Introduction

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The “Big Numbers, Small Numbers” seminar primarily focused on estimation, and calculation with numbers, especially large numbers, that are not known exactly. It is essential to be able to deal with numbers that are known only approximately, because essentially all numbers that come from the real world, from population to production information to income and cost, are in fact not known exactly.

Approximation therefore was a central theme. Accuracy of approximation was captured by the notion of *percentage error*, of an approximate value to a “true” value. In order to relate this to standard base ten notation, the concepts involved in this notation were reviewed, and expressed in five levels of interpretation, or stages, of place value, exemplified by the expressions

352 = 300 + 50 + 2

= 3x100 + 5x10 + 2x1

= 3x(10´10) + 5 x10 + 2x1

= $3x10^2 + 5 \times 10^1 + 2x10^0$.

The first stage is the standard notation. The second stage expresses the number of a sum of parts, namely 300, 50 and 2, labeled by the digits. These were called the “place value parts” of the number. The third and fourth stages dig into the multiplicative structure of the place value parts, and the fifth stage uses exponential notation to exhibit the number as a “polynomial in 10”.

With this structure in hand, we were able to be precise about the effects of rounding a number. In summary:

> The largest place value part of a number always contains at least half the value of the number, and at least 90% about half the time.
>
> The largest two place value parts of a number always contain at least 90% of the number and at least 99% of the number about half the time.
>
> The largest three place value parts of a number always contain at least 99% of the number, and 99.9% about half the time.

From considering typical measurement errors, and especially, looking at placement of numbers on a number line, we concluded that many, probably most, real-world numbers could not meaningfully be reported to more than three figures.

We also discussed the relationship of these ideas to scientific notation. To summarize: scientific notation, such as $A = 1.98 \times 10^8$, focuses attention on the most significant aspects of A from a measurement point of view, namely

a. how large A is (encoded in the exponent of 10, e.g., 8 in our example); and
b. how accurately A is known (encoded in the number of figures reported, here 3).

The Fellows incorporated these ideas into a wide variety of units. A substantial number found the idea of place value parts and the five stages useful, and included some of these ideas. Krystal Smith focused on increasing her students comfort with place value. Lajuanda Bland emphasizes the role of place value in understanding and computing multiplication. Marnita Chischilly uses the stages to explore subtraction, and plans a project that will involve estimation. Tierra Ingram uses base ten notation to analyze the behavior of exponents, with the goal of having her students understand decimal fractions. Charlotte Perry uses the ideas of order of magnitude to help her students understand scientific notation, with a project requiring each of them to create a superhero and compare their powers to ordinary human powers. Lynnette Shouse will have her students perform estimations, using the books *Great Estimations* and *Greater Estimations*, and will use number lines to grasp the relative sizes of place value parts. Aaron Bingea will use number lines heavily to help his students grasp the relative size of large numbers and help them appreciate the significance of order of magnitude. Zachary Meyers will give his students a sense of scale and the meaning of scientific notation, using the metric system and the electromagnetic spectrum. Finally, Lawrence Yee will present his students with a collection of problems that insert combinatorial ideas into everyday situations, to help them understand and appreciate the very large numbers that can arise in dealing with permutations and combinations.

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