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Exploring Perimeter and Area with Third Graders

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Introduction

Every year when I begin to teach perimeter and area, I look through my resources and the internet for engaging ways to teach these topics to my students. There is no end to the activities out there, from using grid paper to make name banners, to creating a city with rectilinear houses, tiling, or using crackers to create a myriad of shapes. All of these activities look good and the students even seem to really enjoy completing them. Yet, I am never truly satisfied with the level of conceptual understanding and mastery my students ultimately achieve. What am I doing wrong? I've come to a decision that perhaps I have put too much emphasis on the activity itself and have not asked myself the harder questions about my goals for the activity and my students' learning. My district's math program also is a good resource for activities and background information, yet the topic of measurement, with perimeter and area within the scope, is practiced more through a procedural focus and an isolated topic. The connections to other mathematical topics and my students' background knowledge are limited, and gaps either begin to form or widen.

Through this unit, I would like to set an emphasis on developing strong lessons with a focus on spending more time in my preparations to make sure my students see this topic of measurement as it relates to mathematics as a whole, and not a separate unit of teaching. I want my students to begin to understand math and not just see it as an arbitrary set of rules and formulas to be memorized with no connection to other parts of their lives. Young children are continuously learning about the world around them and about the ways mathematics is used in their lives. "They show remarkable ability to connect experiences gained from a variety of contexts in order to make increasing sense of what they have observed. An abrupt change occurs on entry to statutory schooling when an emphasis is placed upon more formal and abstract ways of understanding and representing mathematical thought."[1] To that end, there are many questions that I will need to ask and answer about my own understanding and what I want my students to gain. By determining what skills or knowledge the students have learned prior to my introduction of this topic, I can ultimately decide what knowledge and connections I can draw on to facilitate these new lessons. Identifying what misconceptions students are holding will also help guide instruction. Furthermore, it is essential to distinguish between a misconception and an error, the former needing careful re-teaching while the latter needing focus on precision and sense making.

Demographics

The Red Clay Consolidated School District is the largest school district in Delaware and it is located in Northern New Castle County. The district has a combination of schools situated in both urban and suburban settings. The district is comprised of 29 schools servicing approximately 17,000 students About half are White, slightly over a quarter are African American, with the other quarter being mostly Hispanic, along with 4% Asian. The needs of the students are varied, with almost 15% receiving Special Education services, and 10% receiving English Language support. Additionally, more than 2 students out of 5 are classified as coming from Low Socio-Economic households. There are over 1000 teachers supporting the students in the district.

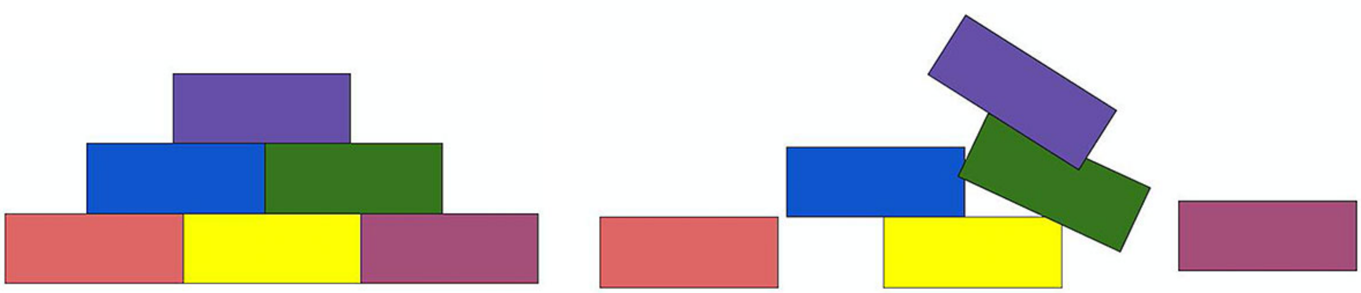
I am a third-grade teacher at Highlands Elementary in the Red Clay Consolidated School District. My school is an urban school located in Delaware's largest city, Wilmington. We are one of the smallest schools in the district with an average enrollment of just over 300 students in Kindergarten through Fifth grade. About 1 in every 6 of our students is white, with the other 5 out of 6 belonging to minority groups. Over 87% of our families are classified as coming from low socio-economic households. Most years, I am the principal educator of 20 - 24 students. The number of students that have IEP's, 504's, and ELL classifications varies each year, and I consistently differentiate my instruction and work collaboratively with specialists to meet all students' needs.

Currently my school uses the Math Expressions program for our math instruction. The most recent 2018 version has just been adopted by the district.

Background

I think of math as a bunch of blocks laying around in a big messy pile. Each block represents a topic that helps mathematical understanding begin to take shape, as students become proficient with a topic, a block gets added to the structure. If the base of this structure is solid, the blocks that are added on top are stable. If gaps or holes emerge early in a student's learning, the entire structure becomes wobbly and unpredictable.

Figure A



If I focus my instruction solely on procedures and formulas, my students' understanding can get wobbly, quickly. This is why I allow students to investigate, manipulate, and contextualize math. I believe that by providing students with rich and varied representations of mathematical concepts, I am helping students make sense of the math problems. I need to connect the mathematical concept to my students and give them a way to understand the process for determining perimeter and area. My students often confuse the concepts of perimeter and area because the two topics are almost always presented together as a set of procedures and they misinterpret the measurement ideas. In this unit, my students will develop an understanding of perimeter as they construct pens for pets, create frames for their art work, and determine how much fencing we need to enclose our recess yard to prevent the ball from rolling into the street. I will highlight the concept of length unit to measure the length of a path that fits around a shape. Additionally, students will develop an understanding of area as they tile rectangles, create posters to cover a variety of classroom surfaces, and move to measuring the base and height of rectilinear shapes.

Foundations

Students begin in kindergarten to count to tell the number of objects. They are developing one-to –one correspondence, as they point to an object and they state a whole number, the stated number thus represents the number of objects present. They move on to master the skill of counting forward beginning from a number within a known sequence instead of having to always begin at 1. As first graders, they move to express the length of an object as a whole number of length units by laying multiple copies of a length unit end to end with no gaps. This foundational work occurring in kindergarten and first grade helps to set the stage for measuring the side length of an object by using non-standard units and then standard units, using tools such as rulers, as second graders. When these students arrive in my third grade classroom, they should have experience with using tools such as rulers to measure the length of an object. They can compare the lengths of two objects using both non-standard units such as paperclips, or standard units such as inches. One object that has more paperclips or inches is longer than the other object. I will build on these understandings or provide activities and re-teaching lessons to sure up these essential base building blocks. If my students have gaps in their learning with these topics, it will lead to larger gaps, misconceptions, and frustration.

Perimeter

Perimeter is made up of the Greek prefix “peri” which means “around”, “about’, ‘enclosing, “surrounding’ and the Geek root “meter” with the meaning “measure”, thus making up the definition, to measure around. Therefore, this leads us to the understanding that perimeter is a measure of the length of a side or path that surrounds or outlines a two dimensional shape. To find the perimeter of a figure, one measures the lengths of all the sides, and adds them up. My students are going to explore this topic using a variety of manipulatives and different polygons. As they participate in the investigation, they will be using problem solving steps to practice Mathematical Practice 1; make sense of the problem and persevere while solving it. As students work through these steps, they are connecting to the problem, making and testing conjectures, and determining what works or doesn’t work and why. This process is what is helping them make connections to their previous learning and experiences and is helping to solidify understanding.

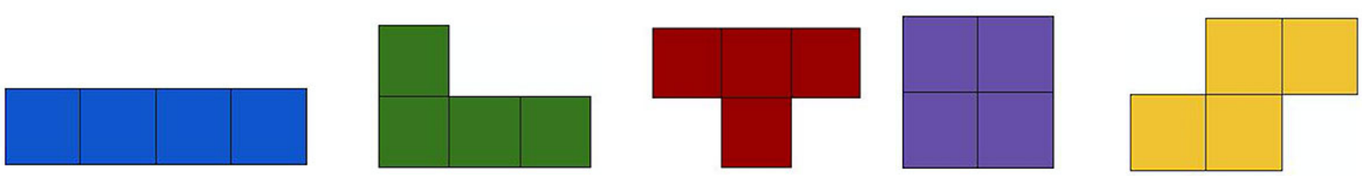
Our initial investigations will involve different manipulatives that will enable students to measure side lengths of a variety of items found around our classroom. Students will choose an item and a manipulative and work to measure the item by placing these non-standard units end to end and then counting the manipulatives used. This exploration time will help me to assess students’ ability to understand the process of measuring end to end without any gaps, it will give students time to freely investigate and begin to form theories about their findings. After a set amount of time, about 10 minutes, students will participate in a class discussion that is student led. Prior to this unit, students in my classroom have participated in many math discussions, which I will describe more thoroughly later in this unit. Another activity that will help to introduce the concept of perimeter to my students uses dot grid paper. Students will be asked to draw a horizontal line segment connecting two dots. This line segment represents 1 unit of length. The students will draw a horizontal line segment 4 units long. Next they will draw a vertical line segment 3 units long. Students will participate in math discussions to explain how many line segments they have drawn. Students will be provided with an addition chart and will utilize this tool to add the line segments. Then students will be instructed to draw to more line segments to form a rectangle. Again a math discussion will take place as students determine the total length of line segments used to draw the rectangle. Some students will add each length as they make their way around the rectangle, 4+3+4+3=14 units. Others may group the horizontal line segments together then the vertical line segments, 4+4+3+3=14 units. Some students may add the horizontal line segment to the vertical line segment and then multiply by two, 4+3=7 7x2=14. Or, students may double the horizontal line segments and then the vertical line segments and add the two products together, 4x2+3x2=14. All of these possibilities will help to solidify that students are calculating the sum of the line segments.

I will also have my class work with polyominoes. A polyomino is a collection of unit tiles, put together using some simple rules:

1. Any two tiles that touch either touch only at a corner, or along a whole edge of each; and
2. The figure is connected, in the sense that you can go from any tile to any other tile by passing through edges (not corners).

A domino has 2 tiles in the collection, a triomino has 3 tiles in the collection, a tetromino has 4 tiles in the collection, and a pentomino has 5 tiles in the collection (that is as far as we will go in this unit but the possible number of tiles is limitless). The purpose of using these “ominoes” allows us to create figures of a given area, but possibly different perimeter. Students can create polyominoes with different shapes. I will model the activity using four tiles to create tetrominoes, with students following along at their work stations. This will give me an opportunity to discuss some geometric terms like symmetry and translations, yet this will not be a focus of the lesson. Shapes that can be made to coincide using reflections, translations and rotations are considered the same. In Figure B, you will see the five different possible tetrominoes.

Figure B: Tetrominoes



Students will investigate and calculate the perimeter and the area of each of these figures and discuss similarities and differences they notice. Students then will continue the investigation with 5 tiles, pentominoes. After the investigations with the ominoes, students will move on to measuring items with rulers to determine the side lengths and then perimeter. As we make this switch, it is important to assess the students’ abilities with these tools and provide re-teaching activities to students who need it. One common error is to measure starting at the 1 on the ruler and not at the 0.

Area

Area is the amount of space inside the boundary of a flat, two dimensional object, such as a rectangle or a polyomino. For most of the figures in this unit, this can be calculated by tiling a shape with unit squares; the area is the number of unit squares enclosed by the figure. The tiles should not overlap, and should completely fill up the figure, leaving no uncovered parts. I will present my students with larger polyominoes of the same area but different shapes and different perimeters, so that they can see that very different looking shapes can have the same area. I will also have them find perimeters and see that they are different. This will help establish that area is distinct from shape, and also from perimeter.

After some work with irregular shapes, I will have my class study rectangles with small whole number side lengths. We will see that they can be paved with regular arrays of squares. I will have students find the side lengths of the rectangles, and we will establish that the area is the product of the side lengths. One of my major intentions is to connect much of the work of third grade to multiplication and develop their conceptual understanding and fluency. Area of rectangles and squares can be helpful tool toward this goal.

Jo Boaler encourages teachers to use an inquiry version of common area tasks, "Instead of asking students to find the area of a 12 by 4 rectangle, ask them how many rectangles they can find with an area of 24."[2] Doing this makes students think about spatial dimensions and relationships and then begin to think about what happens when one dimension changes. Students are making connections, conjectures, and are engaged and excited about their thoughts. I try to use this approach as much as I can.

Connection to Multiplication Fluency

As I have mentioned several times above, understanding multiplication is a major focus of mine for my third graders. However, I do not force my students to memorize facts or practice through timed tests. In contrast, I focus on helping my students develop number sense as they interact with numbers and learn to use numbers flexibly. If memorization and speed are in control, students begin to cling to formulas or formal procedures. A student who relies on memorization may have no problem recalling the fact 8x6=48, however, they then struggle to solve 8x16=128. A student who has a strong number sense mindset may quickly see one solution as 8x10+8x6=128.

In Jo Boaler's book Mathematical Mindsets, she explains a brain process called compression.[3] Learning new aspects of mathematics that you have no previous experience with takes up a large amount of space in your brain. You need to spend a lot of time thinking about how it works and how these new ideas relate to each other. Mathematics that you have learned completely and know well, such as addition, take up small, compact space in your brain. You can use it without thinking about it. (insert citation) My goal is to help my students see mathematics conceptually and to enjoy "playing" with numbers. Compression will happen with increased familiarity and understanding.

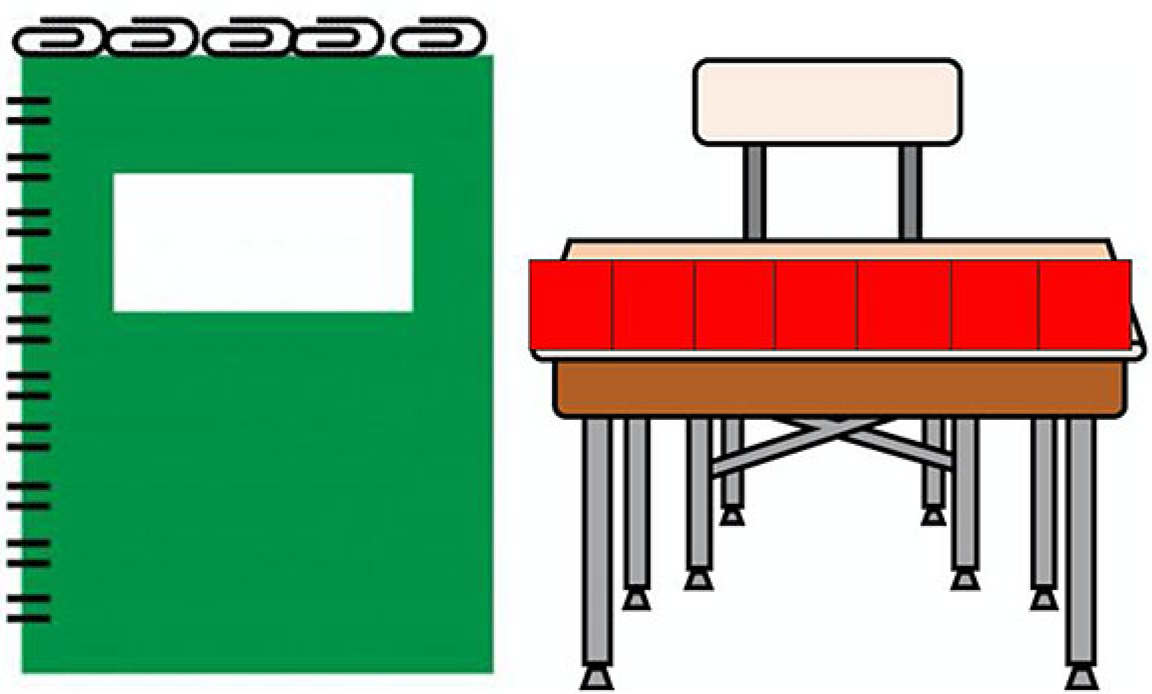
Classroom Activities

Perimeter

Hands on manipulatives

Manipulatives can be essential to providing engaging lessons that have proven effective. Learning is enhanced when students can gain knowledge and understanding while actively participating in the lessons. The physical movement and interactions with the manipulatives enable students to build their conceptual understanding as they move from concrete representations to abstract thinking and reasoning. This is why I will begin this unit with having students measure a variety of objects using non-standard units. Students will be placed into small groups of 2 to 4 students. For this activity, I will not place too much attention on the composition of the groups. They will be exploring measurement and there will be multiple points of entry for understanding. Before beginning, I will review the proper way to measure with the units available, placing each unit end to end without gaps or overlapping. Students will have a recording sheet. On this sheet, they will record the items that they measured, the units they used to measure, and the number of units that were used to measure the item. For the purpose of this unit, we will focus mainly on whole numbers of units. They will need to find at least three items to measure, they can use the same non-standard unit for each or they may change if they wish to. A mixture of the non-standard units will be placed into bins, a package of paper clips, 2” craft sticks, 4.5” crafts sticks, 1 inch tiles, and students will be allowed to choose any other materials they are interested in using. Each student in the group can measure their own three items or they can measure the items as a group on this first round. There will be a time limit of 5 minutes for this initial investigation.

Figure C: Non-standard measuring

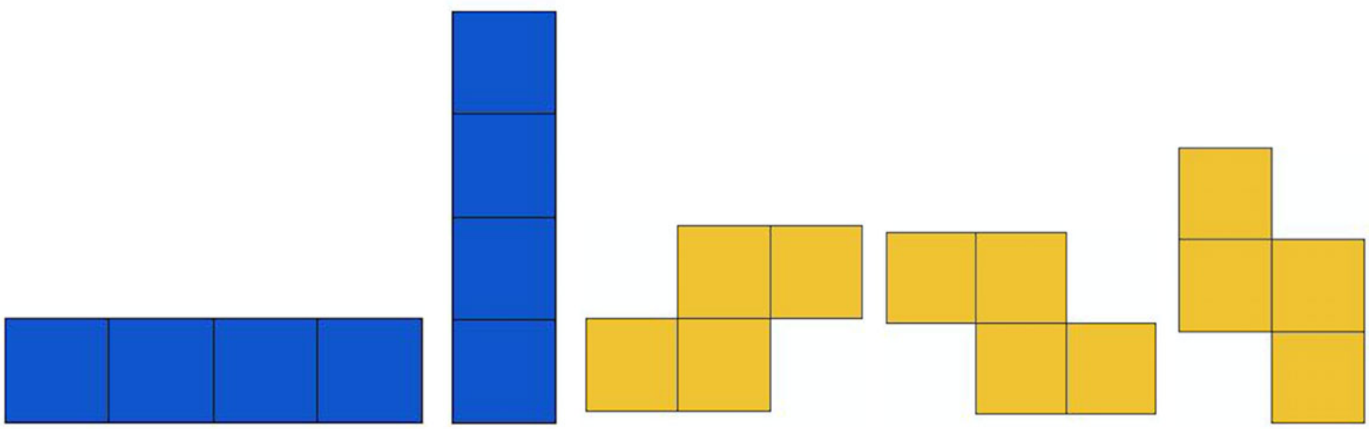


At the end of the five minutes, the group members will look at their papers and discuss what they notice. Do they see any patterns? Which items were larger/smaller? What units did they use, why? They will also discuss what questions they encountered as they worked and what questions they still would like answers to. Then each group will measure the exact same items they measured in the first round, but during this round they will use different measuring units. They will again have a time limit of 5 minutes. At the end of the time, they will again discuss with their group members what they have noticed. If a group uses the same unit to measure different object, they can make statements about comparing lengths. If two different groups measure the same object using different units, but the same unit for each object, their length comparisons should agree' although their numbers will not. Were any measurements different? Why? The final round will then take place, the group will decide on just one measuring unit and will measure all three of the same items from the last round. Again they will have a time limit of 5 minutes. They will participate in a math discussion to reflect on what they have learned through the three rounds. I will be guiding them to make the connection for the need of standard units. Each of the groups will present their investigations and their findings to the class. At the end of the lesson, students will write in their math journal. They will explain their process for their investigation. They will record something that they have learned for the day and/or something they are confused about and want further investigation time or instruction time.

Polyominoes

As stated above, I will be using polyominoes to further explore perimeter with my students and to bridge into the concept of area. I will present a whole class lesson on using the tetrominoes (4 pieces). Students will come and sit on the rug in the front of our classroom. I will have large 1-inch grid chart paper. Each child will have their math board and four tiles. The math boards I use are a part of our Math Expressions math program and provide a flat surface for the students to work on. As I draw a tetromino on the grid paper, students will create the same figure using their tiles. I will explore the translations of the figures and quickly discuss symmetry, rotation, and reflections and show the fact that the figures are congruent and have not had any physical size changes. See Figure D.

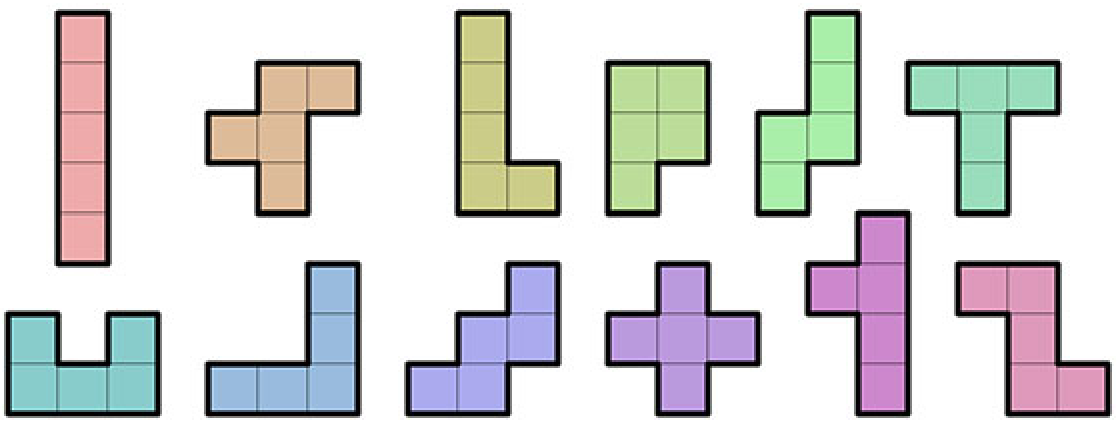
Figure D: Tetrominoes



I will proceed to make all of the five different possibilities for the tetrominoes following the same procedure. I will then pose the question, "What similarities do you notice in these figures?" I will record what the students share. If no one shares any information about the perimeter, I will ask, "How could we tell what the distance around these figures is?" We will then determine the perimeter or each figure, focusing on the length of the sides of the squares that are on the outside of the figure. Since two of the tetrominoes are rectangles, they can use their old method for them, but counting for the others. They should check that for the rectangles, counting gives the same result as before.

The next step will be for the students to investigate the different figures they can create using five tiles, pentominoes. Each student will be given one more tile and their own grid paper. They will work in pairs to try to find all of the combinations for the pentominoes, there are 12 distinct figures that are not transformations of others. See Figure E. After about 15 minutes, I will begin to invite teams to the 1-inch grid chart paper in the front of the room for them to record a pentomino. This process will continue until we have discovered all 12 figures or they are stuck and then I will provide any remaining figures. Students will then record the perimeter for each figure on their grid paper. Students will again participate in our math discussions. They will discuss the procedure they used to find new figures, what they have noticed, what they wonder, and explain any patterns they see.

Figure E: The 12 Pentominoes

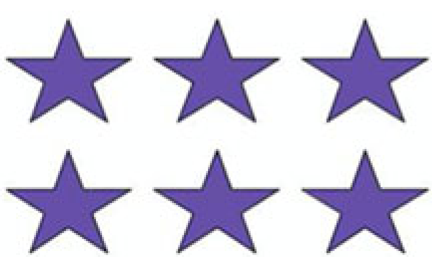


Area

Creating Rectangles

As part of our multiplication study, students have become familiar with creating arrays. Students can make connections to this concept when they have a visual representation of the multiplication expressions. An array arranges pictures into rows and columns. In Figure F the stars are organized into 2 rows and 3 columns. It can also be described as a 2 by 3 array. Students learn to associate the equation 2x3=6 with this array.

Figure F: Arrays



Using this knowledge, students will participate in an activity in which they create many arrays of quadrilaterals on grid paper using dice. Initially I will have them work with their partners and begin to investigate the relationship between the numbers they roll and the arrays they create. The goal of this activity is for the students to make the connection that they can calculate the area by multiplying the number of rows by the number of columns, and finally move to a more exact understanding that the number of squares in a row, with the length of the horizontal sides, is the length of the base and the number of squares in a column, with the length of the vertical sides, is the height. The procedure for the activity is, students will work in pairs. Pairs will need one piece of grid paper, two different color markers or crayons, two number cubes, and a recording sheet. Partner 1 will roll the number cubes and use the numbers to make an array and draw it anywhere on the grid paper. The goal is to fill the grid paper and have your color arrays take up more area than your partner’s. After player 1 draws the array, they will record the equation on their recording sheet. Students will be able to practice their multiplication facts in an authentic way that enables them at the same time to make a connection to the measurement of area. Additionally, this type of activity enables students to think about the meaning of the number facts and what these facts represent visually and spatially.

Relationship between Perimeter and Area

Perimeter of 24

Students will be given a rectangle with a perimeter of 24 units. For differentiation, students can use tiles, grid paper, create arrays, or just draw rectangles with the dimensions written on their drawings. I will ask them to make as many rectangles as they can, keeping the perimeter at 24 units. Students will record the area of all of the rectangles they create. This activity is an excellent place to incorporate the addition chart. As students search for combinations of 24, they can find these patterns using this tool. In Figure G, you can see how students would be able to find side lengths that have a sum of 12.

Figure G: Addition Table

+	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12	13
2	2	3	4	5	6	7	8	9	10	11	12	13	14
3	3	4	5	6	7	8	9	10	11	12	13	14	15
4	4	5	6	7	8	9	10	11	12	13	14	15	16
5	5	6	7	8	9	10	11	12	13	14	15	16	17
6	6	7	8	9	10	11	12	13	14	15	16	17	18
7	7	8	9	10	11	12	13	14	15	16	17	18	19
8	8	9	10	11	12	13	14	15	16	17	18	19	20
9	9	10	11	12	13	14	15	16	17	18	19	20	21
10	10	11	12	13	14	15	16	17	18	19	20	21	22
11	11	12	13	14	15	16	17	18	19	20	21	22	23
12	12	13	14	15	16	17	18	19	20	21	22	23	24

Area of 24

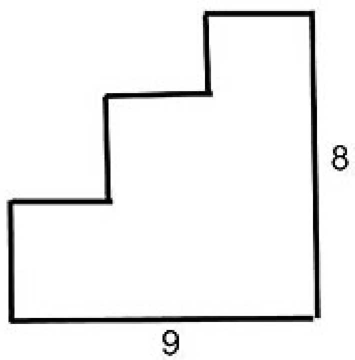
Students will be given a rectangle with the area of 24 square units. For differentiation, students can use tiles, grid paper, create arrays, or just draw rectangles with the dimensions written on their drawings. They will need to make as many rectangles as they can, keeping the area at 24 square units. Students will record the perimeter of all of the rectangles they create. This activity is similar to the activity that was completed with our perimeter study, that activity kept the perimeter constant and this one will keep the area constant. I have given some thought to completing the activities together and having students compare and contrast their findings but have not yet made a final decision.

How Many Rectangles

Finding areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas has been a difficult standard for me to address with my students in an engaging, authentic way. The following activity aims to rectify this problem. Students will be given a sheet of paper with Figure H. They will be given this problem to solve.

Divide the figure below into three different rectangles. Students must use the entire figure and can have no blank spaces. Once you create your rectangles, determine the perimeter and area for each of the designated rectangles. All sides will be whole numbers, you will need to make your best estimate and determine if your estimate is correct be using strategies you have learned.

Figure H



Tessellation

This activity will bring together our work with tetrominoes combining it with a focus on area. Each student will receive a piece of grid paper. They will be making an art piece in which they will tessellate the tetromino figures. A tessellation of a flat two dimensional figure occurs when you use tiles of one or more geometric shapes with no overlaps or gaps. Students will create a tessellated pattern using the tetromino figures. They must use as least two different types of the tetromino figures but can use more if they want. Their finished mosaic must be in the shape of a rectangle. Students will draw a frame around their mosaic and then will record the perimeter and area of their art piece. Students will compute the area, which will be the sum of the areas of the included tetrominoes. Students will compute the perimeter, which they will notice will be much smaller than the sum of the perimeters of the included tetrominoes. There are many possibilities for extensions and differentiations, for example, I could challenge students to make a specific sized rectangle; Can anyone make a 3x4 rectangle?

Comparing Rectangles

I am fortunate enough to work in a school that has one-to-one technology integration. Each of my students has a chromebook to use every day. This activity will be described with that in mind, however there are ways to adapt it to fit a classroom with limited access to technology. Students will work in pairs. They will choose something in the school to measure. I will allow some students to go to other locations in the school to find something they want to measure and return to the classroom, but this activity could be limited to items in the classroom only. Students will be asked to measure to find the perimeter and the area of their chosen object. They will use a standard unit of measurement of their choice. Once they record their measurements on to the form, they will take a picture of their object and upload it into our classroom google slide presentation. I will take the form with the measurements and post them at the front of the room. Once all partner teams have completed the activity, I will show a picture that one team measured. All of the other students will be asked to look at the measurement forms posted on the board and they will need to decide which measurements fit the object being shown. This will continue until all items have been shown. We will then check and discuss which measurements go with which picture. This activity will help students to understand solve real world mathematical problems involving perimeter and area.

Multiplication Charts

I will make some explicit use of the multiplication table to help my students make the connections between multiplication and geometry and measurement. As students create rectangles on the multiplication chart, they can see the dimensions of the rectangle as well as the area. Side length of 4 units with a side width of 2 units will create an array with 4 columns and 2 units in each column for a total of 8 units.

Figure I: Multiplication Chart

×	1	2	3	4	5	6	7	8	9	10
1	1	2	3	4	5	6	7	8	9	10
2	2	4	6	8	10	12	14	16	18	20
3	3	6	9	12	15	18	21	24	27	30
4	4	8	12	16	20	24	28	32	36	40
5	5	10	15	20	25	30	35	40	45	50
6	6	12	18	24	30	36	42	48	54	60
7	7	14	21	28	35	42	49	56	63	70
8	8	16	24	32	40	48	56	64	72	80
9	9	18	27	36	45	54	63	72	81	90
10	10	20	30	40	50	60	70	80	90	100

If this is combined with use of the addition table, then students can see how rectangles with the same perimeter can have very different areas.

Teaching Strategies

Math Discussions

Engaging student in math discussions has a multitude of benefits.

Re-voicing: by teachers- sometimes students are unclear in their descriptions and explanations, if they're unclear, other students remain unclear. If only students who can explain themselves clearly participate, it does not build participation or understanding. I need to clarify in a way to continue to engage and include my students. One way that I have found useful in math classroom is to restate what has been said and then ask my students to respond and or verify if that is what they stated: "So you're saying". This helps me to determine whether the student does or does not understand, and provides an opportunity for me to guide the discussion. - It also allows me to clarify when a student adds to the discussion but is not clear or not understood by the other students. This makes ideas of one student available to others, give students time to hear what is being discussed more than once, helps students to develop a critique or viable argument, and allows students to change their thinking. By asking students, "Can you explain what he/she said in your own words?"-- I give students another opportunity to hear discussion, more time to process, improves engagement and the likelihood that students are listening. Over time, students come to realize that others are listening closely to what they say, and begin to make more effort to contribute in positive way.

Math Circles

I had a big ah-ha moment as I wrote another geometry unit a few years ago. I frequently use literature circles in my English/Language Arts classes. Literature circles are an excellent strategy that infuses student centered inquiry with collaborative learning. Using this strategy encourages students to take responsibility for their learning based on the plan and choices they make. Students choose their own reading materials, lead the discussion, and engage with the texts and each other in a positive, authentic way. Why can't we create the same experience in math class?

I have now developed Math Problem Solving Circles where small groups of students engage with a problem and have designated roles that will help students to make sense of the problem, facilitate discourse, and decide which tools or strategies to use to help solve the problem. I create student groups of 3 to 4 students, then present the problem, and assign roles. I have developed four roles to best suit my classroom needs, they are; Reporter, Questioner, Strategizer, and Reflector. Each role has a purpose, yet the purpose is not to isolate the problem into pieces, the purpose is to provide all students the opportunity to access a problem, comprehend what the problems is asking, discuss a variety of possible strategies, and make connections to other mathematical concepts. The following descriptions provide a starting point and have suggestions for students' engagement. Students will be using a KWCSR chart while participating in a Math Problem Solving Circle. What a KWCSR is, is explained in detail below.

REPORTER: The reporter's role is to read the problem to the group and to fill in the "What do we KNOW" portion of the chart. The reporter will also lead the group's presentation to the rest of the class.

QUESTIONER: The questioner's role is to lead a discussion about what the problem is asking, what needs to be solved. The questioner should fill in the "What do we WANT to know" portion of the chart. The questioner should be asking throughout the process, does this solution make sense? Are we solving the question the problem is asking? Have we used the correct units? Do we need more information? Have we ever solved a problem like this before?

STRATEGIZER: the strategizer's role is to lead the discussion about which strategies would be most efficient to solve the problem. The strategizer should fill in the "What STRATEGIES will we use?" portion of the chart. They will help to ensure that group members are using a variety of strategies.

REFLECTOR: the reflector's role is to lead the discussion about following the plan the group is setting up and to think about its efficacy. Some questions the reflector can ask the group are; Will this plan help find a solution to the problem? Is there another way we could solve this problem? If there are multiple ways to solve the problem, is one way better than the other? The reflector should fill in the "What is the ANSWER?" section of the chart. After the problem is solved, the reflector will help lead the discussion about what each group member learned and how this new information can be used again by asking questions like, What have we learned? How can we use this information again? I believe this type of approach to problem solving will enable more students to become successful as they tap into their collective knowledge. I believe a barrier to problem solving for many students is the lack of experience and the issue for students of not knowing where to start

Problem Solving Process

In my experience, when I talk about problem solving, many of my colleagues think I am speaking of word problems. I take a minute to explain to them the difference; word problems are math exercises that embed numeric equations into a variety of questions, and problem solving involves implicitly teaching students strategies to solve a variety of problems. Yes, word problems are an important part of problem solving and students do need to that can be useful to follow in order to become successful when they begin the problem solving process. They were most famously formulated by George Polya.[4]

In my classroom, I have found that Polya's heuristics[5] are appropriate and useful tools for my students in order for them to begin to understand a problem and what is being asked from the problem. These tools can be: draw a picture, look for patterns, make a chart or graph, guess and check, work backwards, make a list, choose an operation. Each strategy is introduced along with several problems that lend themselves to that specific strategy. I also provide my students with a graphic organizer to help them organize and make sense of the problems. I am not a big fan of teaching key words because there are always many, many problems that do not fit the key word rules, and it short circuits the thinking process involved in understanding the problem. I think this also teaches students to focus on a set of words and not to think holistically of the problem. This approach is both inadequate and misleading.

Polya's ideas include: Understand the Question: students need to read the questions carefully and develop an understanding of what the question is asking. This, by far, is the most important step. Students need to understand the question. Many misconceptions and errors began when students answer a different question than what was being asked.

Choose a Plan: as students begin to work with the problem, they need to decide which strategy will best aid them.

Try your Plan: this is the place in the problem solving process that students put their ideas into action. They are thinking about each step as they proceed and continue or make changes if necessary.

Check your Answer: Once students come to a solution they need to ensure their response is accurate. They should ask themselves some questions to guide their thinking. Did you answer the question that was asked? Does your answer make sense? Did you remember to use the correct units? Then they should redo the problem another way and try to get the same answer and check your math work for small errors

After the solution has been determined students should then, Reflect: Think about what you have done and what you have learned. After a reasonable number of problems have been dealt with, this is also an opportunity to deepen understanding by thinking about how this problem relates to previous problems. Also, students should ask themselves if there is anything they are still confused about.

Understanding the Problem using a KWCSRS

Using the Standards for Mathematical Practice as a guide, I have worked to develop strategies that aid my students as they make sense of problems and persevere in solving them. My students use a revised KWL form specifically adapted to help in my math classroom. We call the graphic organizer a KWCSRS chart. The K section asks students; “What do you KNOW about the problem?” This enables students to clarify the information within the problem and provides them a place to record information they will need to solve the problem. They must also make decisions to justify what information is needed to solve the problem and what information is superfluous. The W section asks students; “What do I WANT to find out?” Many times my students get confused as to what they are actually being asked in the problem and this gives them a place to write it down and focus on what they are solving. The C section asks students; “Are there any CONDITIONS, rules or tricks I need to look out for?” The S section asks students to list two to three STRATEGIES that they believe will help them solve this problem. Multiple strategies are listed so students know if one strategy is not working they should try another. The R section is the place where students REFLECT on their solution strategy and record their answer. In this section students will review their solution and make connections to other problems they have seen. I was finding that many of my students would work hard to solve a problem and then never finalize their work. The final S section leaves room fro students to write and explain their SOLUTION. This space reminds them to refer back to the W section and make sure they have answered the question they were asked.

Math Journals

Math journals are an excellent way to provide students with an opportunity to express their thoughts, understanding, confusions, and questions. Some teachers use math journals as a work space for students, they open to a new page and show their work for the problems they are working on that day in class. Other teachers use the math journal as a place to present daily or weekly tasks. I want my students to begin to use their math journals in a more purposeful way and as a tool to build and reinforce their understanding. I will explicitly teach my students early in the year on how to take notes to help them remember important aspects of lessons. I will encourage students to use the math journal as a work place. Let’s face it, the work areas in their workbooks is never adequate. I also envision this as a reflective space for the students to make connections to what they are learning and voice any confusions they still may have. If students want to share their journals with me, I can respond to their questions and reflections. This will help me to determine who is on the right track and who needs some additional support.

Appendix A

Standards

The main focus of standards addressed in this unit will involve many of the third grade Measurement and Data standards and will include Operations and Algebraic Thinking standards.

Geometric Measurement: understand concept of area and relate area to multiplication and to addition.

As students work with the tetrominoes and pentominoes they will; 3.MD.C.5-Recognize area as an attribute of plane figures and understand concepts of area measurement. Reinforcing, 3.MD.C.5.A-A square with side length 1 unit, called a “unit square”, is said to have “one square unit” of area, and can be used to measure area will enable my students to develop a deeper understanding of the importance of specifying units. In order to ensure students are correctly addressing and calculating area measurement, they will need to understand 3.MD.C.5.B-A plane figure which can be covered without gaps or overlaps by *n* unit square is said to have an area of *n* square units. When introducing area, students will 3.MD.C.6-Measure area by counting unit square (square cm, square m, square in, square ft, and improvised units). In order to understand area and perimeter conceptually, students will need to 3.MD.C.7-Relate area to the operations of multiplication and addition. Many of the exercises and activities developed in the unit will allow students to 3.MD.C.7.A-Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths. As students move forward in their conceptual knowledge, they will move to 3.MD.C.7.B-Multiply side lengths to find areas of rectangles with whole-number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangle areas in mathematical reasoning. Which will then lead them to 3.MD.C.7.C-Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths *a* and *b+c* is the sum of *a x b* and *a x c*. Use area models to represent the distributive property in mathematical reasoning.

3.MD.C.7.D-Recognize area as additive. Find areas of rectilinear figures y decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize perimeter

This is the only third grade standard that mentions perimeter specifically and will be addressed in many of the activities that will enable me to separate the concepts of perimeter and area 3.MD.D.8-Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding and unknown side length, and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.

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Endnotes

1. (Pound 2006)
2. (Boaler 2016)
3. Ibid
4. (Polya 1945)
5. Ibid