Curriculum Units by Fellows of the National Initiative
2019 Volume V: Perimeter, Area, Volume, and All That: A Study of Measurement

A Pathway to Understanding Area and Perimeter

Curriculum Unit 19.05.01, published September 2019
by Lianne Aubert Sanfeliz

Introduction

Measurement is about the methods we use to determine the size of things. Every object has different attributes that can be measured, such as length, area, volume, or weight.[1] Length, area, and volume refer to the shape and extent of an object and might be called "geometric" attributes. Weight, on the other hand, refers to the physical constitution of the body.

The measurement of area and perimeter are widely used in everyday life and can be exemplified with a variety of practical situations such as measuring the size of a room by talking about its floor area, or how much fence to put around a playground.

Even though examples of area and perimeter are present in many "real life" situations, learning about these concepts as part of the mathematics curriculum in elementary school can be confusing or challenging for some students.

When teaching tends to focus on procedural understanding and the use of formulas rather than on the conceptual and relational knowledge, students learn to memorize formulas to "solve" problems without the proper understanding of the concepts. Some students also confuse the concepts of area and perimeter because they experience difficulty understanding the differences between linear (one-dimensional) units and squared (two-dimensional) units or are unable to connect their everyday experience with area and perimeter to what they learn in the classroom.[2]

The close relationship between area measurement with other mathematical concepts, such as multiplication, surface area, and volume, makes area a critical component of the K–12 mathematics curriculum, but without a solid understanding of what area means, students face difficulty in understanding related concepts.[3]

Conceptual understanding is crucial for understanding mathematical ideas instead of just isolated facts and formulas. Through conceptual understanding, students also acquire the bases and ability to transfer their knowledge into new situations, apply it to new contexts, make connections, and reason with supporting arguments about unfamiliar problems.[4] The goal of this unit is to develop a solid conceptual understanding of perimeter, and especially, area in my students. The activities proposed in this unit allow students to construct their understanding of area and perimeter and to appreciate how they differ, with manipulatives and classes.

Rationale

During this seminar Area, Perimeter, Volume and All That, I have had the opportunity of reflecting on my teaching practices regarding area and perimeter while thinking about what are the most frequent misconceptions that my students experience during their learning of this particular topic.

In general, students encounter difficulties to relate multiplication arrays with the area of a rectangular figure. Students often confuse the concepts of area and perimeter when transitioning from the use of manipulatives to pictorial representations. In many cases, they assume that rectangular figures with the same perimeter must have the same area and vice versa.

I also consulted the testing data available for the group of students that were in my 3rd-grade grade in 2017 and compared their performance in the area of Geometry and Measurement. At the end of 3rd grade, 7 % of the students performed below standard, and 66% of the students performed above standards. By the end of 5th grade in 2019, the same group of students scored 24 % below standard and only 27% above standard.[5] Even though these particular test results are probably not enough to establish a solid conclusion about my students' understanding of area and perimeter, there is a pattern in the way many students solve problems about area and perimeter. Students encounter difficulties to relate multiplication arrays with the area of a rectangular figure. Students with limited understanding of the two concepts may fail to realize that it is possible to have rectangles with the same area but different perimeters as well as rectangles with the same perimeter but different areas.

France Machaba, in his research about the misconceptions in the understanding of area and perimeter published in 2016,[6] referred to multiple research studies in the field of mathematics education that have reported that the concepts of area and perimeter are a continual source of confusion for learners. In his report, Machaba suggested that it is perhaps because both area and perimeter involve measurements, or because students are taught formulas for both concepts at about the same time and they get confused due to the lack of conceptual understanding. The confusion between these two concepts results in misconceptions.

I have observed the confusion and misunderstanding in my class, but I do not think area and perimeter should be taught separately. On the contrary, I think it is a powerful instructional strategy to present both concepts simultaneously, but in a way that allows students to understand the contrasting differences between the two.

During our current seminar with Dr. Howe, we learned to approach the distinction between area and perimeter in ways that allow solid comprehension and more in-depth content knowledge. As a result, I plan to develop my unit as a sequence of intentional activities to help my students develop a correct and solid understanding of the relationship between area and perimeter and use appropriate methods to solve problems involving area and perimeter.

Academic context and learners

I teach 3rd-grade Mathematics at Zarrow International School in Tulsa, Oklahoma. Zarrow is a public magnet school that provides language immersion instruction in Spanish. The school's staff includes teachers and teacher assistants from different Spanish speaking countries as well as English native speaking teachers. The current student population is about 500 students from pre-k to 5th-grade.[7] The school admits students based on an electronic, random lottery-selection by geographic quadrant for families living within the Tulsa Public Schools boundaries to give students from all areas of Tulsa equal access to the program. The process also provides for a sibling preference to encourage family engagement with the school community. However, the sibling preference policy is not a guarantee of admission to the school, for siblings do not participate in the lottery with non-sibling new students.[8] The school receives additional funds through the local Zarrow Foundation and school Parent-Teacher Association (PTA). Parent involvement is among the highest in the district.

Based on the school profile data published by our district, the student ethnic composition is as follows:

Ethnicity	Percentage %
White	43.8
Hispanic	30.0
Multiracial	12.9
African American	6.3
Native American	5.4
Asian/P. Islander	1.4

About 30 % of the students are gifted, 12 % are English Language Learners, and 7 % are students with disabilities. 56 % of the students are female, and 46% are male.[9] There are three classrooms at each grade level. Each class has a certified Spanish/English bilingual teacher and a Spanish native speaking teacher assistant. Instruction is delivered self-contained in grades pre-K to 2nd and departmentalized by content area in grades 3rd to 5th. I teach 3rd-grade Mathematics to all 3rd graders at Zarrow (72 students). I deliver instruction mainly in Spanish using Eureka Math, supplemented with a computer-based program in English (Zearn). In addition to the Math teacher, the 3rd-grade team includes a Language Arts teacher and Science/Social Studies teacher, all of whom regularly collaborate through cross-curricular units. Our primary goal is to provide opportunities for developing students' critical thinking skills regardless of abilities and learning styles. All these opportunities also promote second language acquisition more organically because the students learn and use the target language (Spanish) through experimentation and discovery.

I intend to teach this unit to enrich the academic content knowledge of my students during mathematics instruction in 3rd grade.

Learning Goals of the Unit

The focus of this unit is to deepen students' understanding of perimeter and area while reasoning about the relationships and differences between the two concepts. For that purpose, students will explore area and perimeter through manipulation and concrete representation to develop the conceptual understanding of the concepts before the use of formulas.

During the exploration, students will study figures made by combining unit squares and will learn that changing the arrangements of the unit squares in these figures, while not changing the area, may result in changing perimeters. Through a sequence of activities, students will experimentally discover area as the square units on (or inside) a rectangular figure as opposed to the length units around (perimeter). Finally, an extension activity will challenge students to create more than one arrangement for a given area to find the longest possible perimeter. During the unit, students will be able to explore what happens to the perimeter and area of a rectangular figure when the figure is changed in some way by removing unit squares. In particular, they will find that, although removing squares reduces area, the perimeter may stay the same, or even go up! Students will be able to draw rectangles and other rectilinear shapes and determine their area and perimeter. Students will be able to communicate effectively through drawings and numbers, work cooperatively in small groups, and successfully apply their comprehension about area and perimeter in a real-life situation.

Background knowledge for teachers

Bilingual vocabulary

The acquisition of the academic language in Spanish is always a vital learning goal embedded in the teaching of the specific content area in our immersion school. For that reason, I would like to start this section by providing a brief bilingual glossary of the most critical terms that students will need to know to express their ideas Spanish during the development of this unit.

In my teaching practice, when I introduce a new topic, and there are new vocabulary words that could be unfamiliar for my students (at least in Spanish), I present the new word, and explain the definition. Often, students use their background knowledge or the relationship between the English word and Spanish cognates to determine the meaning of the new word. As a group, we establish a definition for the word, a proper illustration, or example if necessary, and I create anchor charts that remain in the classroom for students to reference as needed. At the same time, students write the word, the definition, and the example or illustration in their math journals for future references.

English Terms	Spanish Terms
Area	Area
Square tile	Baldosa cuadrada
Column	Columnas
Square	Cuadrado

Grid	Cuadriculas
Equation	Ecuacion
Row	Filas
Side	Lados
Lateral	Lateral
Length	Longitudes
Array	Matriz
Model	Modelo
Graph paper/Grid paper	Papel cuadriculado
Perimeter	Perimetro
Additive Property	Propiedad aditiva
Rectangle	Rectangulo
Square Units	Unidades cuadradas
Linear Units	Unidades lineales

Important definitions: Area and Perimeter

Even though there are plenty of definitions of Area and Perimeter available, I include the definitions here to maintain the organization and sequence of this unit.

The area of a two-dimensional figure is defined as the amount of space inside the boundaries of the figure. It is a physical quantity that indicates the number of square units occupied by the two-dimensional object.

The area of a two-dimensional surface is measured by finding the total number of same-size units of area required to cover the shape without gaps or overlaps. When that shape is a rectangle with whole-number side lengths, it is easy to partition the rectangle into squares with equal area.

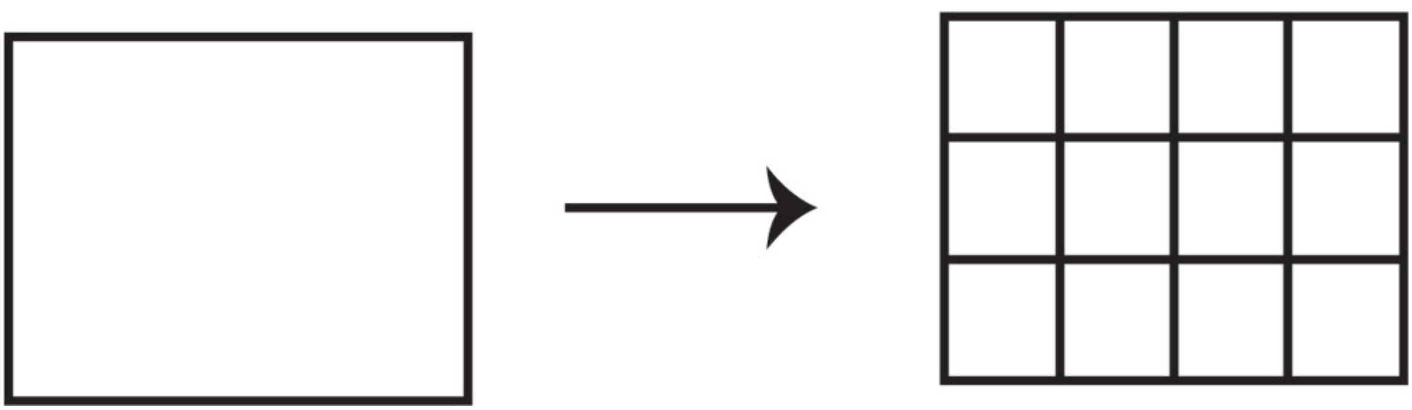

Figure #1: In the example shown, the area of the square is 12 square units because 12 unit squares are needed to cover the surface enclosed by the square.

The perimeter is defined as a measure of the length of the border that surrounds a closed geometric figure. The term ‘perimeter’ derivates from the Greek words, ‘Peri’ and ‘meter’ which mean ‘around’ and ‘measure.’ In geometry, it refers to the continuous line forming the path outside the two-dimensional shape.

As mentioned earlier, the primary purpose of this unit is the implementation of activities for students to understand the differences between area and perimeter. As a result, teachers need to keep in mind the fundamental differences between area and perimeter as listed below.

Area refers to the measurement of the surface of the object while perimeter refers to the outline that surrounds a closed figure. The area represents the space occupied by the object, while, perimeter represents the boundary of the shape.

The measurement of the area represents two dimensions and is expressed in square units such as square kilometers, square feet, or square inches. The perimeter of a shape represents one dimension and is measured in linear units such as kilometers, inches, or feet.[10]

The sequence of Eureka Math curriculum from multiplication to Area. Where is the confusion?

Hong and Runnalls in 2019 analyzed three different series of Common Core-aligned textbooks and found those popular textbooks present conceptual limitations when introducing the concepts of area and perimeter in elementary grades, and not enough opportunities for students to explore those concepts. Although textbooks are just one component of the instructional process, many school districts require a specific curriculum for instruction. As a result, teachers need to comprehend the limitations in the curriculum to fill in the gaps by implementing tasks that promote reasoning and conceptual understanding. The authors suggest the modification of existing textbooks/workbooks tasks as a way of addressing this need. By modifying tasks, teachers can work within the boundaries of curricular materials while still providing opportunities for students to develop conceptual understanding.[11]

As Eureka Math is the “district adopted curriculum” for Mathematics, my instruction will follow the sequence and scope suggested in Eureka, but I will modify the way I deliver the content by adding new knowledge and strategies to address students’ misconceptions about the difference between area and perimeter. I will incorporate this unit at the beginning of module 4 (Multiplication and Area).

In 3rd grade, students start the year by learning multiplication facts and the properties of multiplication (Eureka, modules 1 and 3). Then, they transition from making equal groups (equal amount of objects in each group) to rectangular arrays, and finally to area models. Module 2 is about the measurement of time, weight, and capacity, and students get exposed to the metric and customary systems of measurement.

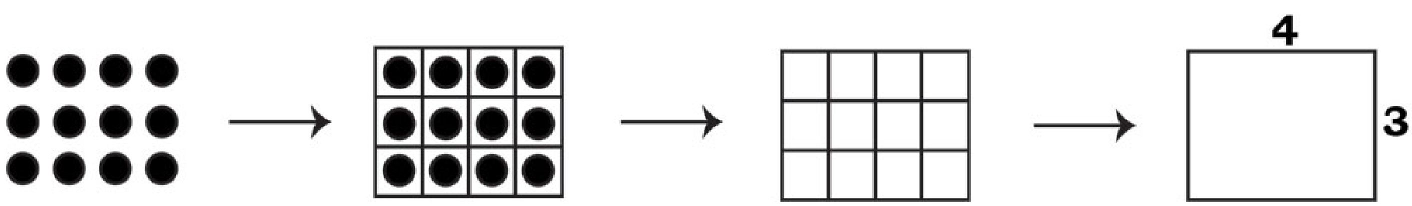

Figure #2. This sequence shows the progression from a rectangular array to area model in 3rd grade.

Equal groups of items, and later a grid representing those items in rows and columns are introduced in modules 1 and 3 to explain multiplication and its properties. In module 4, as the concept of area is presented, items inside the grids are taken away, and students will count the individual squares to determine the area of the rectangle. By the end of the module, the grid system is eliminated, and students will determine the area of the rectangle by multiplying the sides.

Modules 1 and 3 in Eureka Math are designed to set the foundations for the understanding of rectangular arrays to prepare students for area in Module 4. In modules 1 and three students transition from grouping items and adding the groups (repeated addition) they used in 2nd grade to organizing the items in rows and columns or arrays. In those two modules, they also learn the commutative property of multiplication by manipulating the arrays. For example, the array of the example in figure #2 can be presented as three rows of 4 items each for a total of 12 items, or as four rows of 3 items each with the same total.

In module 4 of Eureka, the concept of area is introduced, but the discussion is restricted to rectangular figures, and students start utilizing the grids without items inside. Although the grid with the items inside and the empty grid are both pictorial representations, the blank grid surely requires a more abstract understanding. In general, students don't face any significant difficulty to count squares and determine that the total of squares equals the area or surface inside the rectangle once the concept of area is defined. The misunderstanding occurs more often when we eliminate the grid and students need to find the area of rectangular figures given the units of one or two side lengths as in the last rectangle in figure #2. At that point in the module, students are expected to be able to make a connection between counting squares and multiplying sides to determine area, as well as with the multiplication arrays they already learned, and to be able to distinguish the concept of perimeter (superficially covered in the module) using the same side lengths they use to calculate area.

In addition to all the above, as module 4 continues, students have to compose or decompose combined rectangles to determine the total area to conclude that area is an additive property. Although Eureka Math includes workbooks for students with an extensive amount of practice problems, the majority of the problems are not designed for students to be able to understand the differences between area and perimeter, or linear and square units, as the focus is more about the repetition and practice of the same skill. I think there is a need for students to be exposed to problems that provide more opportunities for them to explore area and perimeter in the same problem to reason about the differences between the two concepts. To me, that should happen before students are asked to find the area of rectangles or combined rectangles without a grid. Not because rectangles are challenging, they are straightforward, but precisely because of the progression of the content, the way it is presented in the curriculum creates confusion and misunderstandings.

Outhred and Mitchelmore in 2000 published research consisting of observing and analyzing 150 students from grades 1 to 4 while they solved array-based tasks to determine the different strategies students applied and the success of those strategies.[12] Their rationale for this research was the consideration that students' misunderstandings and poor performance when solving area problems are due to the tendency to introduce the area formula too early during elementary school. The authors consider that when students memorize formulas without the understanding of the concepts, they have difficulties generalizing procedures. They noticed that although students were able to calculate the area of a rectangle when given both dimensions, they could not transfer that understanding when finding the area of a square when given only the length of one side. The authors also found that students often struggle when transitioning from physically covering a rectangle with unit squares, an action that suggests an additive process, to the use of the rectangle formula, which is multiplicative. Other common misunderstandings include confusing area and perimeter, applying the formula for finding the area of a rectangle to plane figures other than rectangles, and the use of linear instead of square units for area measurements. The research recommends that teachers should allow students enough time to develop an understanding of the concepts of area and perimeter without memorizing the formulas.

I agree that elementary-age students need concrete manipulation to understand mathematical concepts before the introduction of formulas. Children's success in understanding area and the perimeter is related to the resources they use during problem-solving. They need objects (tiles, bricks, folded or cut paper) they can fit, fold, match and count so that they can work to develop a conceptual understanding of area and perimeter and their differences. These are my working assumptions in creating this unit.

Sensory Paths

The final project students will complete in this unit involves the design of a sensory path because our school PTA approved the funds to build a sensory path for our students at Zarrow.

Sensory paths are "activity or movement paths" that help kids to build neural pathways or connections in the brain that are responsible for the senses. The high-energy nature of the exercises included in the activity sequence of sensory paths, help kids to complete complex, multi-stage tasks.

A typical sensory path consists of a path in which kids need to complete exercises specifically designed to develop motor skills. The activities require kids to hop, step and jump to get the blood pumping, which helps children to sit still and focus for longer periods in the classroom. Sensory paths are a great "brain break" throughout the school day or during indoor recess.[13]

Usually, those sensory paths are made with stickers that can be stuck to any surface. At the end of this unit, 3rd graders will apply their understandings of area and perimeter to choose one of the school hallways that may be the best option to build our sensory path. To complete this activity, students will need to consider the available area (enclosing rectangle) of the hallways and the arrangements that offer perhaps the largest perimeter for safe indoor exercising.

The sequence of the Unit and Implementation

Keeping in mind that my 3rd graders need to have experiences in which they can manipulate the spaces they measure to construct deep understanding, the activities and teaching strategies proposed in this unit will include the use of manipulatives and "real life" experiences.

The focus of the unit is to address the misunderstanding that rectangular figures with the same perimeter must have the same area and vice versa. It is not rare that students in elementary grades think that when the area of a figure decreases or increases, the perimeter will also decrease or increase. Students don't realize that it is possible to have many rectangles with the same area, but different perimeters. On the contrary, if the perimeter is the same in a set of rectangles, the area of those rectangles does not have to be the same. For example, rectangles with side lenghts 2 by 4 and 1 by 8 both have an area of 8 square units, but their perimeters are 12 units and 18 units respectively. Likewise, rectangles with the same perimeter can have different areas. For example, 3×4 and 2×5 rectangles with side lengths 3 by 4 and 2 by 5 both have a perimeter of 14 units, but their areas are 12 square units and 10 square units respectively.

Through the activities in this unit, students will be able to generalize about the relationship between area and perimeter. For example, given a rectangle with whole-number side lengths, the dimensions are factors of the area. When the difference between the dimensions of a rectangle with a given area is the smallest, you will have the smallest perimeter. When the difference between the dimensions of a rectangle with a given area is the largest, you will have the largest perimeter. Given a fixed perimeter, the rectangle with the largest area will be the one with the dimensions that are closest together (a square). Given a fixed perimeter, with whole-number side lengths, the rectangle with the smallest area will be the one with the dimensions farthest apart.[14] One of the side lengths will be 1. During the activities of this unit, although students will tile rectangles of various dimensions to determine area and perimeter, and will make generalizations about the relationships between area and perimeter, the focus is to experiment with non-rectangular figures to better understand the differences between the two concepts.

Beginning with a particular rectangle, students will explore different possible shapes obtained by eliminating one unit square at the time. This type of exploration will generate a variety of options for students to analyze and generalize. In all cases, students will keep their new arrangements within the area of the original rectangle, also known as “enclosing rectangle” by making sure students do not eliminate an entire row or column of the original rectangular array. The sequence of activities will culminate with a “real world” experience in which students will need to use the school hallways as “enclosing rectangles” to arrange possible options to design sensory paths.

The activities included in this unit will require the use of tiles as manipulatives as well as drawing on graph paper and recording results in charts.

The figure below represents some examples of possible arrangements that students may find when manipulating rectangles by eliminating subsequent unit squares. Not all the possible arrangements have been included in the figure.

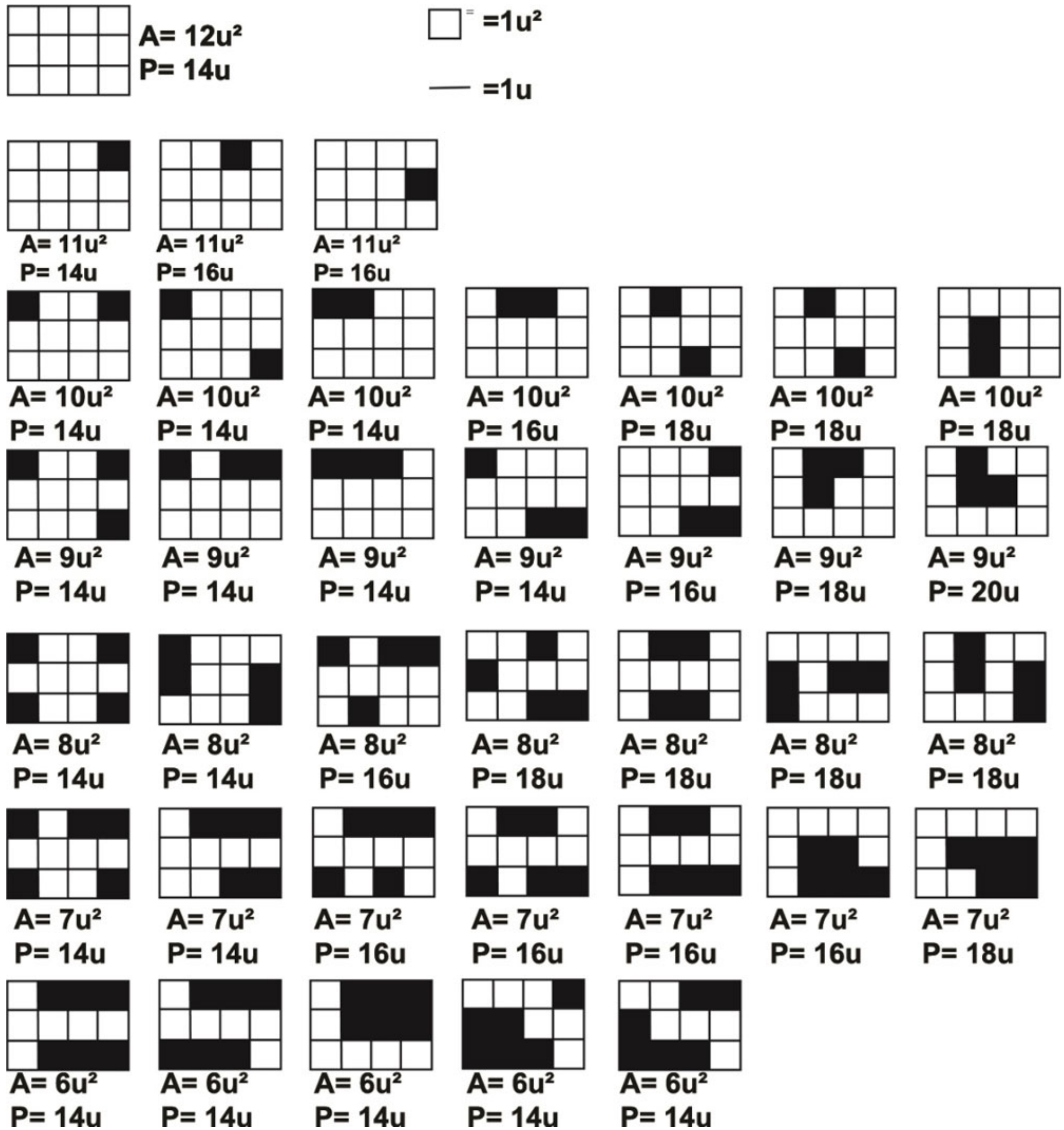

Figure #3: Example of unit square elimination to differentiate area and perimeter. Not all possible arrangements are represented here. As the number of possible arrangements will be quite large, I recommend to restrict this activity to small rectangular arrays.

The sample chart shown below could be used for students to record their findings as they eliminate one unit square at the time within the dimensions of the original rectangle. Each elimination will require a chart.

Original rectangular array dimensions (length of the sides):______ Area of original rectangle: ______Number of units taken away:_______	
Number of possible arrangements: ______ Restrictions: ● Unit squares must be connected through their sides ● The elimination of a whole row or column is not allowed	
Area	Perimeter

Figure # 4: Sample chart for recording area and perimeter of resulting arrangements during unit elimination activity.

Teaching strategies

In this unit, I would like to incorporate a variety of instructional strategies to meet the needs and the different learning styles of my students.

Although Eureka Math in Spanish will constitute the primary resource to guide the scope and sequence of the Math instruction, the activities suggested in this unit have the purpose of facilitating a better understanding of the differences between area and perimeter.

The activities will be taught in a sequence of mini-lessons with specific objectives.

The main teaching strategies I plan to implement are the following:

Hands-on Learning: Students will use concrete models or manipulatives (unit squares tiles). The use of manipulatives during mathematical instruction help students in the process of building their cognitive models to represent abstract mathematical concepts. Using manipulatives, students communicate with one another and with teachers. Through engaging activities with manipulatives, students can express their thinking while gaining confidence in their mathematical abilities.[15]

All the activities included in this unit will have a “hands-on” component. From the first activity in which students need to use tiles to determine the area of rectangles, to the final project in which they need to determine the arrangement with the best possible “walkable perimeter” to design a sensory path using sections of the school hallways as “enclosing rectangles.”

Cooperative Learning: During the completion of the activities proposed in this unit, students will have the opportunity to work with partners and in cooperative groups. They will collaborate to find solutions, sharing their methods and ideas, and promoting discussions and analysis. During this process, I will make sure to provide clear instructions and scaffolding for students to be able to successfully transition from working with a partner to small group collaboration to whole-class discussions and presentations. This strategy helps students support each other as learners because it allows productive struggle. As a result, students learn from their mistakes through explanations from their peers and the teacher. When students are in control of their learning, they have time to try out ideas, listen to one another, and gain confidence among their peers and the whole class.[16] All the activities proposed in this unit are designed for students to work in partners or small groups to promote collaboration and peer teaching.

Read-Draw-Write: This strategy is widely used in problem-solving situations in Eureka Math. The idea behind the plan is to approach mathematical problems by drawing pictorial representations of the given information to clarify students’ understanding of the problem. This strategy is appropriate not just for “word problems,” but it is also beneficial for approaching area and perimeter questions. This strategy consists of three specific steps:

1. **READ:** Students will read the problem or question. It is always a good practice to read the question at least two times and provide opportunities for students to explain their understanding of the question being asked.
2. **DRAW:** Students will draw a picture that represents the information given. During this step, students need to analyze what can they draw from the data presented in the problem and what could be the best model to use. For students to be able to accomplish this step, they need to be familiar with a variety of pictorial representations. It is expected that they have been exposed to a variety of graphic models in mathematics such as bar models, arrays, equal groups, groups of the same size, partitions, tiled surfaces, etc.
3. **WRITE:** Students will be able to draw conclusions based on the drawings in the form of a number sentence, an equation, or a statement.

With the RDW approach, students can draw a model of what they are reading, and the drawing helps lead to the understanding of the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. While using RDW, students have to put in practice several Standards for Mathematical Practice and in some cases, all of them. Some of these would include: make sense of problems and persevere in solving them, model with mathematics, use appropriate tools strategically, and look for and make use of structure.[17] Most of the activities of this unit require pictorial representations for students to express their thinking.

Project-based learning: The unit’s culminating activity is a “real-life” application project in which students will be able to integrate their comprehension from the previous activities of the unit. In this project, students will be able to integrate hands-on and cooperative learning through the completion of the project.

Project-based learning is a pedagogical method based on the idea of John Dewey of “learning -by doing.” In this educational strategy, the teacher sets the goal and acts as a facilitator providing scaffolding and guidance when necessary. Students, working in groups, create a product to present their gained knowledge in a particular topic. The final product may include a variety of media such as writings, art, drawings, three-dimensional representations, videos, photography, or technology-based presentations. The basis of PBL lies in the authenticity or real-life application and is considered an alternative to paper-based, rote memorization, teacher-led classrooms. Some of the benefits of implementing project-based learning strategies in the classroom include gaining a greater depth of understanding of concepts, broader knowledge base, improved communication and interpersonal/social skills, enhanced leadership skills, increased creativity, and improved writing skills.[18]

Classroom Activities

The following are suggested activities to help students to better understand the differences between area and perimeter in 3rd grade.

Important considerations:

Before students engage in the activities, it is important to define what a “unit” is. In this case, one square unit will be defined as one square tile with side lengths of 1 inch.

While tiling, the teacher will guide students in the process of using the side length of the tiles around the linear figures to determine the perimeter.

Later, after students master the differences between area and perimeter, it would be a good idea to explore similar activities incorporating different sizes of tiles to compare unit squares made of one in^2 and 1 $cm.^2$

Working with units of different sizes will help students to better understand that the size of the unit we use to measure something affects the measurement. In other words, if we measure the same quantity with different units, it will take more of the smaller unit and fewer of the larger unit to express the measurement.

I will introduce the topic by reviewing multiplication arrays and how are they related to the area of rectangular figures. It is important to refer to the previous units (Modules 1 and 3 in Eureka Math) for students to be able to apply their background knowledge to the study of the concepts of area and perimeter.

Activity 1:

It is a preliminary activity for students to make connections between multiplication arrays and the area of rectangular figures. Students will work in partners for this activity.

Materials:

- Set of cards with rectangular arrays that I will prepare ahead and laminate.
- Pencil or marker
- Grid paper

Directions:

One student will hold a card showing a rectangular array; the other student needs to write the multiplication fact that corresponds to the array shown on the card, draw the rectangle on the graph paper, identify the length of the sides, write the multiplication equation, and the solution of the multiplication inside the rectangle. Each partner with taking turns to continue the game. As the activity progresses, I will question students about the connection between multiplication arrays and a rectangular area.

Variations to this activity may include that one partner shows a card with a multiplication fact, and the other partner draw the rectangular array on grid paper and the solution inside the rectangle. Both partners can continue until they ran out of cards or fill the grid paper with rectangles. To make this activity as dynamic as possible, I will select multiplication facts that generate arrays of reasonable sizes for students to be able to draw them without spending too much time. It is important to keep in mind that the goal of this activity is for students to make a connection between multiplication and area, and not for students to engage in the drawing of large multiplication arrays.

Once all the rectangles are represented, students will determine the perimeter of each rectangle by counting the units around the edges. I will lead a classroom conversation with guiding questions about the relationship between the area and the perimeter of rectangular figures.

During the classroom conversation, I will emphasize the difference between a square and linear units.

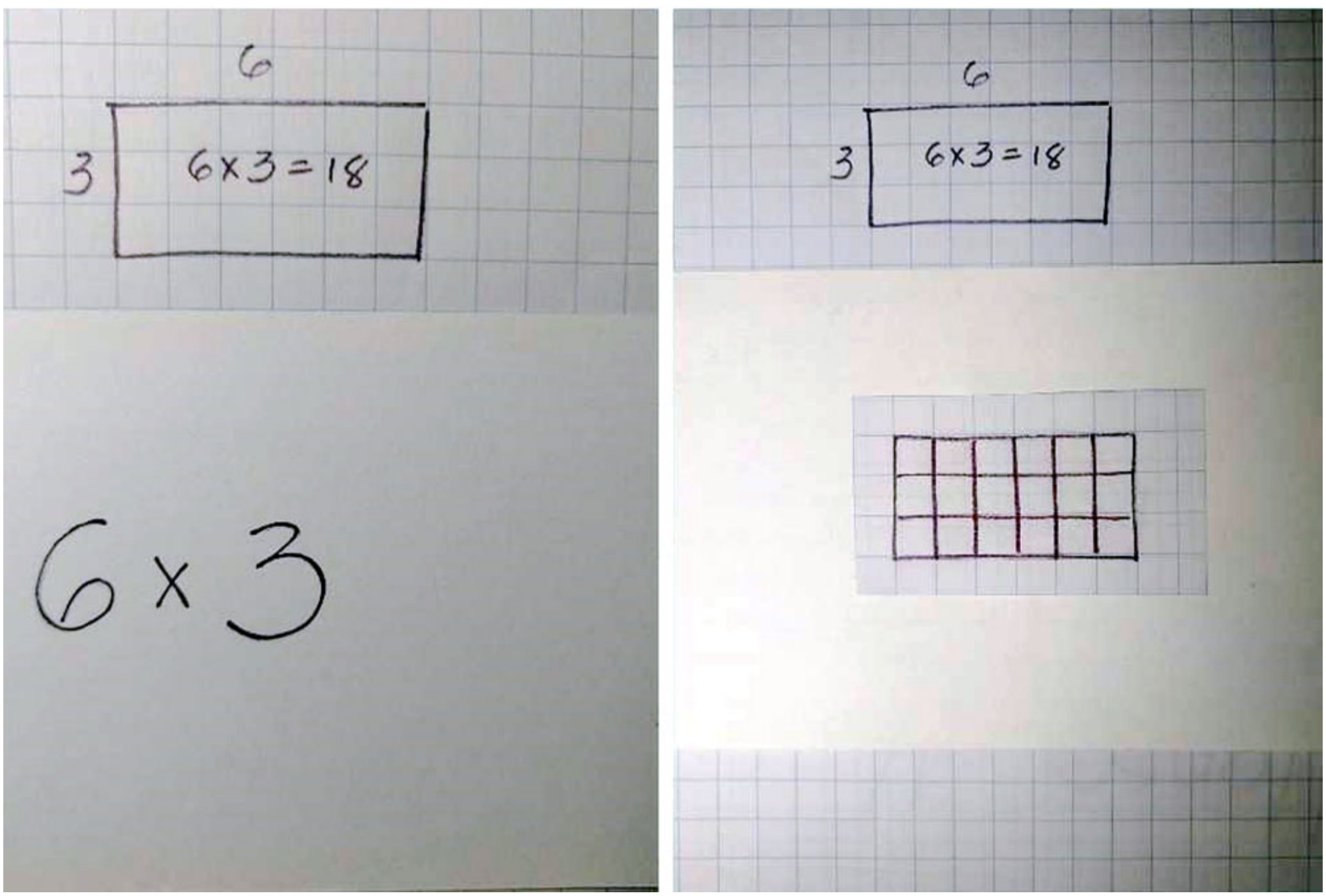

Figure #5: Example of activity #1.

Activity 2:

This second activity could be presented as a continuation of the previous activity to emphasize the concepts or as an alternative instead of the first one. If students are in a classroom setting with “Math centers,” they may have the choice of completing one or the other. The goal of the activity is for students to be able to calculate the area and perimeter of rectangular figures.

Materials:

Each pair of students will need the following:

- Blank grid paper. Each square on the grid will represent 1 unit square.
- Two dice
- Markers or color pencils

Directions:

Students will divide the paper in half and decide which half each partner is going to use.

Each partner will toss a die, and whoever gets the highest number will start the game. Both students will use the same sheet of grid paper for their drawings, but each partner will start drawing in opposite corners of the paper.

The first player rolls the dice and will draw a rectangle with side lengths equal to the numbers rolled on the dice. For example, if the numbers were 2 and 4, the student will draw a rectangle measuring two units by four units. Then, the student will write the corresponding multiplication equation and write the value of the area. The second partner will repeat the procedure, creating a rectangle on his or her half of the grid paper.

Partners continue taking turns and creating rectangles. Rectangles must touch one another from any “free” side. The game continues until one partner is unable to draw a rectangle with the remaining space within half of the paper. Each partner each add up the total area of their rectangles. The student with the largest area is the winner.

This activity could be differentiated by providing dice numbered at appropriate levels for each student. At grade level, students may work with standard 1–6 dice. Students ready for a challenge can use polyhedral dice. Students could also calculate the area of the “free space” on each side. In addition to finding the area, which is the main purpose of the activity, students will determine the total perimeter around the compound figures in each side of the paper and will compare their areas and perimeters. As an introduction for the subsequent activity, the whole class will compare their results noticing the cases in which the “side” with the largest area also has the largest perimeter or when, on the contrary, the “side” with the smaller area has the largest perimeter. Those findings will open the discussion about the relationship between area and perimeter. Do rectilinear figures with the same area are expected to have the same perimeter? Does the perimeter increase with the area and vice versa? I will use the results of this activity to guide students in the process of thinking about the possible relationships between area and perimeter. Based on the engagement of the students, I will even suggest they find the perimeter of the individual rectangles in each side of the paper to identify rectangles with the same area that have different perimeters, and rectangles with the same perimeter but different area. This exercise will generate awareness of the differences between area and perimeter and will scaffold the transition to the next activity.

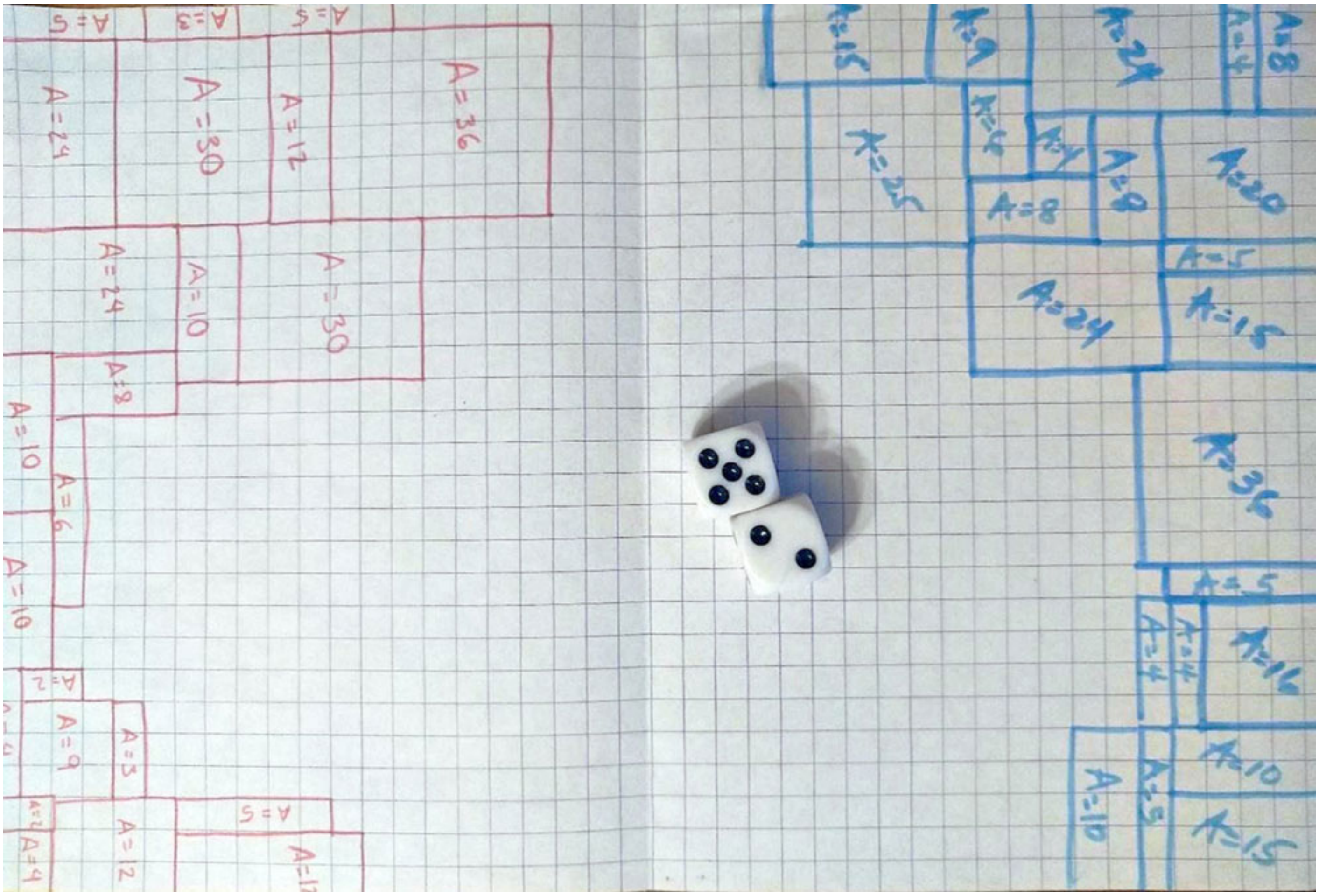

Figure #6: This figure shows an example of the progression of activity 2.

Activity 3:

This activity, although very simple, serves the purpose of illustrating variations of area in rectangles with the same perimeter. It could be added as an alternative for activity 2 or after it to emphasize concept development and the understanding of the differences between area and perimeter.

Materials:

Each pair of students will need the following:

- Blank grid paper. Each square on the grid will represent 1 unit square.
- Pencils or markers

Directions:

Students will work in pairs. Each pair will get assigned a value of perimeter and the task will be to find all possible rectangles with whole-number side lengths for the given perimeter and to find the area of each.

Students will have a discussion of the results to conclude that for a given perimeter, the area is the smallest when the rectangle is long and thin, and largest when the rectangle is as near as

possible to being a square. Long thin rectangles can have less area than nearly square rectangles with the same perimeter.

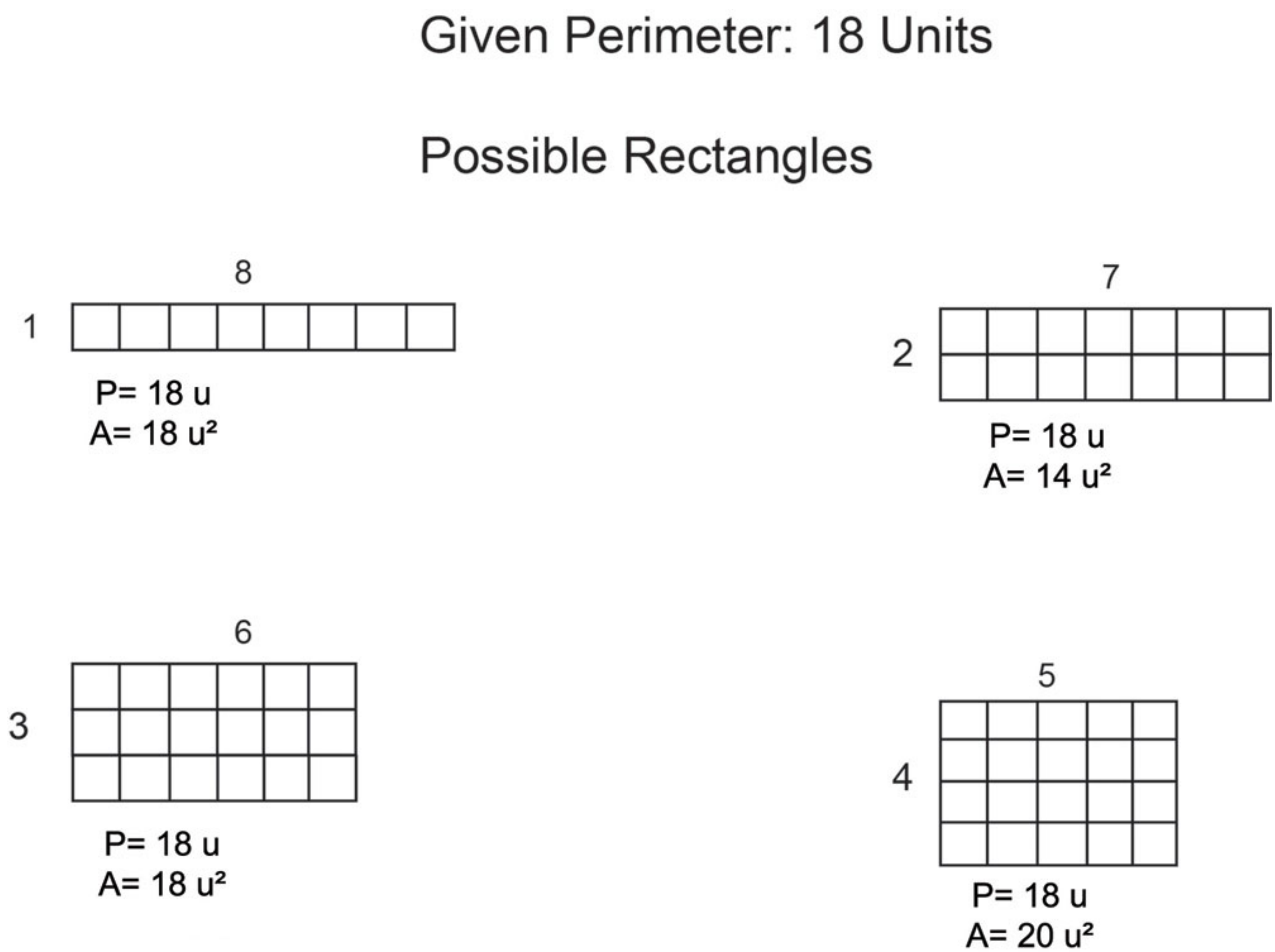

Figure #7: This figure illustrates an example of activity 3. We can observe that the longest and thinnest rectangle with side lengths 1 by 8 units has the smallest area. On the other hand, the rectangle with side lengths 4 by 5 units, has the largest area in this series of rectangles with the same perimeter.

Activity 4:

In this activity, students will consolidate their understanding of the differences between area and perimeter. The exploratory nature of the activity allows students to deal with both concepts at the same time rather than separately. In my opinion, students should be allowed to compare area and perimeter “side by side” to gain better conceptual understanding. In general, the curricular material available in 3rd -grade mathematics is extensive in the number of exercises presented for students to solve problems about area or perimeter separately, but they offer a very limited amount of examples for students to compare both concepts at the same time, and those examples are often limited to rectangles. In this activity, we will move away from just rectangles, and students will explore what happens when they remove unit squares one at the time. What are the possible arrangements, as well as what happens to the area and perimeter.

To complete this activity, students will work in groups of three.

Materials:

- One sheet of laminated grid paper (1 unit square = 1 in^2)
- Foam or plastic tiles (1 in^2). Tiles should be the same size as the unit square on the laminated grid paper
- Grid paper to draw on (½ or ¼ in^2)
- Pencil and eraser
- Set of recording charts similar to figure #4
- Document camera
- Smartboard

This activity could go in different directions, I may assign the same initial rectangle to the whole class, I could assign different rectangles to different groups, or I may let students choose their initial rectangles keeping in mind that the size will be limited by the size of the laminated grid paper and the number of available tiles. Although the grid paper for classroom use comes in sizes of 11 by 9 inches with a "usable" area of 10 by 7 inches approximately, I will propose that students explore rectangles with smaller dimensions such as 3 by 4, 4 by 5, 6 by 4, etc. to keep the activity manageable during a single classroom period.

Directions:

Students will select "jobs" within their groups. One person will oversee placing the tiles on the laminated grid and removing them (one at the time) to create different arrangements. Another student will replicate (draw) each arrangement on the other sheet of grid paper to keep track of all possible arrangements as they remove the tiles one at the time. The third student will be in charge of recording the possible arrangements in the charts to facilitate further classroom discussion and conclusions.

I will ask my students to observe some conditions when they are removing squares. The main one is, that the squares that are left at any time are connected with each other in the sense that you can move from any square to any other square by passing through the middles of the sides (and not through corners). Also, the enclosing rectangle should stay the same and not get smaller. Together, these conditions guarantee that there is always one square in any row, and one in any column. In other words, they cannot remove any whole row or any whole column. Figure #3 presented earlier shows an example of this activity of a rectangle of 3 by 4.

To make this activity as time-efficient as possible, I will assign the same enclosing rectangle to the whole class. After each group finishes their exploration, they will share all the possible arrangements they found for each unit subtraction (-1 unit, -2 units, -3 units, etc.) with the rest of the class to make sure they have all the possible combinations. Another alternative is assigning a different unit subtraction to each small group of students. As a result, each small group will be responsible for finding all the possible combinations for one type of unit subtraction only. For example, group 1 will work eliminating one unit, group 2, two units, and so on. Once that step is completed, students will make sure there are no missing arrangements for any possible elimination and will determine area and perimeter for each resulting arrangement.

To do this step efficiently, I suggest using grid paper, the document camera, and the smartboard. Students will remain in their small groups, and take turns to draw one arrangement on the grid paper under the document camera for everybody to see the drawing on the smartboard. As the whole group will be following along, the rest of the students will be keeping track of the remaining possible arrangements. Each student that goes to the document camera will draw a new one to avoid repetitions. They will sequentially draw a picture for each possible unit elimination.

Then, students will engage in small group conversations to discuss their findings and establish as many generalizations as possible about the relationships between area and perimeter for their particular examples. I will guide the classroom discussion with questions to scaffold the process. For example, what happens to the area and perimeter when a 'corner square' is removed? What happens to the area and the perimeter if the removed square is in a different position? What happens as they start removing two or more squares? Is there a pattern?. The purpose of the guiding questions should be to promote the understanding that area and perimeter represent different concepts and types of measurement.

Activity 5:

It is the culminating activity of this unit. I chose it because it represents an application problem related to our school life at Zarrow. As I explained previously, our school PTA will sponsor the creation of a sensory path, and I believe my students will feel motivated to be able to apply their knowledge in a “real life” situation at school. I already spoke with my principal about this activity, and she was very receptive about the students’ participation.

For this activity, students will be organized into four groups. Each group will be responsible for one hall as follows:

Group 1: Kinder hall

Group 2: Main hall

Group 3: 1st and 2nd grades hall

Group 4: Library hall

I have included a diagram of the layout of the school to show the location of the halls that have been considered for the sensory path.

To introduce the activity, I will review the previous learning with the whole class to make students understand that what we are about to do is like the previous activity they completed earlier, but now with an application component.

I will also show a couple of videos (there are plenty online) about different designs of sensory paths in schools, to increase background knowledge and promote idea sharing among the students.

Materials:

- Grid paper to draw on (½ or ¼ in^2)
- Pencil and eraser
- Masking tape

Once students get in their groups, and each group has their assigned hall, they will go ahead and measure the side lengths of the halls. The dimensions of the available space will constitute their “enclosing rectangles.” As a result, their designs need to fit within the area of the initial rectangle for each hall. Students will place masking tape to make those limits. Our school is all tiled in the common areas. So, the halls are tiled, and the dimensions of the tiles are 1ft. by1ft. For this activity, 1-unit square equals to 1 ft^2 or 1 tile.

When the teams determine the area that they have available, they will go back to the classroom to start working on their design. As a whole class, we will discuss what constraints students have to consider for this project. For example, people will be walking on those tiles, so the designs should allow movement from one side of the hall to the other side. As in the previous activities, the tiles on the design need to connect by their sides, not by the corners, and full rows or columns cannot be removed.

Now, students are ready to outline the rectangle on grid paper, and then, they will start working on the possible arrangements or designs. They will proceed similarly to activity #3, by "removing" tiles from the original rectangle until they determine what arrangement offers the best layout for walking and movement. Students need to draw their options on grid paper like the example in figure #3.

Based on the number of students I usually have per class; each group will have about six students. It allows each student to work with one type of elimination only. For example, one student will try all the possible options of eliminating 1-unit square; another will work eliminating 2, and so on. However, I will not impose that restriction because some teams may decide they want to skip 1 or even 2-unit square elimination and start by eliminating three tiles right away.

Although it will be interesting to see how far students can persist in the unit square elimination process, the main purpose of this activity is not to find all the possible combinations, which would be huge because the halls are large. The main goal of the activity is for students to explore as many options as possible for each hall to finally determine what combination of area, perimeter and design provides the "best" option for a walking path that will be potentially used to design the sensory path.

Once each team is satisfied with their choice, students will outline the design on the hallway floor with masking tape. Each team will have the opportunity to explain their design and the rationale behind their choices.

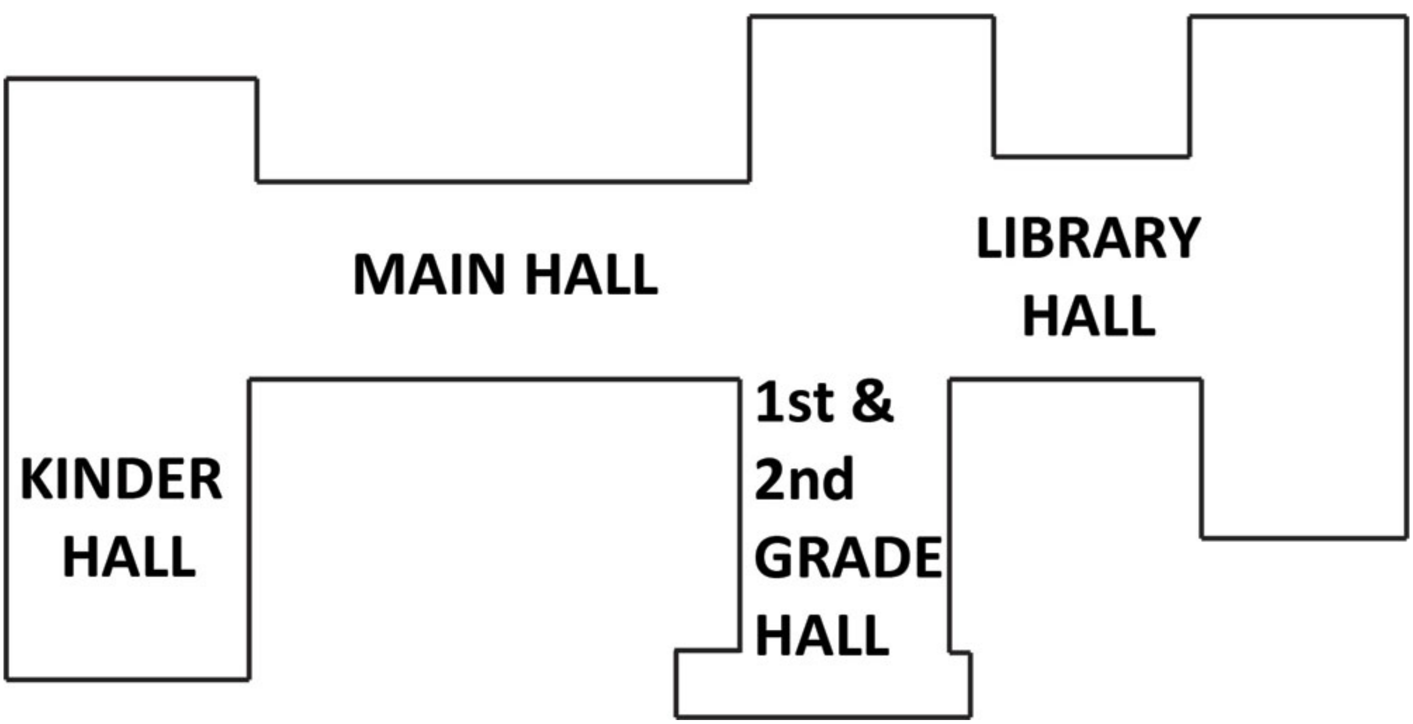

Figure #8: Diagram representing Zarrow International Elementary School and the four areas that can potentially be used to build our sensory path.

Bibliography

Oklahoma School Testing Program - Login. https://oklahoma.measuredprogress.org/. This is the website of the Oklahoma Department of Education to find information about state testing results.

/@eurekamath. "Read, Draw, Write: A Better Strategy for Problem Solving." Medium. May 05, 2015. https://medium.com/eureka-math/read-draw-write-a-better-strategy-for-solving-746300f5cdb8. Information about the RDW problem-solving strategy.

Amidon, Joel, Ann Monroe, and Mark Ortwein. "Planning & Teaching Strategies." Lumen. https://courses.lumenlearning.com/educationx92x1/chapter/project-based-learning/. Description of Project-based learning strategies.

Beckmann, Sybilla. *Mathematics for Elementary Teachers with Activities*. MTM, 2018.

"Cooperative Learning Strategies." National Council of Teachers of Mathematics.

https://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Cooperative-Learning-Strategies/. This book is a useful resource for teachers. The selected chapter

describes cooperative learning strategies.

Huang, Hsin-Mei E., and Klaus G. Witz. "Children's Conceptions of Area Measurement and

Their Strategies for Solving Area Measurement Problems." Journal of Curriculum and Teaching.

http://www.sciedu.ca/journal/index.php/jct/article/view/2041/1230. A study of children's

understanding of the relationship between multiplication, and area, and the differences between

area and perimeter.

Machaba, M. "The Concepts of Area and Perimeter: Insights and Misconceptions of Grade 10

Learners." Pythagoras. https://pythagoras.org.za/index.php/pythagoras/article/view/304/476.

Analysis of misconceptions students may have when learning about área and perimeter. The

study focuses on 10th graders but includes information applicable to the elementary level.

"Magnet Schools." Tulsa Public Schools. https://www.tulsaschools.org/parents-

students/enrolltransfer/magnet-schools. Information about magnet school application.

"Read "Adding It Up: Helping Children Learn Mathematics" at NAP.edu." National Academies

Press: OpenBook. https://www.nap.edu/read/9822/chapter/1#xv. An interesting study about how reforms in education through the years have changed the way students learn mathematics.

S, Surbhi. "Difference Between Area and Perimeter (with Comparison Chart)." Key Differences. September 01, 2017. https://keydifferences.com/difference-between-area-and-perimeter.html. Concept development about area and perimeter.

"School Profiles." Tulsa Public Schools. https://www.tulsaschools.org/our-schools/school-profiles. Profile information about Zarrow International

"Standards for Mathematical Practice." Standards for Mathematical Practice | Common Core State Standards Initiative. http://www.corestandards.org/Math/Practice/. The Common Core official website.

Walton, and Tamela Randolph. "Alternative Methods for Understanding Area Formulas." Illinois Mathematics Teacher. http://journal.ictm.org/index.php/imt/article/view/89. This article emphasizes the importance of having a solid conceptual understanding when learning mathematics instead of formulas memorization. The report also provides ideas of activities for students to gain conceptual understanding.

"Why Teach Mathematics with Manipulatives?" Manipulatives in Math: Why Teach Math with Manipulatives | ETA Hand2mind. https://www.hand2mind.com/resources/why-teach-math-with-manipulatives. Use of manipulatives in mathematics.

"The Difficulty of "length × width": Is a Square the Unit of Measurement?" The Journal of Mathematical Behavior. April 11, 2006. https://www.sciencedirect.com/science/article/pii/S0732312306000125?via=ihub.

Endnotes

---

1. Beckmann, Sybilla. *Mathematics for Elementary Teachers, with Activities*. Pearson, 2018.
2. https://www.open.edu/openlearncreate/pluginfile.php/134950/mod_resource/content/3/EM10_AIE_Final.pdf. (2019).
3. Walton, and Tamela Randolph. "Alternative Methods for Understanding Area Formulas." Illinois Mathematics Teacher. http://journal.ictm.org/index.php/imt/article/view/89.

4. "Read "Adding It Up: Helping Children Learn Mathematics" at NAP.edu." National Academies Press: OpenBook. https://www.nap.edu/read/9822/chapter/1#xv
5. "Oklahoma School Testing Program - Login." Oklahoma School Testing Program - Login. https://oklahoma.measuredprogress.org/.
6. Machaba, M. "The Concepts of Area and Perimeter: Insights and Misconceptions of Grade 10 Learners." Pythagoras. https://pythagoras.org.za/index.php/pythagoras/article/view/304/476.
7. "School Profiles." Tulsa Public Schools. https://www.tulsaschools.org/our-schools/school-profiles.
8. "Magnet Schools." Tulsa Public Schools. https://www.tulsaschools.org/parents-students/enrolltransfer/magnet-schools.
9. "School Profiles." Tulsa Public Schools. https://www.tulsaschools.org/our-schools/school-profiles.
10. S, Surbhi. "Difference Between Area and Perimeter (with Comparison Chart)." Key Differences. September 01, 2017. https://keydifferences.com/difference-between-area-and-perimeter.html.
11. Hong, D., and Cristina Runnalls. Runnalls, "Filling in the Gaps." *Teaching Children Mathematics*25, no. 5 (2019). doi: 10.5951/teacchilmath.25.5.0274.
12. Outhred, Lynne N., and Michael C. Mitchelmore. "Young Children's Intuitive Understanding of Rectangular Area Measurement." *Journal for Research in Mathematics Education*31, no. 2 (2000): 144-67. doi:10.2307/749749.
13. "Sensory Paths! What the Heck Are Those?" Fit and Fun Playscapes. April 04, 2019. https://fitandfunplayscapes.com/sensory-paths-what-the-heck-are-those/.
14. Machaba, M. "The Concepts of Area and Perimeter: Insights and Misconceptions of Grade 10 Learners." Pythagoras. https://pythagoras.org.za/index.php/pythagoras/article/view/304/476.
15. 15. "Why Teach Mathematics with Manipulatives?" Manipulatives in Math: Why Teach Math with Manipulatives | ETA Hand2mind. https://www.hand2mind.com/resources/why-teach-math-with-manipulatives.
16. "Cooperative Learning Strategies." National Council of Teachers of Mathematics. https://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Cooperative-Learning-Strategies/.
17. /@eurekamath. "Read, Draw, Write: A Better Strategy for Solving - Eureka Math." Medium. May 05, 2015. https://medium.com/eureka-math/read-draw-write-a-better-strategy-for-solving-746300f5cdb8.
18. Amidon, Joel, Ann Monroe, and Mark Ortwein. "Planning & Teaching Strategies." Lumen. https://courses.lumenlearning.com/educationx92x1/chapter/project-based-learning/.
19. https://sde.ok.gov/sites/ok.gov.sde/files/OAS-Math-Final%20Version_3.pdf. (2019).
20. "Grade 3 » Measurement & Data." Grade 3 » Measurement & Data | Common Core State Standards Initiative.http://www.corestandards.org/Math/Content/3/MD/.

Appendix

The academic standards this unit is intended to cover are listed and described below.

All the standards refer to 3rd-grade mathematical content and mathematical practices.

As the Oklahoma Department of Education didn't adopt the Common Core State Standards (CCSS), I have included in this appendix, the Oklahoma Standards for Geometry and Measurement that pertain to this unit, as well as the Common Core State Standards, and the Mathematical Practice standards.

Oklahoma Academic Standards for Mathematics 3rd Grade.

Geometry and Measurement:

3.GM.2.1 Find perimeter of a polygon, given whole number lengths of the sides, in real-world and mathematical situations.

3.GM.2.2 Develop and use formulas to determine the area of rectangles. Justify why length and width are multiplied to find the area of a rectangle by breaking the rectangle into one unit by one unit squares and viewing these as grouped into rows and columns.

3.GM.2.8 Find the area of two-dimensional figures by counting the total number of same size unit squares that fill the shape without gaps or overlaps.

Mathematical Actions and Processes:

Develop a Deep and Flexible Conceptual Understanding. Students will demonstrate a deep and flexible conceptual understanding of mathematical concepts, operations, and relations while making mathematical and real-world connections.

Develop the Ability to Make Conjectures, Model, and Generalize. Students will create, identify, and extend patterns as a strategy for solving and making sense of problems.

Develop the Ability to Communicate Mathematically. Students will discuss, write, read, interpret, and translate ideas and concepts mathematically. [19]

Common Core State Standards for Mathematics 3rd Grade.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

3.MD.C.5 Recognize area as an attribute of plane figures and understand concepts of area measurement.

3.MD.C.6 Measure areas by counting unit squares.

3.MD.C.7 Relate area to the operations of multiplication and addition.

Geometric measurement: recognize perimeter.

3.MD.D.8 Solve real-world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.[20]

Standards for Mathematical Practice:

CCSS.MATH.PRACTICE.MP2 Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the units involved; attending to the meaning of quantities, and knowing and flexibly using different properties of operations and objects.

CCSS.MATH.PRACTICE.MP4 Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace.

CCSS.MATH.PRACTICE.MP7 Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure.