Curriculum Units by Fellows of the National Initiative
2019 Volume V: Perimeter, Area, Volume, and All That: A Study of Measurement

SmArt Math: Paper Polyominoes and Ceramic Tetradic Cuboids

Curriculum Unit 19.05.02, published September 2019
by Tina Berry

> “One important reason to study geometry is that it promotes the ability to visualize and mentally manipulate objects in space. This is a necessary skill for many professions.”[1]

Introduction

How do we teach our students to measure perimeter, area, and volume and then how do we teach them to actually USE them in real life outside of the math classroom? We must “learn to transfer learning and perform well in challenging situations, and that requires strategy- particularly in the face of obstacles and uncertainty”.[2] In art, mathematics and the knowledge of measurement quite often come into play and students are at a disadvantage when they don’t know how to apply what they have learned in math books. It is the rare occasion that half (or even 1/4th) of the class knows how to use a ruler other than to draw a straight line (which is also sometimes a challenge). If I ask my students in 7th and 8th grade art to fold a paper into quarters, it is likely a few will know what to do and many more will catch on quickly enough. If I ask them to fold the paper in half and measure out two inches from the center line...it will take a while longer. Asking students how big of a piece of paper they want by length and width measured in inches has also been an issue because the students don't always think solidly about the numbers. I do a bit of math tutoring on the side and many students have issues with perimeter, area, and volume because they don't understand how the concepts relate to each other and they don’t know their real-world applications. I often hear, “I’m never going to use this in my life, so why do I have to learn it?” Even as an art teacher, I tell my students that I use math all the time and most people either use it without realizing they are because they understand it so well, or they don’t use it because they don’t know how.

Overall, in this unit I plan to have my students engage in a hands-on math experience that integrates creative ideas that are normally minimal in a typical math class setting. I want students to understand that this is not a math lesson in art class, but an art lesson that involves math. Our biggest hurdle is to make sure that all students understand the math behind the lesson to be able to finish the final project properly. The hope is that students will feel empowered by the lesson well enough to be able to face harder problems in math class and feel comfortable with inherent learning risks in any classroom setting.

Student and School Background Information

At our suburban school, Memorial Junior High, we have approximately 400 students each year. Our school demographics have changed over the past few years. We have, in past years, had a fairly even distribution of Caucasian, African American, and Hispanic students, but each year the Caucasian population has decreased. This last school year we had approximately 18% Caucasian, 32% Hispanic and 33% African American. We did have a larger portion, 11% that identify as multiracial, which was not an identity marker previously. 90% of our students are considered socioeconomically disadvantaged and receive free lunches. 6% are identified as Gifted and Talented. While about ⅓ of the population is on an IEP, only about a quarter of the students on an IEP are in Special Education classes. We offer various electives: band, choir, technology, art, and PE. The school has 4 special education classrooms, three of which are self-contained.

Our students come from a wide variety of home backgrounds. Many students are bussed in from a notoriously dangerous area of our city, mainly living in poverty in Section 8 (government supplemented) apartments. Others are bussed in from (mostly) apartment complexes in slightly better areas. Then there is a smaller percentage that live in the middle-class neighborhood where the school is located.   Many of the students that live in the school's more affluent neighborhood go to magnet schools in the district or private schools outside of the district rather than attend our school because we have so many students bussed in from rougher areas. This, in turn, does support some segregation in the district.

Our art room is exceptionally large and well equipped compared to others in the district. We have a Promethean board, Apple tv, a class set of iPads, green screen set up, sewing machines, screen printing materials, a clay throwing wheel, kiln, and a great set up of materials (a lot of which I have acquired through grants). Students are allowed quite a bit of freedom in creating projects within given parameters for each assignment. For example, we may investigate and experiment with watercolors using various materials such as salt, oil, baking soda, and vinegar, and students will then be assigned to create an artwork of their choice using techniques and materials they have learned. We do a lot of integration with other subjects such as NASA lessons to utilize the seven simple machines, sewing effigies of Revolutionary War soldiers, and writing and video recording project self-critiques in the form of news broadcasts in front of the green screen. We often use math in our art projects (scale models using ratios and fractions, proportions of bodies and faces, and using the body as a measuring tool) but I have not created a lesson based specifically on math until now.

Unit Learning Goals

In this unit we will explore the concepts of 2D shapes and 3D forms and discuss why they are important in art, math, and real-world scenarios. This unit aims to solidify students' conceptual understanding of basic measurement and physical mastery of basic length measuring tools.

The specific objectives for students in this lesson include:

1. Demonstrate how to measure properly using a ruler, yard stick, and a tape measure, and which method to use for specific needs.
2. Understand and apply knowledge of measurement of perimeter, area, and volume and the nature and names of the units associated with each of them.
3. Use measurement to produce figures on a 2D surface (of the type known as “mathematical nets”) that students will fold up to create a 3D art piece.

Important points that I wish to convey to students:

1. Unit consciousness is essential in measurement and other applications of math.
2. Rectangles with whole number side lengths are simple shapes that can be broken down into single square units that are then used to measure perimeter and area.
3. Complicated planar shapes can be decomposed, or broken down, into simple shapes.
4. Math does have many true, genuine uses in the practical world, outside of academics.

To reach these ends, my students will be doing various activities to scaffold learning. They will work with the basic measuring tools to measure area, perimeter, and volume. They will practice with manipulatives to create and understand polyominoes, then use their knowledge of polyominoes to draw all the hexomino nets that can make a cube. They will then use the cubes they create to understand tetradic cuboids (solid form made from four cubes). They will use these cuboids to create their own templates for clay planar tiles. At the end of the 5-week project each student will have a finished ceramic box that meets given dimension specifications based on the perimeter of a square, the area of a rectangle, and the volume of a cube.

The Unit

As someone who is proficient in math and art, I sometimes use measurement in very casual ways. If I need a 7 foot of paper to decorate my bulletin board, I go tear off a piece of paper from the large roll. I’m typically quite accurate without using a measuring tool. If I need a piece of fabric, I estimate what I need based on the width of the bolt and the lay of the pattern. When I’m cooking, I often don’t use measuring cups and spoons, unless it is vital for the recipe.

My students are rarely proficient in math and even if they are, they are likely not as adept at estimating the things mentioned above, more due to experience than understanding. In our seminar, we worked on measurement using square units. We studied polyominoes as a potentially effective device to get students to understand the concepts of perimeter, area, and volume using the most basic of units. I would like to include skills on how to use a ruler, as well, to get students to actively use their knowledge of perimeter, area, and volume in real life scenarios and begin to get that needed experience in measurement.

Rules for Rulers

Far too often, students don’t know to start at 0 when using a ruler. When there is a gap at the beginning of the ruler before 0, they don’t understand to skip it. Once a student told me that he always thought since he was supposed to skip the little gap that the teacher meant to skip to 1, so he always had. We want to dispel any misunderstandings and start with the basics of measurement with a short introduction to rulers, yardsticks, and measuring tapes that we will then use to actively measure things large and small to gain more experience. We will emphasize the idea that a number on a ruler is telling you how many unit lengths away from the zero point it is located, these units usually being either inches or centimeters.

Forms, Properties, and Units of Measurement

Forms of Measurement

There are many forms of measurement: time, weight, mass, velocity, atmospheric pressure, temperature, altitude, distance, and depth, to name a few. Even counting is a form of measurement. Then of course, there is our focus- perimeter, area, and volume. These are forms of measurement we see and use almost every day, often without even thinking about it.   For a quick example, “do you want the small, medium, or large drink?” involves a measure of volume.   Asking students for examples of perimeter, area, and volume will give a quick overview of student understanding of these specific forms of measurement.

Properties of Measurement

There are four key properties of measurement that we took note of in our seminar: comparison, combination, repetition, and partition. Though each is very simple, I feel that some students will benefit from going over them, others may feel more confident starting the unit with something they understand well enough, and others who thoroughly understand can be of assistance to others around them that need extra help.

1. Comparison is when we compare two quantities with respect to the same measurable attribute. This comparison can be represented as equal, unequal, less than, or greater than.

A |________|

B |__________| If AC, and A=D, we conclude that B>D and B>C.

C |______|

D |________|

2. Combination is when two of the same form of measurement of varying amounts are combined, or added together, as seen in addition.

A |________| B |__________| A+B= |________|__________|

This may be extended to encompass subtraction, also.

3. Replication is the repeated copying of one unit of the same form of measure, a form of multiplication.

A= |________| 2A= |________|__________| 3A= |_____|______|______|

4. Partition is when a unit or group of units of a single form of measure are subdivided into equal parts, a form of division.

A= |________| A= 1/2 A + 1/2 A |____|____|

These four properties guarantee that we can use numbers to express any measurable attribute of a quantity as a certain multiple of a pre-selected unit of the same attribute. That is, we can express the length of some object as a certain number of inches, or the area of some region as a certain number of square miles, or the weight of a bag of apples as a certain number of pounds, and so forth. But for the number to have meaning, we need to specify the unit.

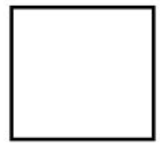
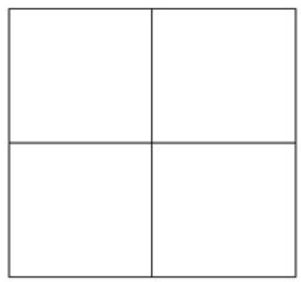
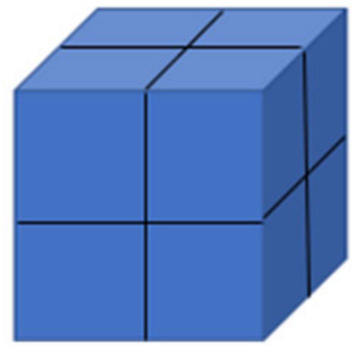
Units of Measurement

Units of measurement will depend on what is being measured and the tool doing the measuring. It is impossible to measure volume in degrees or temperature in ounces, perimeter in minutes or area in kilowatts. Units for length include, but are not limited to, inches, feet, yards, miles, and their metric counterparts- centimeters, meters, and kilometers. A measurement is created when a number is attached to an individual quantity by choosing a unit of measure. The width of a piece of copy paper is 8 ½ inches. The paper's actual width is the individual quantity. 8 ½ is the number, which is an adjective to modify the noun, inches. The important thing to know is, all numbers need a unit in order to take on meaning. If you ask me how many tables there are in my classroom and I say 7, you understand I am saying 7 tables. However, if I randomly tell you I need 7, you will likely have no idea what I am talking about. Is it dollars? Pizzas? Boa Constrictors? You won't know until I say that what I need is actually 7 pencils. Attend to the unit, it matters.

Area, Perimeter, and Volume

Area and perimeter are basic geometry concepts. However, getting students to reliably associate the name with the purpose or the equation is a bit harder than just telling them once. Perhaps a good way to explain is it in everyday terms. If the gym teacher asks students to walk the perimeter of the gym, what does she want them to do? Does she want them to walk randomly around the entire gym floor bumping into each other or does she want them to walk around the outer edge? Most students understand the teacher wants them to walk around the outer edge. As well, when the teacher asks students to clean up their areas, they generally understand it isn't just the edges of the space around them to be cleaned, but the entire desk and floor around them that the teacher wants cleaned up (whether they actually clean it up or not). Perimeter versus area is understood in both instances, however, students may not connect these ideas to geometry concepts in math class. Volume is the three-dimensional space that something occupies, such as air. Students can easily recognize the difference between a 12 oz can of soda vs. a 20 oz bottle vs. a 32 oz drink cup at the gas station as a physical example, but may not fully comprehend the numbers associated with the containers. We will be dealing with rectangles and squares in this unit- the most basic of shapes, and whole numbers, so that students will become more comfortable with the ideas and methods presented rather than being wrapped up in algebra and fractional arithmetic.

Relation of Units of Length, Area, and Volume

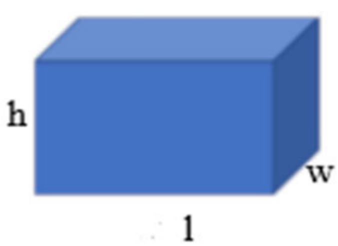
1 Unit of Length (To measure perimeter)	1 Unit of Area (1 square unit)	1 Unit of Volume (1 cube unit)
		
1-Dimensional	2-Dimensional	3-Dimensional
		
2 units	4 $units^2$	8 $units^3$

Perimeter is the sum of the lengths of all the edges of a shape. It measures around the shape.

$$P = x + y + x + y = 2x + 2y \text{ (with a square, use } P = 4x\text{)}$$

Area measures all the space within the shape.

$$A = \text{Length} \times \text{Width} \quad \text{or} \quad A = lw$$



Volume measures the capacity of the form

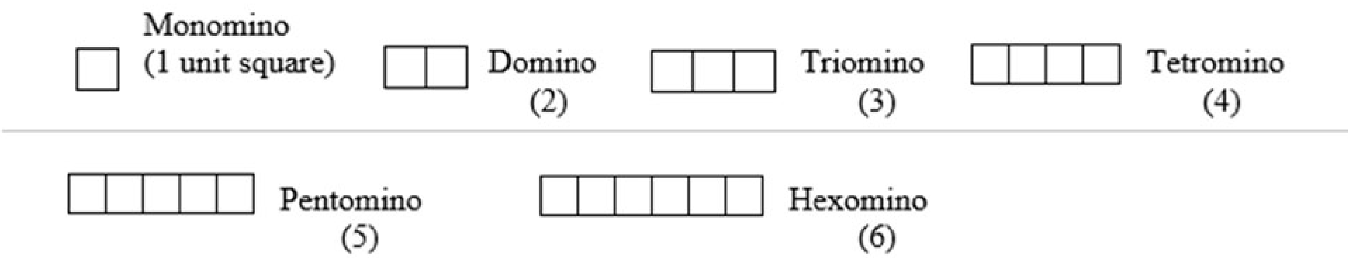
$$V = \text{Length} \times \text{Width} \times \text{Height} \quad \text{or} \quad V = lwh$$

Note: For irregular shapes and forms it may be necessary to decompose, or break down, the shape or form before calculating area or volume. To get perimeter of an irregular shape, simply add the lengths of every side together.

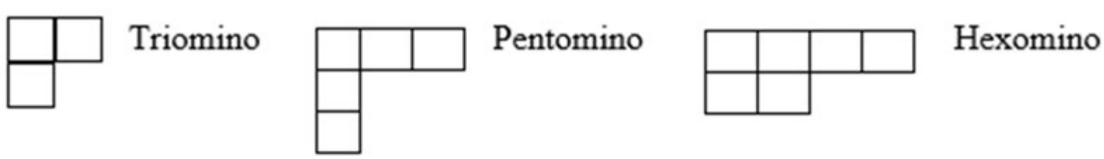
Polyominoes

"Polyominoes are the shapes made by joining squares edge to edge. They were named and first studied by mathematician Solomon W. Golomb, starting in 1953."[4] Most people know what dominoes are, though they may have been called biominoes if Mr. Golomb had named them. His work with polyominoes has changed the world forever in many ways. "He developed a game (polyominoes) that was the later basis for the later computer game Tetris."[5] Many students will immediately recognize the polyomino shapes as Tetris pieces. Golomb's mathematical research was "regarded as vital to the digital communications revolution…His concepts known as Golomb rulers and Golomb sequences were used to make possible such digital communications as internet and cellular telephone networks."[6]

Polyominoes show how complicated planar shapes can be and how simple rectangles are.[7] This is why we break down complicated shapes into simple shapes to find area and perimeter. Below are polyominos up to number 6, they do however go on and on. We will only be using up to the number 6 for our unit. Students can challenge themselves by going higher.



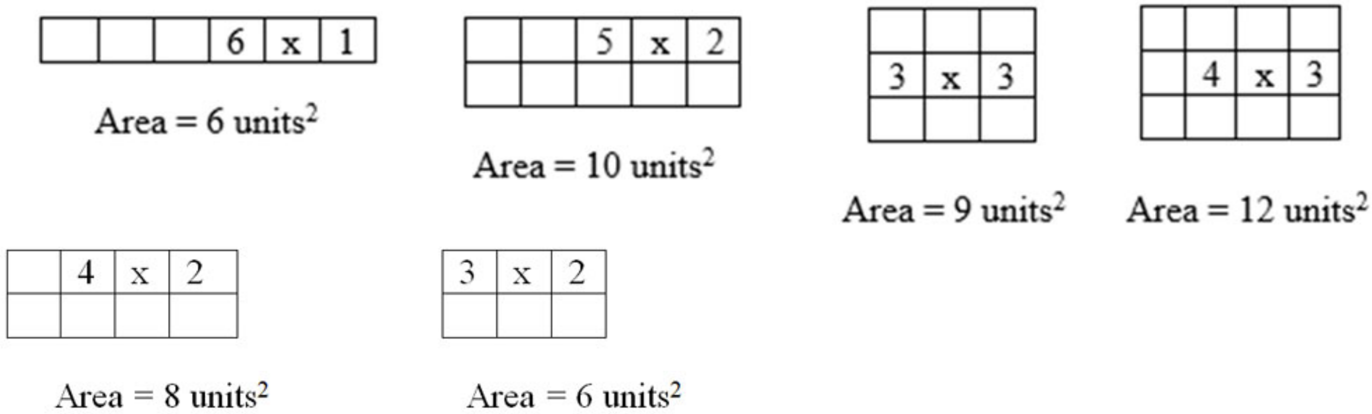
Polyominos can be rearranged so long as every square is touching another square, edge to edge.



If each square unit is not edge to edge with another unit it becomes a separate unit.

When introducing polyominoes, I will show students, one unit, then the domino and explain how it only has the one way to present it, side by side. Next, the triomino, side by side or as an L form (as seen above). Students will work together (see Polyomino activity) to find the polyominoes of the 4-unit tetrominoes (5 variations) and 5-unit pentominoes (12 variations) and present them on the board by teams.

Next, we will look at hexominoes, the six-unit polyominoes. Hexominoes will always fit into one of the six areas of an enclosing rectangle shown below.



There are 35 combinations within these enclosing rectangles. It is a challenge to find them all, and flips, rotations, and mirror images may be hard to eliminate, but do not count. It is important to note that the perimeter and area of the enclosing rectangles are different, as are the perimeters of the created hexominoes within the enclosing rectangles. Hexominoes will all automatically have an area of 6 units^2 because they all consist of 6 individual unit squares. Students will investigate and find all the Hexominoes in small groups and as a class.

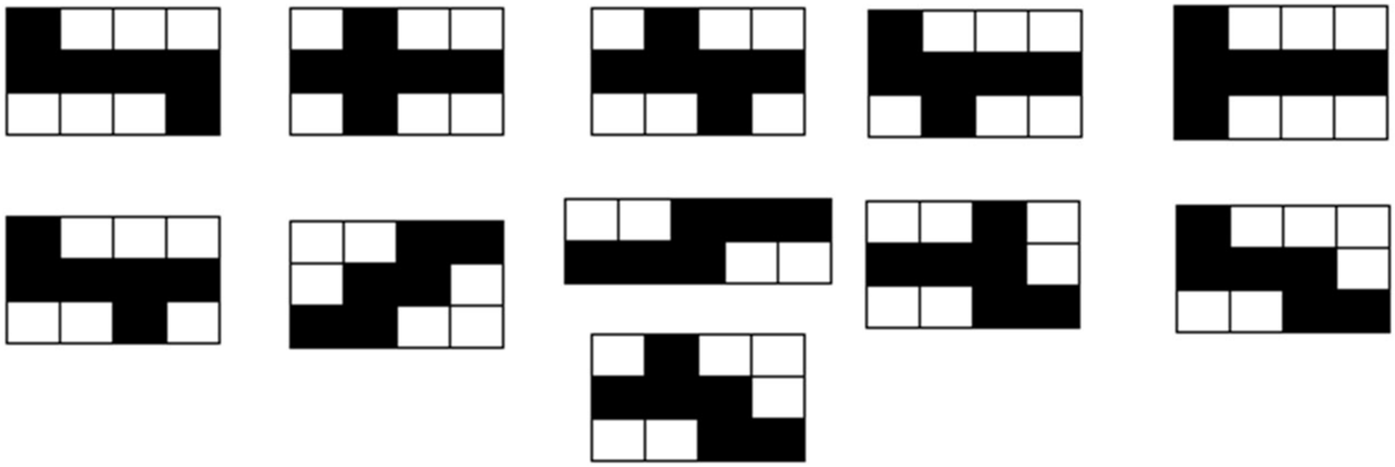
Hexomino Nets and Tetradic Cuboids

A net is a 2-D shape that can be folded up to make a 3-D form. Nets are also defined in geometry as “A pattern that you can cut and fold to make a model of a solid shape.”[7] The area of a net is equal to the surface area of the form. Some hexominoes are nets for a cube. A tetrad is any group of four things and a cuboid is a form made by putting cubes together, in ways similar to the formation of polyominoes in the plane. When four cubes come together to create a form, where each cube must have one face completely touching the face of another cube, they form a tetradic cuboid.

Creating Nets Out of Hexominoes

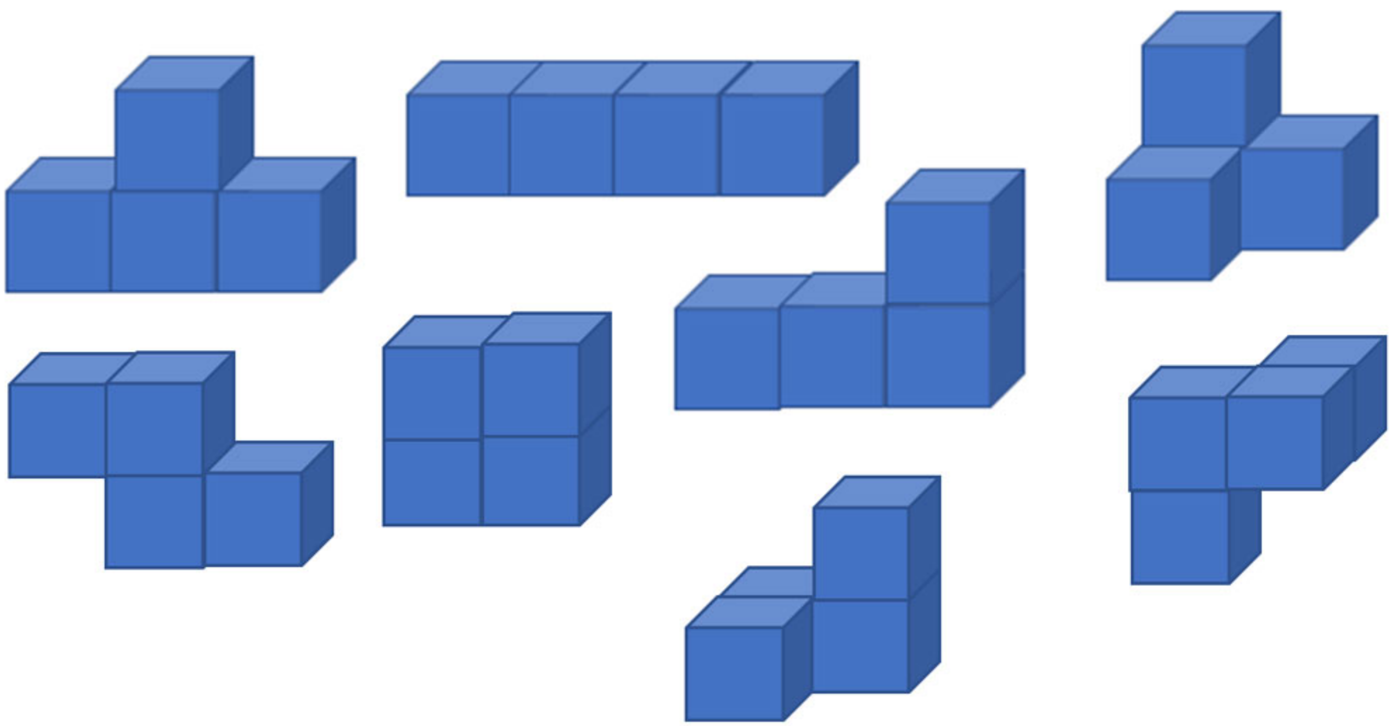
Of the 35 hexominoes there are 11 that can be cut out and folded to make a cube; these are called cube nets. The challenge that will be presented may be done visually, though some individuals may need to cut out the shapes of the nets and fold them up to determine if they will form a cube or not. In the mathematics textbook, *Mathematics for Elementary Students,* Sybilla Beckman notes that “One important reason to study geometry is that it promotes the ability to visualize and mentally manipulate objects in space. This is a necessary skill for many professions.”[8] She also mentions a report from the U.S. Department of Labor that states “being able to ‘see things in the mind’s eye’ is a foundational skill for solid job performance”.[9] Students will work in pairs to discover which of the 35 hexominoes fold into cubes, as described in the Activities section. Students will each pick a cube net of their choice and recreate it.

The following are the 11 hexominoes that can form a cube.



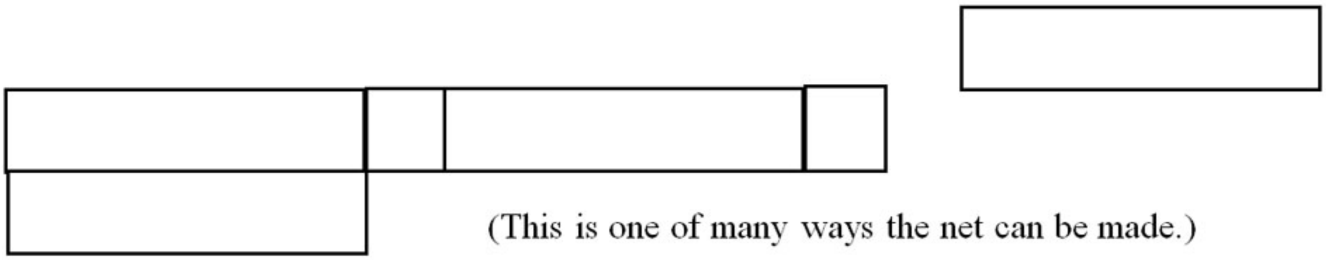
Tetradic Cuboids

When 4 students get together, with the cubes they created from their hexomino nets, they will work together to assemble tetradic cuboids. I will challenge each group of 4 students to make as many different tetradic cuboids as they can. When they make one, they will sketch it, then disassemble their cubes to make another one. These cuboids will be the models for the final product in this unit, a tetradic cuboid made of clay slabs.

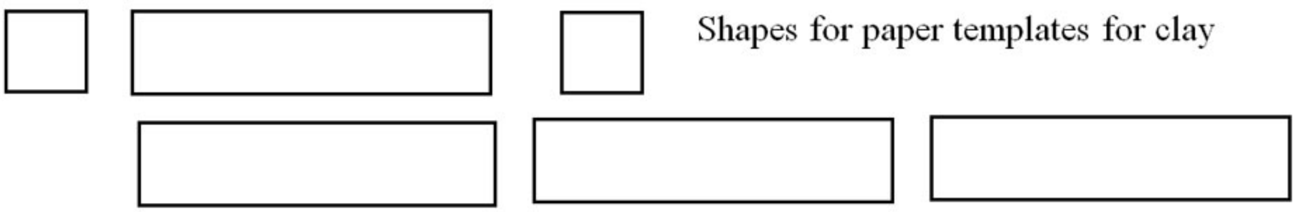


The clay slabs will be made from paper templates of the planar surfaces of the cuboids. For example, the square prism would have two 1-unit squares for the ends and four 4-unit straight slabs for the sides. If these are assembled into a plane figure in preparation for making the cuboid, the plane figure will be a net for the cuboid. Students can also create a cardstock net that will be folded and glued so they can use it as a model of the tetradic cuboid they will build out of clay.

This is the Form



(This is one of many ways the net can be made.)



Shapes for paper templates for clay

Strategies

Experiential Learning

Learning is an "active and social production". For this we need to engage students in "active discussions to help them to create meaning" from their learning.[10] There are many psychologists, sociologist, and educators who agree that hands on learning is essential for learning for most students. David Kolb is an educational theorist and author who states that "learning is the process whereby knowledge is created through the transformation of experience".[11] His theory consists of four stages that we will use in our unit. We will actually cycle through the experiential learning model multiple times through this unit. This unit is so hands on that the strategies are largely student based rather than what I will be doing. I will be more of a guide. I believe that it is so important for us, as teachers, to provide authentic practice in "real life situations, not in isolated exercises".[12] This method is helpful to not only have the real life experiences, but to make theories of what will happen, plan for action, actively participate in the experience, and then reflect on the experience.

Concrete experience

This is the "do" step. In the first stage the students are actively experiencing an activity. Students will first be measuring using rulers, yardsticks, and measuring tapes. In the next activity they will be using manipulatives to create hexomino patterns. Then they will be making nets and cubes, followed by paper templates, and then finally making their clay tetradic cuboid. These are all of our "do" items. This step will actually occur third, more details to come in the third stage below!

Observation and reflection

In the second stage (to be completed last) students will practice reflective observation, in other words, they will reflect on the various stages of the project, and observe how they were connected, and what came out. We will have group discussions about what we measured, how we measured, what problems came up, what our results were, and what we learned using rulers, yardsticks, and measuring tapes. After creating hexomino patterns, students will take turns posting their forms on the board to see what shapes they did and didn't find on their own. We will reflect on how we went about getting the different shapes and how they fit into the area of the enclosing rectangle. After making nets, cubes, and tetradic cuboids we will discuss why there are only five possible Tetromino forms, but eight possible tetradic cuboids. Note: We do not count reflections or rotations. After creating our clay cuboids, we will measure and discuss why the size is smaller after firing (shrinkage).

Forming abstract concepts

The third stage is about thinking. The learner is meant to create a theory or model of what has been or will be observed. Even though this is the third step, Kolb makes it clear that the steps can start anywhere as long as they are followed in order from there. I would like to have students think about what is going to happen before starting working, after a short introduction from me in the form of a mini lesson. (As I wrote previously, this unit is very student focused and they will be actively involved in each step so my lessons will be short and concise for each activity.) Making predictions and stating expectations may take away some of the nervousness and anticipation anxiety in students and give them a path to follow. Even if it is the wrong path, they will discover that and make corrections. I always tell my students; we have to learn from our mistakes. Sometimes in art we make mistakes that can't be fixed and it just leads us down a path we didn't intend on taking, but may enjoy nonetheless. Don't be afraid of change.

Testing in new situations

Although this is the fourth step, it will happen second. This is the "plan" stage. Here the students will plan how they will get through the activities. In the measurement activity, students will know in advance that they are measuring the pool deck, for instance. In their small group they will need to plan on how that will happen. Will they break down the deck into smaller pieces to measure or measure the entire area and subtract the area of the actual pool? Who will hold which end of the measuring tape (if that is the tool, they decide they are using)? Planning can sometimes be hardest part of getting a job done right, and a job can be completely sabotaged if the planning is not done right from the start.

Differentiated Instruction

> "A fundamental reality in teaching is that our students vary (sometimes widely) in their prior knowledge, skill levels, and experiences, in their interests, and in their preferred modes of learning and way of showing their achievement."[13]

In my classes I have students who don't speak English, students who are extremely talented artists, students with outstanding test scores and intelligence levels, students who are delayed in their learning, and even students with physical disabilities that hinder their ability to actively participate in some projects. However, all these students are able to work and enjoy art class. This is all due to differentiated instruction. Not everyone works at the same level, and that is why I must plan for these different levels of ability, comprehension, and interest. Sometimes, it is just a matter of pairing students with lower abilities with students of higher abilities that are willing to help them out. Sometimes it is a matter of matching personalities to encourage students that aren't interested. Having pictures matched with words in English and Spanish is essential in my room. For this project we will be using manipulatives (a strategy in itself) so that students with lower skill levels can manipulate the pieces to make shapes rather than being expected to draw them on graph paper (as higher level students will be expected to and all students are welcome to try). When working with the nets and making cubes, students with physical and mental handicaps can receive a helping hand to get them done. Most students enjoy working with clay, but varying skill levels means that some students will need exceptions for the forms they create. Some students may need to be assigned a single cube rather than a tetradic cuboid.

Activities

The activities in this unit flow from one to the next but will be separated into 5 general steps. I estimate that, with the final art project, it will take three to four weeks to complete the unit. These steps are meant to scaffold hands-on use of measurement and the understanding of units, shapes, and forms in math and art. We will look at the size of art pieces and how they compare to each other and how they compare to our typical sized work in class. Most students are familiar with and like sports, therefore we will briefly discuss the size of a basketball court, an Olympic pool, and a football field's markings. I feel it is a way to get students to better relate what they are learning to the real world.

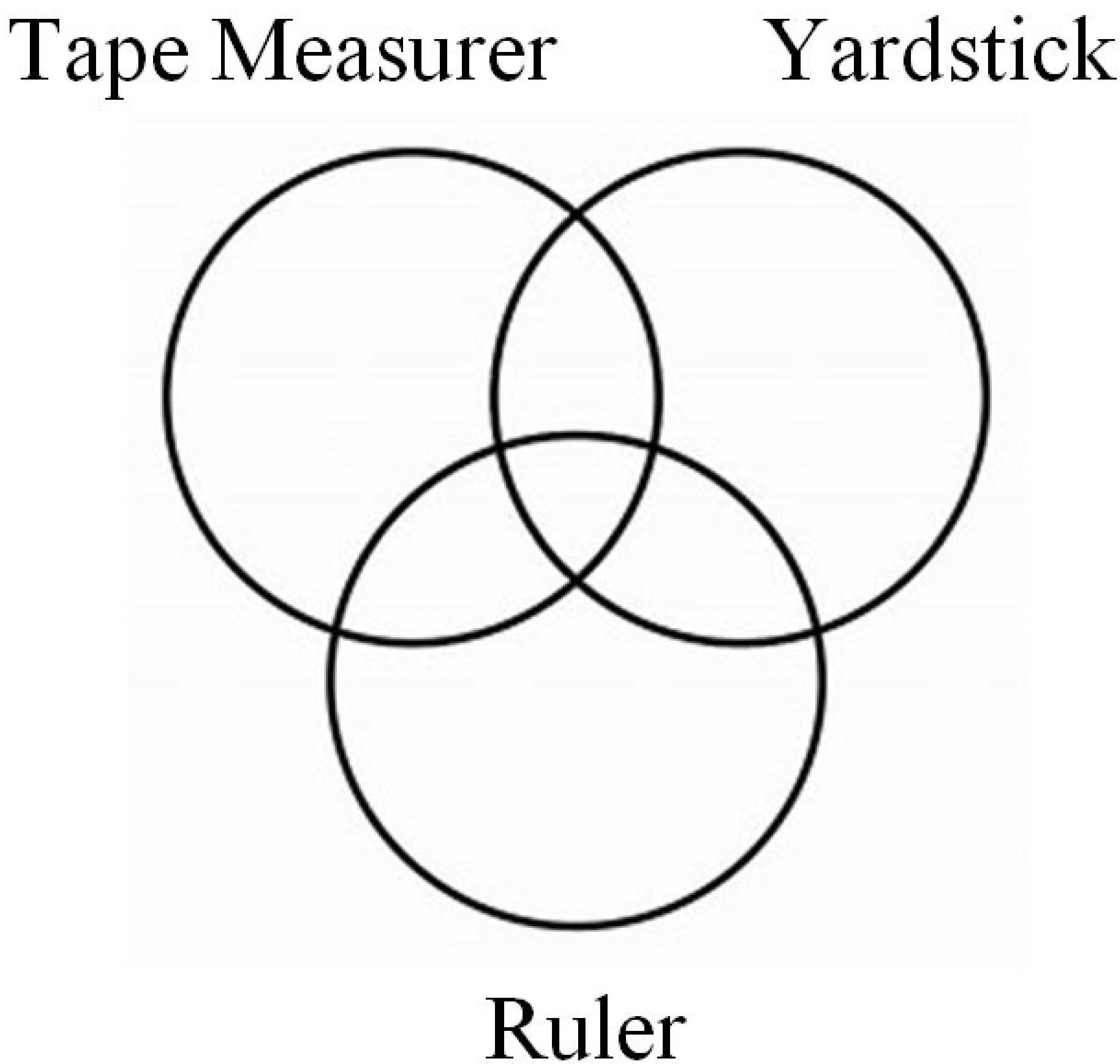
> "All areas of human experience are measured and counted, but few as visibly as sports. Sports are ruled and regulated by numbers. Playing fields and courts are laid out, scores are counted, records are broken- all of which demand a basic level of numeracy."[15]

Andrew Elliot states in his book, *Is That a Big Number?*, "The glamorous sister of memory is imagination."[14] He impresses on the reader that imagination not only lets us create, but also enables us to "see the future", visualize possibilities, work out issues, and plan a complex process that we can later follow. This is something that I will share with my students at the beginning of our unit when showing them the final product that they will be working toward so they will perhaps be a little more open to the process and mindset of the unit.

Rulers and Measuring

Basics of Standard Measuring Instruments

After showing students an example of the final pieces, they will make, we will start off with a review of the use of rulers. This will be basic so that students who don't have a solid understanding will gain a little competence here, but not taking so long as to bore those who already have a solid grasp of measuring. The time it takes to finish this will depend on the individual class, of course. I will show my students a giant version of a ruler. We will discuss the importance of starting at 0 and using the lines to determine whole numbers. We will briefly discuss halves and quarters, though we won't dwell on them since we aren't going to be using them in this unit. The activity will help students understand how to properly use a ruler, yardstick, and measuring tape through cooperative learning and reciprocal teaching strategies. Students will get in 5-6 groups of mixed understanding of measurement and each group will be given a ruler, a yardstick, and a measuring tape. Items will be presented for students to measure in their groups using the three tools (straw, dowel rod, pipe cleaner/chenille stem, etc.). The answers will then be given for students to check their answers against. Student volunteers will come show the class how they found the correct answer, so that groups that did not get it can understand how the answer was found. As a group they will then compare the three items in a three circle Venn diagram. Given a short period of time to write answers, we will then discuss the answers as a class. Some important points for students to understand are when to use each tool, how to use each tool, and difference between standard and metric measurements.



Using Our Measurement Tools

For this part of the activity we will look at two art pieces: Kehinde Wiley's "Louis the XIV of France"[15] and "The Mona Lisa"[16] by Leonardo Da Vinci. After discussing the pieces, we will use our large roles of paper to measure and cut out the sizes of paper that match the two pieces to compare to the size of paper that we typically use for our own art pieces in class (12"×18"). The importance of the unit attached to a task will be emphasized in measurement. We will stick with standard measurements and students will be advised to "choose your units wisely to allow yourself to work with numbers that are modest in size, balanced by your ability to visualize the unit you're working with".[17] It will be observed that "The Mona Lisa" should be measured in inches, while "Louis the XIV" should be measured in feet.

We will then work in small groups to measure an artwork in a frame with a ruler and a yardstick for perimeter, the school's pool deck with a measuring tape for area, and the classroom sinks with a ruler to determine volume, and finally measuring out a walking track on the school field using a walking tape measurer to find the volume of cement needed for a track and determine the amount of cement that would be needed for a 3 foot wide, 4 inch thick track around the entire field. After doing these measurements we will discuss the varying sizes of the sports arenas that we know and love, for swimming, basketball, football, and soccer.

Finally, we will look at the use of the body for measurement to relate to the visualization of size, such as a fathom that "originally came from a word meaning 'outstretched arms', it becomes easier to remember that a fathom is 6 feet (equally, 2 yards, the yard being more or less as long as nose to tip of outstretched arm)."[18] Students will work with their partners to measure their own wing span to compare it to the 6 foot estimate. Being as most of the students are not full grown, and everyone's arms are different lengths, they will see the variance, but understand the point of the exercise. We can also look at the fingers for inches, between elbow and wrist (or fingertips) for feet, etc.

Fun with Polyominoes

I will work with students on the board (as a class) and on grid paper (individually or in pairs) to show the build-up of polyominoes from 1 to 5 using the grid method shown in the Polyominoes section in the unit, and discuss. Students will calculate the area and perimeter of given shapes (polyominoes) by decomposing the shapes and will apply this experience to understand that a simple rectangle is an array of unit squares. Working in pairs students will then create hexomino (6 square units) nets based on square inches using a ruler and paper, noting the perimeter and area of each shape, which will be presented on the board as a class (these will be used in the next activity). Students will be challenged with an exit ticket to create three 8-unit polyominoes, and to answer the question: "What is the area and perimeter of each shape you created?" As an extra challenge, students can create an octomino with a perimeter as large as possible, and one with a perimeter as small as possible. There will be many of the large perimeter shapes, but only a couple of the small perimeter shapes. This will be an informal assessment to insure understanding of the activity. The reason for this activity is to practice measurement and to better understand our next step, nets, which are polyominoes.

2-D Net to 3-D Cube

In pairs, students will pick out the hexomino nets (from the last activity) that can be folded and put together to make a cube. Some students will be able to visually figure this out, others may have to cut out individual nets and fold them to check the legitimacy of the cube. After a given period, students will share out their discoveries and we will discuss why some work, and some don't.

Each of the two partners will choose a net that will fold into a cube properly and recreate it on cardstock based on 3 by 3-inch squares, with area of 9 $inches^2$ (and 12-inch perimeter) and a final cube volume of 27 $inches^3$. They will need to add tabs along one side of the edges that touch (I will demo this before students start). Before folding up the net and putting gluing it together, students will be required to add texture and color to each square. They will have 6 different sides featuring textures and color wheel properties- primary, secondary, tertiary, cool, warm, analogous, monochromatic, tints, tones, or shades. This could be used as an opportunity to review or teach the properties of the color wheel in an art class, if it hasn't already been done, or in a math class the students could just be given the opportunity to color and design the cube as they wish. When finished coloring, students will fold and glue the net into cube form.

For an exit ticket, students will find the surface area of their individual cube. "We can calculate the surface area of a prism by adding together the area of the base(s) and the lateral faces."[19] With a cube, it is easiest to find the area of one square and multiply it by 6, however, it will be up to the students comprehension of what we have learned so far to determine if they understand this or not.

Discovering Tetradic Cuboids

Each pair of partners from the previous activity will join with one other pair so that they have 4 cubes between the four students (for groups of three a spare cube can be given to finish the tetrad needed or a group may need 5 students, but will still only use 4 cubes). Students will then create all variations of tetradic cuboids they can. For a tetradic cuboid each of the four cubes must have a planar surface that matches up to at least one other cube planar surface as shown in the Tetradic Cuboids section of the unit discussion. With each tetradic cuboid discovered, students will draw it on paper to the best of their ability (they will need these drawings for the final project). Some students may be interested in the Soma Cube Puzzle. It is a puzzle cube that is created by fitting together all the possible combinations of three- or four-unit cuboids (soma cubes), joined at their faces, such that at least one inside corner is formed.

Tetradic Cuboids in Clay

Finally, as the culminating art project, students will pick their favorite tetradic cuboid and use their tetradic cuboid drawings (and possibly cardboard tetradic cuboids) to create a tetradic cuboid net. They will each create templates for the individual faces of their chosen form (as shown in the Tetradic Cuboids section of the unit above). These templates will be used on clay slabs to cut out the shapes that will then be joined to replicate the paper tetradic cuboids. They can then add texture and design to their pieces before firing and glazing. It could be interesting to see if pieces made by students (along with a - unit cuboid made in advance by me) could form a Soma Cube and if students could then find the area and volume of the cube.

Appendix

Being as this is a math project in an art class (or used as an arts integration project in a math class), I have given the standards for art and math, focusing in on the seventh grade.

7th Grade Art Standards

STANDARD 3: Visual Art Expression: "Creating" The student will observe, select, and utilize a variety of ideas and subject matter in creating original works of art.

Students will design and color their nets before folding and gluing them to create cubes. After creating tetradic cuboids out of clay students will use additive and subtractive techniques, as well as painting, to create an original final design on their ceramic cuboids.

STANDARD 4: Visual Art Appreciation: “Connecting” The student will appreciate and utilize visual art to make interdisciplinary connections and informed aesthetic decisions.

This unit is all about interdisciplinary connections. Students will use math and art to create and design their individual ceramic pieces.

7th Grade Math Standards

7.GM.1 Develop and understand the concept of surface area and volume of rectangular prisms.

By moving from hands on learning with the area of a square to the area of a net, to the volume of a prism, students will get experience in developing and understanding the concepts of surface area and volume of a prism.

7.GM.1.1 Using a variety of tools and strategies, develop the concept that surface area of a rectangular prism with rational-valued edge lengths can be found by wrapping the figure with same-sized square units without gaps or overlap. Use appropriate measurements such as cm2.

This will be accomplished by using rulers, yardsticks, and a tape measure to gain measuring concepts then moving on to creating nets which measure surface area of a prism.

7.GM.1.2 Using a variety of tools and strategies, develop the concept that the volume of rectangular prisms with rational-valued edge lengths can be found by counting the total number of same-sized unit cubes that fill a shape without gaps or overlaps. Use appropriate measurements such as cm3.

Using individual cubes students will see how the cubes work together to form unit cubes of a larger prism. This can be shown easily using the Soma Cubes Puzzle.

Bibliography

Baker, Ann C., et al. *Conversational Learning: An Experiential Approach to Knowledge Creation*. Quorum Books, 2002.

This is the book that Kolb’s “Experiential Learning” uses. I checked it out and found it interesting even though I never quoted it directly. The overall point of the book being, even though we have profound differences, we can still learn from each other. I found this to be paramount to my being comfortable as an art teacher in a math class.

Bauer, Patricia. "Solomon Golomb." Encyclopædia Britannica. December 31, 2018. https://www.britannica.com/biography/Solomon-Golomb. Accessed July 12, 2019

Solomon Golomb’s work with polyominoes is an interesting look at how mathematicians have fun with math. His life’s work has changed the world of math, science, telecommunications, and space exploration.

Beckmann, Sybilla. *Mathematics for Elementary Teachers, with Activities*. Pearson, 2018.

I have used two different versions of the same book for information available in the previous version that is still valid, but not available in the later edition.

Beckmann, Sybilla. *Mathematics for Elementary Teachers*. Pearson, 2008.

Dunham, William. *The Mathematical Universe: An Alphabetical Journey through the Great Proofs, Problems, and Personalities*. Wiley & Sons, 1994.

Elliott, Andrew C. A. *Is That a Big Number?* Oxford University Press, 2018.

This book was suggested by our seminar leader and proved to be a very enlightening read. Elliot puts numbers and measurement in a unique light using comparison and relating ideas to many different topics of interest.

Foundation, National Science. “Mathematician and Engineer Solomon Wolf Golomb, Is a 2011 National Medal of Science Laureate.” *YouTube*, YouTube, 5 Feb. 2013, www.youtube.com/watch?v=DZ24iQ26mis. Accessed July 27, 2019.

Hanson, Helene M. *Di - Differentiated Instruction: Enhancing Teaching & Learning*. Pro-Ed,   2015.

Howe, Roger. “Area, Perimeter, Volume, and All That: A Study of Measurement”. Lecture, Yale National Initiative, New Haven, Connecticut, July 2019.

"Kehinde Wiley Studio KW Studio." Kehinde Wiley Studio. “Louis the XIV of France”. http://kehindewiley.com/works/haiti/. Accessed Jun 15, 2019.

Kolb, David A. *Experiential Learning: Experience as the Source of Learning and Development*. Prentice-Hall, Inc., Englewood Cliffs, N.J., 1984.

McTighe, Jay, and Grant P. Wiggins. *Essential Questions: Opening Doors to Student Understanding*. ASCD, 2013.

"The Mona Lisa - by Leonardo Da Vinci." https://www.leonardodavinci.net/the-mona-lisa.jsp. Accessed June 15, 2019

"Net." Math Is Fun. https://www.mathsisfun.com/definitions/net.html. Accessed June 11, 2019.

Peterson, Altair, Ouida Newton, and Chris Pearce. *Everything You Need to Ace Math in One Big Fat Notebook: The Complete Middle School Study Guide*. Workman Publishing, 2016.

Picciotto, Henri. *Polyomino Lessons*. Creative Publications, 1986.

Weinstein, Lawrence, and John A. Adam. *Gue??timation: Solving the World's Problems on the Back of a Cocktail Napkin*. Princeton University Press, 2008.

Notes

---

1. Beckmann, Sybilla. *Mathematics for Elementary Teachers*, 3rd Ed., 426.
2. McTighe, Jay, and Grant P. Wiggins. *Essential Questions: Opening Doors to Student Understanding*, 24.
3. Beckman??? Needs to be inserted.
4. Picciotto, Henri. *Polyomino Lessons,* 1.

5. Bauer, Patricia. "Solomon Golomb." Encyclopædia Britannica. December 31, 2018. https://www.britannica.com/biography/Solomon-Golomb. Accessed July 12, 2019
6. Ibid.
7. "Net." Math Is Fun. https://www.mathsisfun.com/definitions/net.html. Accessed June 11, 2019
8. Howe, Roger. “Area, Perimeter, Volume, and All That: A Study of Measurement”
9. Beckmann, Sybilla. *Mathematics for Elementary Teachers*, 426
10. Hanson, Helene M. *Di - Differentiated Instruction: Enhancing Teaching & Learning*, 1.
11. Kolb, David A. *Experiential Learning: Experience as the Source of Learning and Development*, 38.
12. Hanson, Helene M. *Di - Differentiated Instruction: Enhancing Teaching & Learning*, 2.
13. McTighe, Jay, and Grant P. Wiggins. *Essential Questions: Opening Doors to Student Understanding*, 25.
14. Elliott, Andrew C. A. *Is That a Big Number?,* 34
15. "Kehinde Wiley Studio KW Studio." Kehinde Wiley Studio. “Louis the XIV of France”. http://kehindewiley.com/works/haiti/. Accessed Jun 15, 2019.
16. "The Mona Lisa - by Leonardo Da Vinci." https://www.leonardodavinci.net/the-mona-lisa.jsp. Accessed June 15, 2019
17. Elliott, Andrew C. A. *Is That a Big Number?, 82*
18. Ibid, 38.
19. Peterson, Altair, Ouida Newton, and Chris Pearce. *Everything You Need to Ace Math in One Big Fat Notebook: The Complete Middle School Study Guide*, 326.