, and Math) activities, abuse prevention, conflict management, therapeutic services, a college prep program, and academic and tutoring support. Despite having school services and community resources such as a community college branch campus, churches, a YMCA, restaurants, convenience stores, barbershops, and other facilities and businesses, the community where my school is located is impoverished. Nearly 90% of the students in the building are considered economically disadvantaged. The median income is approximately $29,000 which is barely half of Pittsburgh's median of $56,000.[5] The community has high unemployment, poverty, & crime rates. In the 2016-2017 school year, 84% of the third graders, 87% of the 4th graders and 95% of the 5th graders scored below grade level in mathematics.

This year, I anticipate a similar group of students, but the difference is, I know many of the students, have a better understanding of how the school works, who the students and their families are, and what the community has to offer. These are things I had to learn this past year. I now have time to prepare myself, and I am working to set the stage up for my students' success.

Content Objectives

This 2 to 3-week unit is designed to focus on having my students correctly interpret, use, and compare base ten numbers, especially decimal fractions. Arithmetic will not be covered directly in this unit. In “Place Value as the Key to Teaching Decimal Operations,” Judith Sowder explains,

> More recent research on decimal-number understanding confirms that many students have a weak understanding of decimal numbers. […] The children in these studies were primarily from classes where the introduction to decimal numbers was brief so that sufficient time would remain for the more difficult work of learning the algorithms for operating on decimal numbers. But time spent on developing students' understanding of the decimal notation is not time wasted. Teachers with whom I have worked claim that much less instructional time is needed later for operating on decimal numbers if students first understand decimal notation and its roots in the decimal place-value system we use.[6]

In my experience, this is true. It has also been my experience that reteaching my students anything, is challenging, and reteaching students how to operate on decimal numbers in deep, meaningful ways is no exception. My students are often reluctant or angry when they must relearn what has already been taught, and what they thought they already knew. This unit will attempt to alleviate some of the reteaching and frustration that may usually occur when working on operations of decimal numbers.

The topics in this unit will follow the Pennsylvania Core State Standards, the Assessment Anchors and Eligible Content Aligned to the Mathematics Pennsylvania Core Standards, and the Pennsylvania Core State Standards for Mathematical Practice Standards (see Appendix), but the standards are very similar to the Common Core Standards. Additionally, the topics will address the Pittsburgh Public School District’s Scope and Sequence. The unit is divided into two sections:

- Understanding the base ten place value system
- Comparing Decimal Numbers

Because I feel it is important for students understand where math, and all subjects, fit into their lives, this unit will also empower and elevate 5th grade students’ conceptual understanding of decimal place value by:

- Being culturally responsive;
- Implementing goal setting using pre-assessment data;
- Using concrete representations such as base-ten blocks, straws, and meter sticks;
- Using visual representations such as hundredths grids and number lines;
- Using abstract representations such as place value charts;
- Differentiating instruction in small groups to ensure mastery;
- Completing a culminating project-based learning activity and post assessment;
- Using portfolios.

By the end of this unit, students will be able to:

- Explain and illustrate, in various ways, how a digit in one place represents 10 times of what it

represents in the place to its right and 1/10 of what it represents in the place to its left, using place value charts, hundredths grids, and meter sticks. (With adjacent digits such as 6.55 and in separate numbers such as 6.15 and 5.61).
- Read and write decimals to the thousandths using base-ten numerals in various forms including, word form, and expanded form (using decimals, recognizing the multiplicative make-up of the place value pieces in decimal form and decimal fraction form).
- Compare two decimals to thousandths using the comparison symbols >, =, < to record the results of the comparisons.

Understanding the Base Ten System

The key to understanding the base systems is realizing that digits "roll over" like a clock or an odometer when they are full. Our decimal system is Base Ten, meaning that it uses a system of units, each of which is ten times as large as the next smallest one, and conversely, one tenth as large as the next largest one. Consequently, we never need more than nine (one less than ten) of any unit to express a number. When we get to ten of any given unit, it "rolls over" and creates one of the next larger units. Our method of measuring time is similar, but with time, we wait 60 seconds before "rolling over" to a new minute.

Back in the day, Babylonians used base 60 and Mayans used base 20, but base 10 is what we use today. Perhaps because we have 10 fingers. The Egyptians, Greeks and Romans also used base ten, but they did not have the idea of place value, in which the position of each digit indicates the unit is multiplying. The Egyptians and Romans had a separate symbol for each of the first few units. The Greek system was very limited and used their alphabet.

The base ten system is also known as the decimal system. Although the concept is not simple, the system is simply a way of writing numbers. It is called a positional system because each place value increases by a factor of 10 from the place to the right! Prior to developing this unit, I did not know the decimal or base-ten place value system was based on the Hindu-Arabic derived symbols for the digits: 0,1,2,3,4,5,6,7,8, and 9. It was introduced in Europe around the 12^{th} century, and because it was such an organized way to work with numbers, that allowed for counting and calculations in a more efficient way, it replaced the Roman Numerals, which had been the main method for writing numbers. Place value notation is more efficient than Roman Numerals (compare MMXVII with 2018), but the big advantage of place value notation was the way it supported computation, especially multiplication!

The Egyptians used Hieroglyphic writing, which included symbols for writing whole numbers up to 1,000,000. This decimal based system allowed for the additive principle: a number was represented by a collection of symbols whose values added up to the number. This is a way in which numbers can be counted. There was a special symbol for each power of 10. See figure 1 below. To see the Egyptian Base Ten System in its entirety, please go to the notes section.[7]

It is important to mention this historical background here because my students have failed so much that they believe they can't do math. Even many of their parents and family members make snide comments such as, "I wasn't good at math either, so I don't expect him/her to be that good either." This belief is extremely deep in many African American students, so it's important to acknowledge the rich history of their ancestors and show that math is already in their blood. Again, I am aiming to empower my students to believe they can excel as young mathematicians, and one way to do this is to include the history of math of their ancestors and other multicultural aspects of mathematics. In the movie, Stand and Deliver, Jaime Escalante held a high conviction in his students, saying that mathematics was in their blood because their ancestors, the Mayans, were the

first to conceptualize the idea of “Zero.” Because he believed in them, and showed them who they were, his students believed in themselves and achieved at high levels.

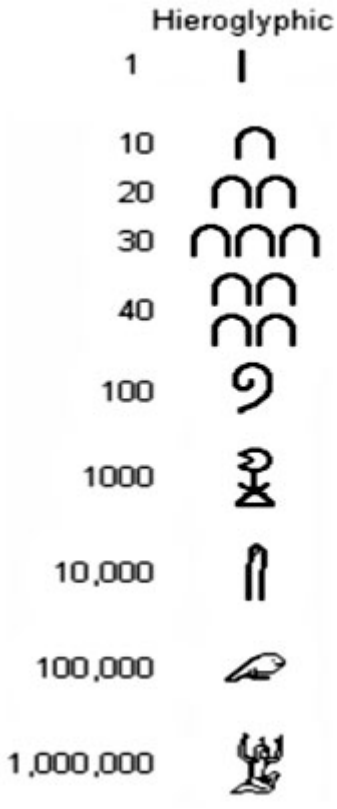

Figure 1.

The difference between the Egyptian Base Ten system and the Hindu-Arabic System is that the Egyptians did not have a positional system. In Egyptian hieroglyphics, each power of ten was represented by a different symbol, whereas 0 played a key role in specifying clearly the positions of the digits in the Hindu-Arabic System. The Base Ten positional system means that the position of a digit gives its place value. This system was extended from whole numbers to include decimal fractions by Islamic scholars in the 10th century. For example, a typical person in the community where my school is located might have a yearly income of $ 28,927.25. A student in 5th grade should be able to explain the value of the digits in a decimal fraction, such as the number above. For example, a fifth grader should know that the 2 at the left of that number represents $ 20,000 = 2 × $ 10,000, but the 2 just to the right of the decimal point represents only 2 tenths or a dollar or 2 dimes, or 2 × 0.1, or 2 × 1/10. They should also be able to explain that the two in the ten thousandths place is 100,000 times larger than the two in the tenths place or that the two in the tenth place is 100,000 times smaller than the two in the thousandths place. Each digit has an explicit value that is determined by its place in the number, and each adjacent digit that is the same is larger or smaller by some power of 10. A major part of the unit, these concepts will be address in every lesson within the unit as well as operations with decimal numbers and decimal fractions later in the year.

The Five Stages of Place Value

This unit will emphasize the five stages of place value as described in by Howe-Reiter.[8] The stages can be summarized in the sequence of the equivalences below:

625

= 600 + 20 + 5

= (6 x 100) + (2 x 10) + (5 x 1)

= 6 x (10 x 10) + (2 x 10) + (5 x 1)

= $(6 \times 10^2) + (2 \times 10^1) + (5 \times 10^0)$

The first stage is what we refer to as the standard form of a number. The second stage recognizes what we call expanded form, and exhibits the number as a sum of pieces, one for each digit, which we will call place value pieces. The third stage can be referred to as the second expanded form and recognizes the multiplicative make-up of the place value pieces. In this case, you can see the base ten units, namely 1, 10, and 100 in this example, being multiplied by non-zero digits. The fourth stage is a stage that often gets left out of textbooks and perhaps many classrooms including mine. But this stage recognizes the multiplicative make-up of the base ten units as a power of 10, or tens being multiplied repeatedly. Lastly, the fifth expression illustrates the structure using the exponential notation for powers of 10. In Pennsylvania, all 5th grade students are expected to interact with each stage based on the PA Core Standards. Focusing on the Five Stages of Place Value will give students a strong background in place value because it allows them to get to know the numbers as made of pieces and to really understand the patterns of the base ten system and the powers of ten. This unit will discuss the first four stages but will leave the fifth stage for later grade as negative exponents are not a part of the state standards. This concept will be addressed and practiced in lesson 5.

Place Value

Like the whole number system, the full decimal system uses place value meaning that the place in which a digit is located determines the value of the decimal number. For example, the 3 in 3.19 stands for three ones, but the 3 in .63 stands for three hundredths. Each digit has a place, and that place determines the value of the digit, or how much it is worth. The chart below shows the value of the decimal places between one thousandth and one million.

Whole Numbers proceed to the left of the decimal point.							AND	**Decimal Numbers** proceed to the right of the decimal point.		
million	hundred thousand	ten thousand	thousand	hundred	ten	one	•	tenth	hundredth	thousandth
1,000,000	100,000	10,000	1,000	100	10	1	AND	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1000} = 0.001$

Table 1.

It is a challenge for many students to correctly translate between word form and decimal numbers. For example, in the number 3.19 a student may say and write three point one nine or three point nineteen which is the literal translation of the symbol instead of the word form signifying the value for three and nineteen hundredths, or three ones and one tenth and 9 one hundredths. Students are unable to understand the full meaning of positional notation. Many adults use the literal translation as well, but it does not allude to place value, which is important when doing calculations and operations with decimals. I will do my best to correct

this misconception by naming the values consistently, accurately, and often, to reinforce the place value and ensure my students do so as well. Students need to verbalize each digit as it relates to the order of magnitude. Another important aspect is the word and. It must be used to show the decimal point in a mixed number.

Decimal Numbers

Decimal numbers are an extension of the whole number system, in which negative powers of ten, which represent proper fractions, are separated from the positive powers, which represent whole numbers, by the decimal point. The decimal point is a symbol that allows the base ten system to express parts of numbers. When describing quantities that have parts less than one, we call them decimal numbers, or decimal fractions. Students are introduced to three new places on the right of the decimal point by fifth grade: the tenths, hundredths, and thousandths places.

Lesson 3 will address how some students can infer and then continue the pattern to smaller decimal places. This is because each place follows the same word pattern of the base ten number system. Meaning that, after they learn about the tenths, hundredths, and thousandths, students may understand that ten thousandths, hundred thousandths, millionths, ten millionths, etc. will proceed. However, this does not mean that the student understands the multiplicative pattern of repeatedly multiplying or dividing by 10. In other words, students may not understand that "each place's value is ten times the value of the place to its right," or 1/10 the value of its place to the left.[9] As a digit moves to the right of the decimal point in a number, the value of each place is divided by 10 or multiplied by 1/10. It is beneficial to imbed this concept into a problem-solving activity. Cognitively demanding tasks that require students to explain their reasoning is an instructional practice that is beneficial for all students.

Comparing decimal numbers should be easy, but when it comes to comparing or even ordering them in racing something interesting happens. Students assume the person with the largest number wins the race. But in racing the smaller number is the winner. A good way to illustrate this concept is through an actual race which lesson 8 will address. Many students will visually and physically see who won the race and know how to order the numbers based on what they have witnessed. Some students will have a difficult time recording and analyzing the times even though they saw the race. Regardless of how students determine the order, it is imperative for my students to learn to compare each place with the same place in the other number(s) and understand that the largest place is the decision maker.

Decimal Numbers and Money

Decimal numbers can be represented in many ways. Children are often aware of them because they have some experience dealing with money. Money is very relevant to my students. Having it means they can get what they want and need to survive. Not having it means they may suffer or perhaps end up hungry and homeless. The value of money can be large, and the value of money can be small. But most students know they need money to get the things they want and need to survive. Simply put, money makes sense (pun intended). But it didn't always.

Money is a great way to "storify" this unit and connect to the past. Between bartering with objects, salt, silver, gold, cloth, cowry shells, animals, etc., an easier currency was bound to be developed. No one wants to carry a baby goat in their pocket. But my students know what it means to trade things. According to Richard Pankhurst in "An Introduction to the Economic History of Ethiopia," "The earliest example of coins minted in Africa comes from the kingdom of Axum, which struck money from the 3rd until the 8th century CE."[10] This is

pre-slavery. Our school curriculum does not often discuss Africans before slavery. My students need to know that their ancestry is rich! My students have stereotypes about what Africa is and is not, and many believe that is a poor region. I have heard negative attitudes and misconceptions shared in the classroom, and believe it or not, I had those same perceptions as a child. Typically, the math in the U.S. is presented in public school materials as the exclusive creation of men from European ancestry. But through my research, I learned the lattice multiplication method was introduced to Europe by Fibonacci. Although he was not the only, he was a major source for the use of the decimal system. He was an Italian mathematician who learned how to use Arabic numerals from a Moorish (African) teacher in Bugia, located in Algeria, in Africa. I will certainly share this knowledge with my students!

While money is familiar to many students, it is not the only way decimals can be represented. Physical representations of decimals using base-ten blocks, meter sticks or other number lines really help students understand the concept of decimal number units of measure. Lesson 4 will feature money examples.

Metric System

Did you know that the United States is only one of three countries in the world that has not adopted the International Metric System?[11] I feel that if I do not take the time so teach my students about the metric system, it will do them a great disservice as they grow to explore the world. Focusing on length in lesson 6, I will introduce my students to the metric system, and share that only the countries of Burma, Liberia, and the US, are the only three countries in the world that have not converted. Imagine how difficult traveling can be when you visit another country without this background knowledge! The metric system is a natural way to help students understand place value and number lines in a concrete way. Also, measurement is a skill that our students struggle with, and do not have a lot of time to study due the time of year in which it is introduced. I feel that if some of the basic concepts of the metric system are taught earlier and included within another standard, students are more likely to remember. Table 2 below illustrates the metric units of length. It uses a place value chart which my students will already be familiar with, as well as the mnemonic device, “King Henry Does Usually Drink Chocolate Milk,” to help them remember how to convert units.

Metric Conversions

	Whole Numbers to the left of the decimal point.				AND	**Decimal Numbers** proceed to the right of the decimal point.		
Length	**K**ilometer	**H**ectometer	**D**ekameter	**M**eter		**D**ecimeter	**C**entimeter	**M**illimeter
Mnemonic Device	**K**ing	**H**enry	**D**oes	**U**sually		**D**rink	**C**hocolate	**M**ilk
Abbreviation	**K**ilo km	**H**ecto hm	**D**eka dam	Base **U**nit		**D**eci Dm	**C**enti cm	**M**illi mm
How many meters in this unit? Word Form	thousand	hundred	ten	one	.	tenth	hundredth	thousandth
How many meters in this unit? Standard Form	1,000	100	10	1	AND	$\frac{1}{10} = 0.1$	$\frac{1}{100} = 0.01$	$\frac{1}{1000} = 0.001$
Bigger units. ⇐						⇒ Smaller units.		

Table 2

Conceptual understanding that connects the notation with the value being represented is key for students to fully grasp decimal numbers.

I will encourage my scholars and young budding mathematicians to think and ask questions such as, “How can I make this easier to deal with?” Generally, mathematicians look for ways to make complicated ideas easier and more familiar for them to deal with.

Expanded Form

The expanded form of a number shows the value of each digit in a number when in a place and decomposes each non-zero digit to represent a sum. For example, let’s consider the following: A gallon of gas at Sunoco costs $3.249. Model this number using base-ten blocks. Then write this number in word form, standard form, and the two expanded forms.

This is a typical problem a student may need to answer. Students in fifth grade are not required to write exponential notation of decimals which result in negative exponents. The value of these digits is 3.249 and can be understood to represent a sum:

3.249 = 3 + .2 + .04 + .009

The expression on the right side of this equation is the expanded form of 3.249. Each number uses the place value system, which means to place a zero in the places where the non-zero digits are located. When writing numbers in expanded form, students often have some misconceptions especially when it comes to fractional parts. They may show this number in two different ways. One way is 3,000 + 200 + 40 + 9. If a student records the number this way, they are disregarding the decimal point and reading these digits as whole numbers instead of a decimal number.

To guide a student that disregards decimal points, it would be helpful to ask them about it. I would ask these series of questions during small group time when I provide differentiated instruction to my students. Do you see a decimal point or a comma? Can you read this number to me? Read it out loud to me. How much money would this be? Can you use base ten blocks to create this number? I would have the student build the number. Which base ten block represents ones, tenths, hundredths, thousandths? Can you show me how to represent this number using money? How may dollars, dimes, and pennies? How come the nine cannot be represented using money? Can you write these digits on a place value chart? What is the value of each digit? How many ones, tenths, hundredths, and thousandths did you write in each place?

Students also have another misconception once they become aware of the decimal point. They will write and recognize that three is a whole number, but two tenths, four hundredths and nine thousandths may be written in the same place. For example: .2 + .40 + .900 instead of 0.2+ .04 + .009. While they are now aware there is a decimal point, they do not understand that each digit has its own special name and place. They believe that these digits have the same quantity or order of magnitude. When digits have the same order of magnitude, they are said to have the same quantity of "powers of 10" that there are in a number. For example, the misconception of the value of the digit 4 in the number 3.249 has a value of 0.4 = 4/10 and has a magnitude of $4 \times 1/10 = (4 \times 10^{-1})$ and the actual value of the 4 in the given number is 0.04 = 4/100 and has a magnitude of $4 \times 1/100 = (4 \times 10^{-2})$. The difference in the order of magnitude in these two numbers is 1, and 0.4 is 10 x greater than 0.04.

This misconception stems from students work with whole numbers, and the belief that all one must do to write a number in expanded form is to annex zeros to the non-zero digit. This is another reason why word form and patterns must be emphasized and practice with models must occur.

Expanded form does not stop here. In fifth grade, students much be able to decompose numbers even further. In this second version of expanded form, students are essentially breaking the numbers down into base ten pieces. Students should be aware of the identity property at this time meaning that if a multiplicand is 7 and the multiplier 1, the product will be 7. In other words, the product of any number multiplied by 1 is always equal to the given number. With that, I want my students to have experience with knowing 10 x 7 = 70, 100 x 6 = 600, but also that 0.2 = 1/10 x 2 and 0.04 = 1/100 x 4. I want my students to realize that the number of zeros in the standard or expanded form of the units tell you the value one should multiply by.

Using the approximate cost of gas mentioned above, here is the second expanded form using multiplication:

$3.249 = (3 \times 1) + (2 \times 0.1) + (4 \times 0.01) + (9 \times .001)$

Please note the following equivalencies

2×0.1	4×0.01	9×0.001
	are equivalent to	
$2 \times 1/10$	$4 \times 1/100$	$9 \times 1/1000$
$2 \times 1/10$	$4 \times 1/(10 \times 10)$	$9 \times 1/(10 \times 10 \times 10)$

I will develop and expect my students to be able to write and read these equivalences. Another version of expanded form explains how many times to repeatedly add unit fractions based on the non-zero digits in the standard or first version of expanded form. Showing students this version will help them connect what they already know about the relationship between repeated addition and multiplication. It is always important to

recognize and build on children's background knowledge and strengths. I will also explain that due to its length, we rarely want to write expanded form in this way.

3.249 = (1 + 1 + 1) + (1/10 + 1/10) + (1/100 + 1/100 + 1/100 + 1/100) + (1/1000 + 1/1000 + 1/1000 + 1/1000 + 1/1000 + 1/1000 + 1/1000 + 1/1000 + 1/1000 + 1/1000)

When my students learn material and are engaged in various ways, they retain math content for longer periods of time. Last year, paper airplanes were a huge disruption across our building. Sometimes when you can't beat the kids, you join them! To "gamify" the unit and connect it to the present, I will have students create various paper airplanes, and measure the distances they fly. Each student would need to measure the distance writing the standard form, word form, and expanded form to the nearest hundredth of a meter (i.e., nearest centimeter). This allows students to use real measurements, be physically active, and apply what they have learned. Let's say one of the paper airplanes flew 3.19 meters in length. I want my scholars to visibly see that it flew three whole meter sticks AND 1 whole decimeter (or 1.10) of the next meter stick, and 9 centimeters of the next decimeter (or 9/10) of a dm or 9/100 of the meter).

In other words, the expanded form of 3.19 = 3 + 0.1 + 0.09

Which is 3.19 = (3 × 1) + (1 × 1/10) + (9 ×1/100)

Number Lines

Number lines can be very helpful for students. As a matter of fact, number lines are an abstract form of length measurement! In this section, I show just how. For example: In lesson 6, we all flew airplanes and measured the distances each plane flew. It was easy to see which plane flew the furthest or shortest distance based on where they landed in the hall and if you saw where it landed. What may not be so easy is if the numbers are close in numerical value, and you were not in the same group as someone with that value. I will choose two numbers from our airplane list to compare. The first student's plane flew 2.7 meters. The second student's plane flew 2.65 meters. Whose plane flew the furthest? Using number lines in lesson 7 will help students see how meter stick are concrete number lines, therefore making another connection go the students and building upon what they have already learned.

Many students have a misconception that the longer the number is, the larger it is. This comes from their experience with whole numbers. A student that believes this would say the second student's plane flew farther than the first student's plane because 2.65 has more digits than 2.7.

I want my students to understand that decimal numbers "fill in" sections on the number line between whole numbers. On the number line below, the whole numbers 0 through 10 are represented. Some students do not realize there are decimal numbers between whole numbers. "You can think of plotting decimal numbers on the number line in successive stages."[12] On this number line, the first stage consists of the consecutive whole numbers from 0 through 10. This would represent the ten meters for our problem. Students would need to estimate where the numbers are to plot them on the number line. Because 2.7 and 2.65 are both greater than two, we would want the student to plot them after two but before three.

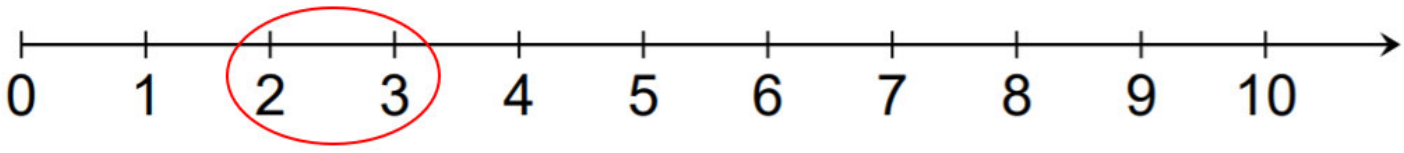

Figure 2.

Let's take a closer look at this same number line by "zooming in" to the whole numbers 2 and 3 with the tenths marked. Why? Because the students that jumped have measurements that are between two and three. Almost immediately we can plot student 1's measurement of 2.7 meters. It is a little more challenging to see student 2's measurement of 2.65. Another important thing to mention is although intervals between the digits 2 and 3 are broken into 10 sections, there are only nine tick marks for the decimal numbers. Digits in the base 10 system "roll over" after 9.

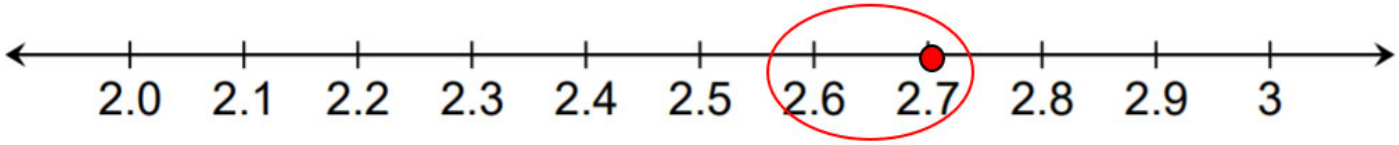

Figure 3.

Let's "zoom in" again. Where? Because 2.65 is greater than 2.6, we will "zoom in" between 2.6 and 2.8 to show the hundredths between 2.6 and 2.8 so students see that all the places between whole and decimal numbers need to be broken up into 10 equal pieces which creates new decimal numbers in between the points previously plotted. Once the hundredths have been plotted, it is easier to see where each number is located on the number line.

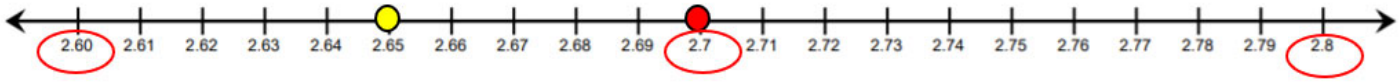

Figure 4.

At each stage in "filling in" the number line, we break each interval into 10 equal pieces which then displays new decimal numbers in between the points previously plotted. A good point to make to students is that like whole numbers, decimal numbers are also infinite. In fact, there are an infinite number of them in any interval, no matter how short! By breaking the meter stick or number line down into sections, this will show that the pieces are shrinking. By comparing the two numbers on the number line, a student is able to see that 2.7m is further away from zero than 2.65m is and therefore, able to show the first student's plane flew farther than the second student's plane; 2.7m > 2.65m. Decimal numbers do not only "fill in" number lines. They also represent distances from zero. This is important because we do not want our students to develop the misconception that decimals are less than zero. While there are negative decimal numbers, fifth grade students are only required to work with positive numbers.

Ordering Decimals

Although ordering decimal numbers is not included in the 5th grade standards for my state, I feel that it is a necessary skill for students to know. Therefore, the first part of this lesson will help to develop that concept. I will "gamify" this lesson by "mak[ing] it social."

Once these place value concepts have been addressed and hopefully mastered by students, they can then be applied to show an understanding of operations as they pertain to whole and decimal numbers. It is important to note that I do not plan to have students do arithmetic in this unit. Immediately following this unit, I will teach a unit on adding and subtracting decimals, with multiplication and division of decimal numbers

occurring later in the year.

Children often think there are no numbers smaller or less than one despite maybe carrying change in their pockets, seeing an ant on the ground, or even eating one slice of pizza out of a box. We encounter small numbers and decimal numbers every day when we see the prices of items such as $3.19. However, the conceptual understanding of decimals requires students to connect decimals to whole numbers and to fractions. Prior to teaching this unit, I will have taught a unit focused on whole numbers. I want my students to see the magic and power of the base ten system and how a good understanding of numbers and mathematics can prepare them for the world and life outside of school.

Teaching Strategies

Do Now

At the beginning of my math periods, to set the tone for the day, part of my daily routine is to have students complete an activity quickly and quietly that they must start immediately. Most times they are a review of previously taught material that takes no longer than 5-7 minutes to complete and an additional 5-7 minutes to review.

Number Talks

Sometimes at the beginning of my math lessons (at least 3 times/week) in lieu of a Do Now, we complete Number Talks. A Number Talk is a 10-15 minute whole group mental math activity where students find answers in their heads, then share the strategies they use to find that answer aloud while also explaining their thinking, justifying their reasoning, and making sense of each other's strategies.

Graffiti Walls

This is a creative way that allows students to record their thoughts, ideas, comments and questions about a topic. It allows students to learn each other's ideas. This will be used to introduce the topic of decimal place value, and to prepare for a class discussion.

Pre-Assessment

Because students come to class with such a wide variety of pre-existing knowledge, skills, beliefs, ideas, and attitudes about numbers, it is important for me to access their prior knowledge. I will give my students a pre-assessment to help me do this, and to determine what they know and what they need more instruction on. Once I know what knowledge and misconceptions they may have, I will be able to begin differentiating my instruction for each lesson within the unit which will help me form small groups.

Differentiated Instruction

My district prefers differentiated instruction to occur most days of the week. This refers to instruction that is tailored to meet my students' needs in small group settings. Personalized computer instruction, remediation, reteaching, enrichment or review fall in this area, but it also can involve such matters as the types of numbers in the problems a given student is asked to solve.

Small Groups

I typically facilitate small group instruction after whole group instruction as a way to differentiate my instruction, but also to reduce the student-teacher ratio. Small groups allow me to give my students more focused attention and a chance to ask specific questions about they have learned or are currently learning. Depending on the number of students in my class and needs of the students, the sizes of the groups will vary, however I prefer to plan groups no larger than 5 students per group.

Read Aloud

Sometimes, I will read text aloud to my students to engage them with mathematical concepts. To connect this unit to the present, it be will "storified" by reading the book, "Little Numbers: And Pictures that Show Just How Little They Are!" by Edward Packard.[13] This book shows how numbers get exponentially smaller than one by a factor of 10 for each place to the right of the decimal point! It is very important for my students to be visually stimulated, and stories are one way to do that.

Video

Showing students math in multiple ways and connecting it to the real world helps them build stronger connections. I will be sure to demonstrate this in lesson two to further illustrate the patterns in the number of zeros of the product when multiplying a number by powers of 10 and what it means to divide by a power of 10. I will show two video versions of the Powers of Ten around lesson 2.[14] Powers of Ten was created in 1977 and takes the viewer on journey traveling into space beginning with a headshot of a couple lounging in Chicago. The camera "zooms out," into outer space showing the distance being traveled in meters. These are big numbers and show the number written with exponents. The video also "zooms in" showing small numbers! The video is quite captivating. The 2nd version is more modern, and students may relate more to it.[15] However, it does not show the exponents, and I think the exponents are important to show the pattern. Nevertheless, both videos are cool and connect to science!

In addition to these two videos, I will also show another that demonstrates the linear measurements of the metric system using the mnemonic device "King Henry Does Usually Drink Chocolate Milk."[16] This mnemonic device will help my students not only remember the units of the metric system, it is my hope that I can help them make a connection with decimal place value and the names of the places. While it is easy for students to look up or use a conversion chart, they are not always permitted in testing situations. Therefore, I also hope to help my students remember how to convert from one metric unit to another which will be taught later in the year.

Formative Assessments

Before during and after each lesson, I will quickly evaluate my students' knowledge and progress using quizzes to determine their level of comprehension of math concepts, learning needs and academic progress throughout the unit. These will mostly consist of classroom observations while students complete the Do Nows, complete assignments during small groups, and exit slips. However, students will be given opportunities to self-assess by setting goals at the beginning of each unit, and by reflecting on what they know for each lesson throughout unit. Formative assessment should inform the teacher and student and be constant.

Exit Slips

I will use exit slips at the end of each lesson to determine each student's level of proficiency. Exit slips are one of the easiest ways to gather information about my students' current levels of understanding. My district prefers us to use exit slips as a way to create a running record of which standards students are understanding or need more support with.

Clock Buddies

To engage students, this fun activity will allow me to create readymade sets of partners for cooperative learning lessons.

Task Cards

I use task cards to keep students engaged a few times a year. They are a set of cards that have tasks, activities, or questions written on them which I will use to reinforce concepts in this unit. They are alternatives to worksheets.

Project Based Learning Activity

Throughout the unit, but mostly towards the end, this project will engage students in solving real-world problems or answering complex questions. This will be another way to gauge what students have learned in the unit in addition to their post assessment.

Post-Assessment

At then of the unit, I will give students a unit test to measure student achievement and the effectiveness of the unit.

Portfolios

Each student will have a personal folder that I will be stored in a crate in the classroom. The folder will show student's work and growth over time. Students pre and post assessments, problem solving tasks, formative assessment data will be collected and placed inside.

Classroom Activities

This unit will be composed of eight lessons. They are:

Lesson 1: What do I know about decimal numbers? (1 day)

Lesson 2: Little Numbers: Comparing Adjacent Digits (1-2 days)

Lesson 3: Little Numbers: Comparing Adjacent Digits within in Word Problems (Day 2)

Lesson 4: The Five Stages of Place Value (Making a Reference to Money) (1 day)

Lesson 5: Reinforcing the Five Stages of Place Value (2 days)

Lesson 6: Using Meter Sticks to Measure the Distance Paper Airplanes Fly

Lesson 7: Comparing Decimal Numbers Using Number Lines (1 day)

Lesson 8: Who wins the race? Cumulative Review/Project (2-3 days)

I have included sample plans that I will use for each lesson below including the objective, materials, procedures, and closure.

Lesson 1: What do I know about decimal numbers?

Objective: Students will: share what they know about place value and decimal numbers using The Graffiti Wall Strategy; participate in a class discussion; complete a pre-assessment.

Materials: Markers. Same group, same colored marker so I can ensure each student has participated from each group. 5 large Post-It posters hanging in the room with the following 5 statements or questions:

- Write any number less than one. Any number less than one is acceptable, but it is important to focus on decimal fractions, and students may need guidance.
- Show as many ways as possible to represent 214 (my homeroom number). Accept non-standard as well as standard ways. Allow students to share their knowledge and creativity, but the primary focus is standard ways of representation.
- Where do you see numbers in your community? List as many places as possible.
- What do you know about decimal numbers?
- The year is 2018. Which is the easiest way to write 2018? I will write 2018 on one of the posters using the Hindu-Arabic Numerals, Hieroglyphics, Hieratic Numerals, and Roman Numerals.

Procedures:

1. Students will be broken into groups of 4-5 students and given the appropriate marker and sent to a poster.
2. Students will have about 3-5 minutes to discuss and respond to the prompt/question.
3. After time expires, students will rotate to the next poster. This time, they can respond to the prompt/question OR respond to a comment left by the previous group.
4. Students will continue rotating until all the groups have visited the posters.
5. Students will return to their seats and the posters will be used to guide a class discussion.
6. After the discussion, students will complete a pre-assessment which will allow me to see where my students are, what they need, and where I should begin. The data I glean from this pre-assessment will allow me to differentiate learning in small groups and my scholars and I to set worthwhile goals. The pre-assessment data will be added to their portfolios.

Closure: Exit ticket. I will use one of the templates in the resources below.

Lesson 2: Little Numbers: Comparing Adjacent Digits (1-2 days)

Objectives: Students will: listen to me read the book, “Little Numbers,” and answer comprehension questions related to vocabulary words and events taking place in the story regarding what is happening to the main

object: the dinosaur; understand that a digit in one place represents 1/10 of what it represents in the place to its left: be able to explain that as a digit moves further away from the decimal to the right, the value of the digit is being divided by 10 OR “shrinking” by a factor of 10 using base-ten blocks (Figure 5), a number line (Figure 2) and a place value graphic (Figure 6).

Materials: The book, “Little Numbers.” A bundle of 100 straws, already grouped into smaller bundles of 10. (This will be from an activity we completed earlier in the year with whole numbers when showing how each place gets exponentially larger than one by a factor of 10 for each place to the left of the decimal point. We will have also read “Big Numbers,” also by Packard). Base-ten blocks.

Procedures:

1. Activate prior knowledge of whole numbers: Students will play a game of “Would You Rather…”

   a. Would you rather have $ 0200 or $ 2000?
   b. A discussion would follow on why and what about the numbers makes them think one number is larger than another.
   c. How much greater is that number than the other?
   d. Teacher will show both numbers using base ten blocks. How many flats make a cube? (10) How many flats make two cubes? (20) (This figure shows how this unit will use base-ten blocks to represent decimal numbers.) Base ten blocks are another way student can manipulate the value and size of numbers. Most teachers have plastic sets of base-ten blocks that can be used to represent whole numbers, but they can also be used to represent decimal numbers. I did not always know this.

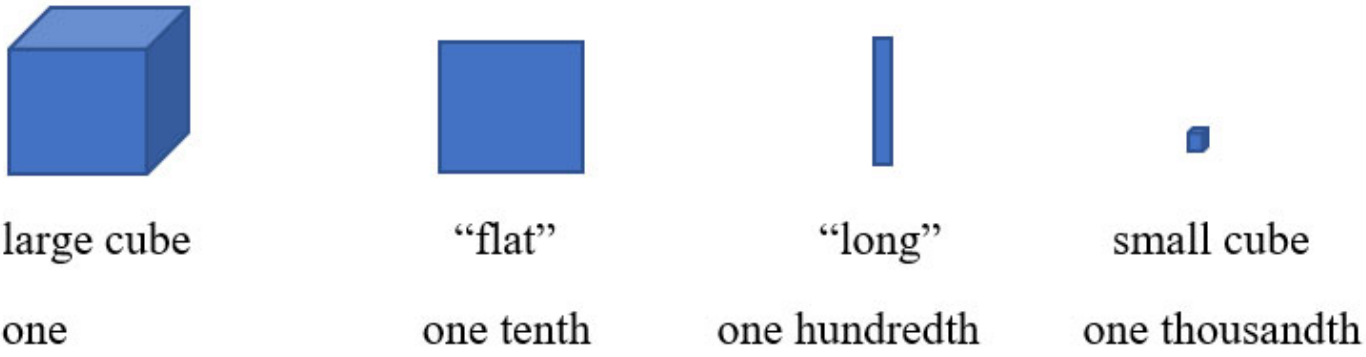

Figure 5.

2. A portion of following figure will be drawn on the board. (The ones to the millions place and no division.) This will be to remind the students of the multiplicative patterns of the place value system.

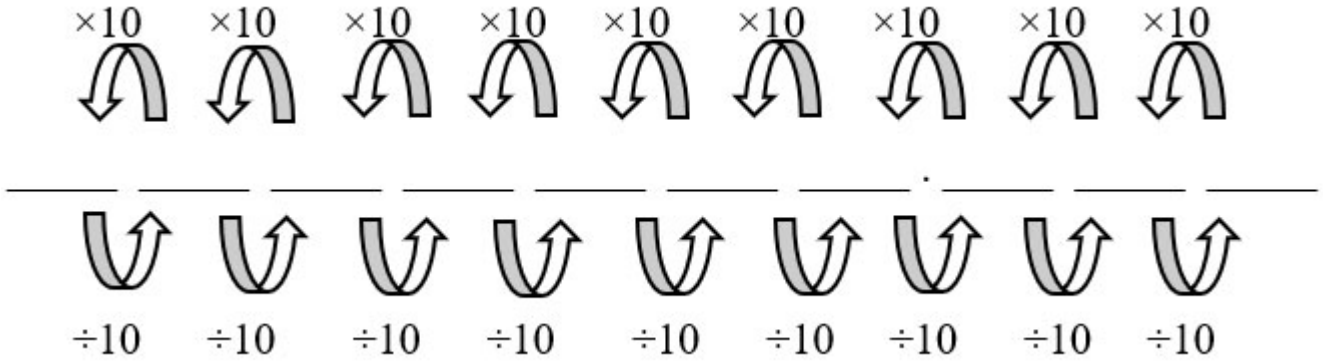

Figure 6.

3. Introduce new concept. Read “Little Numbers.”

4. We know there are decimal places that are not whole numbers. If the relationship is to multiply by a factor of 10 to get to the next place value to the left, what relationship, or what happens mathematically as you move to the to the right in a number? (Students will only see arrows, but no division symbols at this point.)

5. Show one strategy for determining the relationship between places moving to the right of the decimal point by looking at a whole number and modeling it with the straws. I will write all numbers on the board using the model, so the students can see that the numbers are shifting their place and value.

6. Separate the 100 straws back into groups of 10. What is the value of the straws I am holding now? (10)

7. Separate the straws that are in groups of 10 into 1 straw. What is the value of the straws I am holding now? (1) What is happening to the value? (The value is shrinking.) By how much each time? (10)

8. How I show the next place value using this one straw? (Cut it.) Into how many pieces? (10.) Why ten? (Because each place is getting smaller by 10 as you move to the right of the place value model). I will physically cut the straws.

9. What mathematical operation is occurring for this “shrinking” to happen?

10. Show another strategy modeling with base-ten blocks for determining the relationship between places moving to the right of the decimal point by looking at a whole number.

11. We will use 333 to model this using the place value graphic and base-ten blocks. See Figure 3.

a. What is the value of the digit in the hundreds, tens, and one’s place? Write the values.
b. What happens, mathematically, to the value of the digit as it moves to the right from the hundreds place to the tens place? (The tens place value is exactly 10 times smaller than the hundreds place value for the same digit. Students should also explain that 300 divided by 10 equals 30, and I will write this on the board.

12. The same line of questioning will occur with the next example, 7.77. The value of the base-ten blocks will change, and this will be emphasized with an anchor chart I will create using Figure 5. We will keep in mind that the goal is for students to understand and explain that the value of the place to the right is 1/10 or 10 times less than the place to it’s left.

13. Another example may be needed. This will be based on time, student engagement, and student understanding.

14. Following the introduction of the new concept, I will allow students to work with the place value concept in small groups or with a partner by completing task cards. While the students are responding to the prompts and questions of the task cards, I will conduct formative assessment and facilitate learning by providing feedback and asking clarifying and probing questions.

Closure: Exit slip: Students will fill in the following blanks: When digits shift to the right of the decimal point, it is like they are _____ ("shrinking.") When the dinosaur grew smaller from 1.0 to 0.1. How many times smaller did it become? (10 times smaller).

Lesson 3: Little Numbers: Comparing Adjacent Digits using Word Problems – Day 2

Objectives: Students will: be able to understand that a digit in one place represents 1/10 of what it represents in the place to its left; be able to explain that as a digit moves further away from the decimal to the right, the value of the digit is being divided by 10 OR "shrinking" by a factor of 10 using base-ten blocks (Figure 5) and a place value graphic (Figure 6).

Materials: Base-ten blocks, place value graphic, problem solving activity.

Procedures:

1. Using examples like the task cards from yesterday, students will review decimal place value patterns.
2. Students will be broken into 3 groups for differentiated instruction. For 15-20 minutes at a time.
    a. Online individualized learning.
    b. Vocabulary terms will be added to journals.
    c. Problem solving activity with teacher.
        i. Problem Solving Activity: Heaven and Jayden were arguing about the size of two numbers. Heaven thought seven-tenths was ten times larger than seven-hundredths. Logan thought seven-hundredths was ten times larger than seven-tenths. Who is correct? Show and explain how you know. Make sure to refer to place value in your explanation.
3. Students will return to original seats. Students will share out one thing they learned, and one thing they still want to know more about.

Closure: Exit slip will be vocabulary based.

Lesson 4: The Five Stages of Place Value (Making a Reference to Money)

Objectives: Students will: be able to use base-ten blocks to model, read, and write decimal numbers in word form (base-ten numerals), standard form, and the two expanded forms (decimal and fraction). For this unit, the 5th stage of place value will not be taught.

Materials: Base-ten blocks, place value chart, anchor chart illustrating coin images and values places on a place value chart, as well as a five stages of place value anchor chart both created by teacher.

Procedures:

1. I will refer to a local gas stations' cost of gas per gallon. This is because gas prices round to the nearest

thousandth. A gallon of gas at Sunoco costs $3.249. Model this number using base-ten blocks. Then write this number in word form, standard form, and the two expanded forms.
2. Students will work with a partner to model $3.249 using base-ten blocks.
3. Students will use paper base-ten blocks to cut and paste this model in their journals.
4. Students will use their place value charts to write the digits in each place, and then write the word form. This will also be placed in their journals.
5. Students will use the base-ten blocks to write the expanded form and the multiplicative make-up of the base ten units as a power of 10.
6. Students will model and record several more decimal numbers with a partner. (I intend to use numbers from local grocery stores and corner stores. I will have students bring in decimal numbers they see in their neighborhoods, and/or I will take pictures or use items from a circular to add to a PowerPoint to visually represent the numbers in their world.)

Closure: Exit slip: Students will write the word form, standard form, and two expanded forms of a decimal number.

3.249

= three and two hundred forty-nine thousandths

= 3.0 + 0.2 + 0.04 + 0.009

(3 x 1) + (2 x 0.1) + (4 x 0.01) + (9 x 0.001)

= (3 x 1) + (2 x 1/10) + (4 x 1/10) + (9 x 1/1000)

Lesson 5: Reinforcing the Five Stages of Place Value (2 days)

Objectives: Students will: be introduced to the metric system watching a video; learn an acronym to remember the order of the units in the metric system from millimeters (thousandths) up to kilometers (thousands); be able to use meter sticks to measure the length of the objects in the classroom; connect Metric Conversion chart with the place value chart (specifically rows four and five); record the name of the object, and the length of it using word form, standard form, and two forms of expanded form (decimal and fraction).

Materials: Metric conversion chart, meter sticks, The Story of King Henry video, Venn Diagram, and journals for note taking (the video is told in liters, but the lesson will meters, allow students the opportunity to take notes and remind them to focus on meters for this lesson).

Procedures:

1. Review: How many tenths make one whole? (10). How many hundredths make a tenth? (10) How many hundredths make a one whole? (100) How many thousandths make a hundredth? (10) How many thousandths make a tenth? (100 = 10 x 10) How many thousandths make one whole? (1,000 = 10 x 10 x 10). If we move a digit to the left of the decimal point, one place at a time, is the value of the digit increasing or decreasing? (Increasing) How much larger is the value of the digit? (10 x larger) What operation? (Multiplication) What is happening to the value of a digit as it moves to the right of the decimal point? (Decreasing) How many times smaller is the value of the digit? (1/10 x smaller). What operation can we use to show that the value of the digit is decreasing? (Division if we divide by 10 or multiplication if we multiply by 1/10).

2. Today we will use meter sticks that show this concept. Let's look at a meter stick.
3. Each group will have a meter stick.
4. This meter stick is one unit long. The unit is a meter. Like our bundle of straws this meter stick has 100 sections called decimeters. Can you see them?
5. Do you notice any other divisions? Do you notice any smaller groups? Look at your meter sticks and discuss other groups that you notice.
6. Students should notice that the decimeters are broken into smaller groups of 10 and these are the centimeters. They should also notice that the centimeters are broken into groups of 10, and these are called millimeters.
7. There are units that are smaller and larger than the units we see, and some measure large things and some measure small things. My question is, how can you remember all these units and what they measure? (Allow for brief discussion.)
8. Share Metric Conversion table. Allow students to compare and contrast it with the place value chart using a Venn Diagram.
9. Watch "The Story of King Henry" video.
10. Students will take notes while watching the video. A copy of the Metric Conversion Table would be good to paste into their journals.
11. After the video, allow students to go around the room measuring and recording different items using the five stages of place value including word form, standard form, and the two expanded forms. (This is NOT a lesson on measurement conversion, so please be careful). The goal is to reinforce the five stages of place value.)

Closure: Exit slip: Write down an example of one item you measured, and include the name of the item, the word form, standard form, and two versions of expanded form. (I will not allow students to use measurements that are whole numbers.)

Lesson 6: Using Meter Sticks to Measure the Distance Paper Airplanes Fly

Objectives: Students will: create paper airplanes; fly paper airplanes; record the distance their paper planes fly in meters, using standard form, word form, and the two versions of expanded form.

Materials: Metric Conversion Chart, Meter Sticks, (a good idea would be to use tape to mark the meters in the hall, gym, or where ever the planes will be flown and allow the students to use the meter sticks to measure the distances their planes fly. My building has an extremely long and straight hall that allows for straight flights, so I anticipate not having enough meter sticks to measure the length of the hall. Chalk is an option if this activity is done outside. Be mindful of wind and rain); paper to fold airplanes.

Procedures:

1. Review the units of the metric system.
2. Teacher will make an airplane and fly it. This will immediately attract students' attention.
3. Review some ground rules regarding the planes. Planes can only take flight on the runway which is in the hall. Planes may only fly one at a time. All planes are out of fuel once they taxi (land) on the runway and must be placed in the landing zone (a box or bag of some sort).
4. Build Airplanes. Write names on them. Place them in a safe location.
5. It is my goal to have support staff during this lesson. This lesson is intended to be taught in small groups.
    a. Group 1 – Individualized learning on computer or Ipad.

b. Group 2 – Problem solving activity with support teacher – Problem: Part 1: Tay's teacher asked him to write 7.835 in expanded form. Tay wrote: 7 + 0.8 + 0.30 + 0.500. What is Mike's misconception? Use base-ten blocks or a place value chart if for support. Part 2: What is another way to write this number?
c. Group 3 – Flying airplanes, measuring, and recording distances in hallway with teacher. (All students will remain in the hall until it is time to rotate groups.)

6. All students will record the distances of their flights on a large piece of chart paper from top to bottom, in no particular order. (These numbers will be used in the next lessons on ordering and comparing decimal numbers).
7. Students will return to their seats and record the distances their planes flew in standard form, word form, and the two expanded forms if possible (some students' planes may be exact whole numbers.)

Closure: The students' recording sheet will count towards their exit slip for the day.

Lesson 7: Comparing Decimal Numbers Using Number Lines

Objectives: Students will: be able to use number lines to compare decimal numbers; be able to use the comparison symbols, <, >, and = to write expressions.

Materials: Airplane data, open number lines, some closed number lines, dry erase markers and erasers (if you have laminated number lines).

Procedures:

1. Students will discuss what it means to compare two items. Students will compare two whole numbers with a partner. Discussion. How did you determine which number was larger? Which place has the largest value? How much larger is each place as a digit shifts to the left?
2. Students will compare two decimal numbers with a partner. Discussion. How did you determine which number was larger? Which place has the largest value? How much larger is each place as a digit shifts to the left? To the right?
3. We can also use number lines to compare numbers and show which number has the largest value. Using the information from number lines above, will guide this lesson, and can be used as an example.
4. Teacher will choose one or two more sets of numbers from the list and students will work to compare the numbers using number lines. Teacher will take notes as students work and ask clarifying and probing questions as necessary.
5. Students will break into differentiated groups:
    a. Individualized Lessons on computer or Ipads.
    b. Comparing decimal numbers and fractional numbers. Worksheet or task cards.
    c. Problem Solving: Dionne and Don were talking about the numbers 1.253 and 2.351. Part 1: With base ten blocks or number lines, draw a picture of both numbers. Part 2: What is the value of the 2 in both numbers? How does the value of the 2 in the first number compare to the 2 in the second number? Part 3: What is the value of the 5 in both numbers? How does the value of the 5 in first number compare to the value of the 5 in the second number?
    d. Teacher will pull small groups of students to review certain concepts or facilitate the differentiated groups.
6. Students will share one thing they learned or liked in today's small groups with a neighbor. Three to five students will be asked to share with the class.

Closure: Exit slip: Plot 2 numbers on the same or separate number lines to determine which number is larger. Write a sentence explaining number is larger.

Lesson 8: Who wins the race? Cumulative Review/Project (2-3 days)

Objectives: Part 1: Students will: be able to race in a 100-meter dash in small groups; be able to track and calculate their classmates' times to the nearest thousandth (if possible); record all classmates' names and times accurately on a given table; be able to order decimal numbers in order from least to greatest and greatest to least; be able to justify who won the race.

Materials: Stop watches, base-ten blocks, number lines, hundredths grids, place value charts, journals, table, glue sticks, and scissors.

Procedures:

1. Review comparing decimal numbers by using airplane data.
2. Introduce ordering numbers by having students order numbers from least to greatest and greatest to least from within the building such as ages, grades, homeroom numbers, etc. Record and label the orders.
3. Discuss what tools can be used to order numbers if they get stuck. Model the examples.
4. Use and order four numbers from the airplane data using the tools students suggested. Discuss the different ways the decimal numbers were ordered and why they were ordered that way.
5. Explain to students that they will practice ordering decimal numbers in small groups today, and they will get their data from relay races. (Wait for the shouting or pouting to end.)
6. Students will gather journals to cut and paste tables.
7. Students will count off in the number of groups I want there to be based on the number of students I want to be in each group. I would like four students in each group if possible.
8. Students will travel to the gymnasium or outdoor location for the relay races with journals and writing utensils.
9. I will explain directions to students in the classroom as well as review them in the actual location. Each student gets one time to run. All students must record the data accurately. We will cheer our classmates on. We will be good sports. We will sit when it is not our turn to race. We will give as close to accurate race times. We will race fairly. Students may generate some guidelines as well.
10. Students will race against each other in their groups. Classmates will cheer them on, record their times, and calculate their times. It is my goal to have multiple students' record the times, so we can compare the differences in the numbers they get. However, we will agree on the numbers we will record so that our data is consistent.
11. Once all students have completed the race, we will return to our classroom.
12. Students will work in their racing groups to order their times in order from least to greatest and greatest to least. They will record the times to the nearest thousandth of a second, if possible, in their journals. (This information will be necessary for part 2.)
13. Students will write who came in first place and provide reasoning.
14. Students will share with the class in small groups.
15. Discussion on what students noticed about the numbers. Why is the largest number the winner in the airplane activity, but the smallest number is the winner in the racing activity?
    1. Optional: Order the entire class list from who finished the race first to who finished last.

Closure: Exit slip - Students will explain why the smallest number in their group is determined the winner of

the race.

Objectives: Part 2: Students will choose a classmate and use their number to compare similar adjacent digits by stating how many times larger or smaller the same digit is in a different place. Students will show how to compare the value of this same number using a comparison symbol to show which number is greater. Students will also need to state of these two numbers, which friend won the race. Students will use the Clock Buddy Strategy to determine partners.

Materials: Journals, Clock Buddy Template, I will create a template to help students organize information in journals, glue sticks, scissors, place value charts, number lines, base-ten blocks, hundredths grids, and colored pencils or crayons.

Procedures:

1. Distribute Clock Buddy Templates. One for each student. Each student will need to write their name and date at the top of the paper.
2. Students will stand and find a clock buddy for 12 o'clock and then return to their seats. This is to ensure each person has a buddy. The same process will occur until each all 4 time slots are filled.
3. Once all slots are filled, put clocks to the side. They will be used later in the lesson.
4. Students will review what it means to recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left. Sample 1: Explain the relationship between the two 5's in the number 355.921. (Students should be able to discuss and explain the 5 in the ones place is 1/10 the size of the 5 in the tens place OR the 5 in the tens place is ten times the size of the 5 in the ones place. It would take ten groups of 5 to equal the among that is in the tens place OR the 5 in the tens place has a value of 50 and for it to be equal to 5, it would need to divide into 10 equal groups.) Sample 2: I will use numbers from the races.
5. Ask students to refer to their Clock Buddies, choose a time, and students will proceed to begin comparing similar digits in their numbers. Students will also write an inequality to compare their race times, and who won the race based on their time. Each partnership will last about 10-12 minutes.
6. This will continue for four rounds. Students should have four examples of each intended target of this lesson.
7. 3-5 students will share out their comparisons.

Closing: Each student will complete an exit slip independently. I will choose two numbers for students to show their understanding of the intended learning target.

Objectives: Part 3: Project Based Learning Activity- Students will: use their personal racing data to record their individual time in word form, standard form, and the two expanded forms (as decimals and fractions); compare similar adjacent digits by stating how many times larger or smaller the same digit is in a different place in a different number; show how to compare the value of the same numbers using a comparison symbol to show which number is greater; determine and record which friend won the race and how they know.

Materials: Stop watches, paper, gymnasium or outdoor courtyard, poster paper, base-ten blocks, hundredths grids, place value charts, number lines, markers, glue, or chalk.

Procedures:

1. Students will record their individual time in word form, standard form, and the three expanded forms (as decimals and fractions) independently. I will circulate around the room to clarify or probe students to think, I will also record who needed support and with what.
2. Once all students are complete. I will introduce the Decimal Place Value Poster Project.
    a. Title
    b. Group Member Names
    c. Individual Group Member Data using the four stages of place value. (May be directly written on the poster or written in a provided table and glued to the poster).
    d. Order group members numbers form greatest to least.
    e. Order group members numbers form least to greatest.
    f. Write 2-4 inequalities comparing the racing times.
    g. Write a sentence explaining who won the race and how the group knows.
    h. Poster is neat and organized.
    i. Class presentation.
3. A rubric will be used to evaluate the students.

Closure: Students will present their project to the class. (I anticipate this taking a few class periods).

Resources

Reading Lists for Students

Packard, Edward. *Big Numbers: And Pictures that Show Just How Big They Are!* Brookfield, Connecticut, Millbrook Press. 2000.

Perkins, Useni Eugene. *Hey Black Child*. New York, Boston. Little Brown and Company. 2017.

Schmandt-Besserat, Denise. *The History of Counting*. Morrow Junior Books, New York. 1999.

Materials for Classroom Use

Pre-Assessment option - https://teachingtoinspire.com/2015/09/teaching-decimals.html

Decimal Place Value Quiz or Review - https://www.teacherspayteachers.com/Product/FREE-Decimals-Place-Value-Quiz-or-Review-2041350

Decimal Place Value Pre-Assessment - https://printshop.katyisd.org/DSF/PreviewPdf.ashx?FileId=u1L4E7dlN3M-&SITEGUID=d8f13d95-6ad8-4f21-886c-0faa5ba553f9&SITEGUID=d8f13d95-6ad8-4f21-886c-0faa5ba553f9&WMK=0&WMKFID=1488357&ULR=True

Formative and Instructional Problem-Solving Tasks - http://3-5cctask.ncdpi.wikispaces.net/Fifth%20Grade%20Tasks

Problem Solving Assessment Tasks - https://hcpss.instructure.com/courses/108/pages/grade-5-year-at-a-glance#fragment-2

Illustrative Mathematics - https://www.illustrativemathematics.org/content-standards/5

Inside Mathematics -

http://www.insidemathematics.org/common-core-resources/mathematical-content-standards/standards-by-grade/5th-grade

Graffiti Walls - https://www.facinghistory.org/resource-library/teaching-strategies/graffiti-boards

Clock Buddies - http://www.teamstraus.com/SchoolDaysBorder_files/Teacher%20Farm/clockbuddies_Lower_El.pdf

Task Cards - https://www.teacherspayteachers.com/Product/Read-Write-and-Compare-Decimals-Task-Cards-Freebie-for-5th-Grade-2042564

Exit Slip Templates - https://www.nbss.ie/sites/default/files/publications/exit-entry_slip_-_comprehension_strategy_handout_copy_2_0.pdf

Appendix

Standards

PA Core Standard Integration

The unit will incorporate standards from the Pennsylvania Core State Standards for Mathematics in Numbers and Operations of Base 10. The focus will be primarily on students being able to demonstrate an understanding of place-value of decimals and compare quantities or magnitudes of numbers in my students real-life.

- Understand that in a multi-digit number, a digit in one place represents 10 times of what it represents in the place to its right and 1/10 of what it represents in the place to its left (M05.A-T.1.1.1)
- Read and write decimals to the thousandths using base-ten numerals, word form, and expanded form (M05.A-T.1.1.3)
- Compare two decimals to thousandths based on meanings of the digits in each place using the comparison symbols >, =, < to record the results of the comparisons (M05.A-T.1.1.4)

PA Common Core Standards: Standards for Mathematical Practice Standards

The unit will incorporate standards from the Pennsylvania Core State Standards for Mathematical Practice Standards. These practices support important processes and proficiencies in mathematics education as outlined by the National Council of Teachers of Mathematics (NCTM) and the National Research Council's report Adding it Up. This PA Document has been adapted from the Common Core State Standards for Mathematics.

- Make sense of problems and persevere in solving them.
- Reason abstractly and quantitatively.
- Construct viable arguments and critique the reasoning of others.
- Model with mathematics.
- Use appropriate tools strategically.
- Attend to precision.
- Look for and make use of structure.
- Look for and express regularity in repeated reasoning.

Notes

1. Hammond, Zaretta P “3 Tips to Make Any Lesson More Culturally Responsive,” accessed August 10, 2018, https://www.cultofpedagogy.com/culturally-responsive-teaching-strategies/.
2. Boaler, Jo, and Carol S. Dweck, *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching* (San Francisco, CA: Jossey-Bass; a Wiley Brand, 2016), 27.
3. Delpit, Lisa D, "*Multiplication is for white people": raising expectations for other people's children*, (New York, NY: The New Press, 2012), 21.
4. Boaler and Dweck, *Mathematical Mindsets*, 26.
5. Department of Numbers, Pittsburgh Pennsylvania Household Income, accessed August 13, 2018, https://www.deptofnumbers.com/income/pennsylvania/pittsburgh/.
6. Sowder, Judith, *Place Value as the Key to Teaching Decimal Operations*, accessed August 15, 2018, https://web.stevens.edu/golem/llevine/CIESE/place_value-decimal%20operations.pdf.
7. Number Systems, Egyptian Base Ten Number System, accessed August 14, 2018, http://www.math.wichita.edu/history/topics/num-sys.html#egypt.
8. Howe, Roger and Reiter, Harold, *The Five Stages of Place Value,* unpublished, accessed, July 15, 2018.
9. Beckmann, Sybilla, *Mathematics for Elementary Teachers*, (Adison Wesley, Hardcover) 25.
10. Pankhurst, Richard, *An Introduction to the Economic History of Ethiopia*, (London: Lalibela House, 1961).
11. The World Factbook, Appendix G: Weights and Measures, accessed August 14, 2018, https://www.cia.gov/library/publications/the-world-factbook/appendix/appendix-g.html.
12. *Mathematics for Elementary Teachers* (36).
13. Packard, Edward, *Little Numbers: And Pictures that Show Just How Little They Are!*, (Brookfield, Connecticut: Millbrook Press, 2001).
14. Eames Office, “Powers of Ten.” YouTube Video, 9:01. August 26, 2010. https://www.youtube.com/watch?v=0fKBhvDjuy0.
15. Obreschkow, Danail, “Cosmic Eye (Original HD Portrait Version 2011).” 3:09. January 11, 2012. https://www.youtube.com/watch?v=jfSNxVqprvM.
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Academic Standards for Mathematics. Grades Pre K – High School. Pennsylvania Department of Education, last modified March 1, 2014. file:///C:/Users/Kryst/Downloads/PA%20Core%20Standards%20Mathematics%20PreK-12%20March%202014%20(1).pdf. p. 5. (accessed August 2, 2018).

The Assessment Anchors and Eligible Content Aligned to the Mathematics Pennsylvania Core Standards. Pennsylvania Department of Education, last modified April 2014. http://static.pdesas.org/content/documents/Grade%205%20Mathematics%20Assessment%20Anchors.pdf. p. 3. (accessed August 2, 2018),

Bamberger, Honi, J., Obedorf, Christine, and Schultz-Ferrel, Karren. *Math Misconceptions: From Misunderstanding to Deep Understanding*. Heinemann. 2010.

Beckmann, Sybilla. *Mathematics for Elementary Teachers*. Pearson. 2005.

Boaler, Jo, 1964- and Carol S. Dweck. *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching*. San Francisco, CA: Jossey-Bass; a Wiley Brand. 2016.

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Guyer, Jane I. and Pallaver, Karin. 2018. Money and Currency in African History. http://africanhistory.oxfordre.com/abstract/10.1093/acrefore/9780190277734.001.0001/acrefore-9780190277734-e-144 (accessed July 14, 2018).

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Howe, Roger and Epp, Susanna S. *Taking Place Value Seriously: Arithmetic, Estimation, and Algebra Unpublished*. (accessed July 15, 2018).

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Torres-Velasquez, Diane and Lobo, Gilberto. 2004. *Culturally Responsive Mathematics Teaching and English Language Learners*. http://www.k12.wa.us/BEST/Symposium/2b.pdf. (accessed June 1, 2018).

Ukpokodu, Omiunota. 2011. *How Do I Teach Mathematics in a Culturally Responsive Way: Identifying Empowering Teaching Practices*. https://pdfs.semanticscholar.org/84ee/b1f09761f8d41af7fc32e95723bdfaa33915.pdf. (accessed June 3, 2018).

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