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Pentomino Garden: An Explorative Unit in Measurement, Manipulatives, and Gardening

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by Jamie A. Griffin

Introduction

"With many under served, underachieving minority students; her students invariably believe they are bad at math, and if you looked at their past performance, you might be tempted to think so too."[1] While this quote is not about me or my students, I was able to relate to it. I teach first grade at Ross/Woodward a public school in New Haven, CT. The majority of my students are Hispanic and African American with more than half of them qualifying for free lunch. My young students come to first grade with misconceptions and fears about mathematics. A goal of mine for the last three years has been to help my students overcome these fears and feel confident in math. "I am passionate about equity. I want to live in a world where everyone can learn and enjoy math, and where everyone receives encouragement regardless of the color of their skin, their gender, their income, their sexuality, or any other characteristics. I would like to walk into math classrooms and see all the students happy and excited to learn."[2] Unfortunately, as a girl, I was not encouraged in math while I was in school. I did not feel confident in my math abilities until I became a teacher myself. For a long time math has been treated as an elite subject. Girls and other minority groups are often not expected to be "good" at math. This is unacceptable, all students regardless of race and gender can achieve in mathematics. I realized that a lot of the fears that I had surrounding the topic from when I was in school could be projected onto my students. This math anxiety was likely to stay with my students long after they left my classroom. I realized that in order for my students to feel confident in and enjoy mathematics it would need to be rigorous and relevant to them.

One major misconception that my students have when they come to first grade is that in order to be good at math you have to be fast at math. They directly correlate this to fact fluency in addition and subtraction. "Another serious problem we face in math education is that people believe that mathematics is all about calculating and that the best mathematical thinkers are those who calculate the fastest. Some people believe something even worse- that you have to be *fast* at math to be *good* at math."[3] Students who are not fast at math facts often feel less confident in math and therefore, do not enjoy it as much as other school subjects. We spend most of the year focusing on addition and subtraction. However, the focus of these units is strategies and not speed. The strategies are presented to my students to get them into the right mathematical mindset. I want them to realize that math is much more exploratory than they think. This is the basis of my curriculum; exploratory yet rigorous math.

Measurement is a great subtopic of mathematics that can be exploratory for students. In first grade, students primarily study how to measure length. I began to think about how I could make this more engaging for my students. I realized that perimeter is a linear measurement and began to think about implementing this concept into my classroom. "It makes sense to start by discussing that most basic, most fundamental of measures: distance."[4] When we measure length, we are actually measuring distance. I decided I would begin to use this term when teaching length to my students. I determined that if my students could measure distance and perimeter, then they could explore area as well.

"Our brains need help when thinking about areas because so much of our early learning about numbers and measurement is linear. There are no tools in the set of school mathematical instruments to measure area: areas are almost always the result of calculations."[5] Students learn how to measure length first since it is linear. Measuring distance is easily related to counting as students count the distance between zero and the edge of the object they are measuring. Area is more complex, which is why it is saved for later grades. However, with the proper exploratory activities and strategies young students can complete tasks related to area. Area is commonly taught with multiplication. In first grade we begin to explore "repeated addition", this is the beginning of multiplication. I realized that my students could complete area tasks if they were related to repeated addition.

Area and perimeter are not expectations for first grade, however, they can be related to other tasks that are incorporated in first grade. With the proper guidance, manipulative work, discussions, and tasks, I knew my students could explore area and perimeter. These concepts would prepare them for higher grades and standardized testing. I was excited to give my students a strong foundation in these concepts and provide them with opportunities that they would enjoy at a rigorous level.

Rationale

I teach first grade at Ross/Woodward Classical Studies Magnet School in New Haven. Our magnet theme is Classical Studies where we focus on educating the whole child. "We support a rigorous classical curriculum through which students engage in studies of great work over time. In pursuit of excellence, Ross Woodward focuses learning on that which is timeless and of lasting significance."[6] I knew I wanted to design a unit that would create opportunities for my students to engage in experiences that would last long after first grade. For this reason, I knew that I would want them to actually create real life versions of what we create with manipulative tools. A special feature of my school is that we have a courtyard that can be used for gardening. My students and I have been cleaning up this space and turning into an outdoor classroom. Since we are the ones who have been cleaning the space, we get to determine how to utilize it. For this reason, my students will be able to complete the end of unit activities "Tetris Garden" and "Polyomino Walkways" in our courtyard. If your school does not have this type of space, this unit could be adapted to any available and accessible grassy areas near or around your school with approval of administrative staff for these activities.

Another component of teaching the whole child is that we focus on the importance of Socratic Seminars. "Socratic questioning and student dialogue are the central teaching and learning device at Ross Woodward. All teachers use relatively little didactic teaching; instead we use product-oriented coaching for the majority of our instruction."[7] In the seminar model, students are guiding the discussions and sharing their reasoning with

each other. What I have noticed is that my students struggle the most during our math seminars. They struggle to use math vocabulary correctly and have trouble articulating how they got their answer. Due to this, there is a heavy focus on math talks in my curriculum.

I had three learning goals for this unit. The first was that my students would learn how to use a ruler. While using a ruler is not an expectation based on the Common Core Math Standards for first grade, I felt that my students should learn this skill as a counting strategy as well as a tool for measurement. This curricular unit would come after our regular measuring unit from Singapore Math which focuses on non-standard units of measure. This curricular unit serves as an extension of their learning which focuses on rigor and relevance for my students. The second goal was that my students would be able to comprehend the array structure of a rectangle, with whole number side lengths can be decomposed into an array of unit squares. This is commonly taught as a strategy for students to learn multiplication. We would use repeated addition with the array structure, which would set my students up for success when they begin to learn multiplication. My last learning goal is that students should comprehend and explain the difference between area and perimeter when measuring. Again, this is not an expectation for first grade, but I am confident that my students can grasp them. While these three goals seemed lofty and well above grade level, I knew that I could help my students reach them if I provided the proper strategies to help them.

My unit also had smaller strategies that appear throughout it to help my students achieve the learning goals. These strategies are measuring with manipulatives, math talks, cooperative learning, and gardening. They will help my students reach full levels of understanding. They will ensure that my students can apply the skills in their final activities and projects. My hope is that these strategies will make the math concepts and learning goals relevant to them. The strategies and goals will also help my students develop spatial awareness, which some of them are lacking. This curricular unit will extend the learning from our district given curriculum and make math relevant to their lives.

Measuring Distance as Counting

Young students often struggle to see the connection between math and measurement. A common misconception that my students have is that math is confined to just addition and subtraction. While my students are measuring, they do not realize that the strategies they are using are mathematical. In first grade measuring can be easily connected to counting. “The connection of arithmetic to geometry through measurement both enlarges the conception of arithmetic, and provides concrete and conceptual tools to help students think about arithmetic.”[8] In the primary grades' measurement can be explained as counting: how many units of the relevant measure will fit in this object or quantity? This will help my young learners realize that measurement is indeed related to mathematics. The first grade Common Core Math Standards state that “the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps.” Students determine the number of same-size length units by counting them. They are counting the distance between the start point and the end point. They can then state the number of units when explaining how long the object is.

Roger Howe states that first grade students should make the connection between counting numbers and measurement numbers. “The most basic and probably the simplest type of measurement is linear measurement: measurement of length or distance.”[9] Distance can be measured by counting the number of

inches from one end to the other end of an object. Kindergarten students spend a majority of the year working on their counting abilities and should be able to count from 1 to 100 by the end of kindergarten.[10] Unfortunately, many students come to my first grade classroom being unable to count to 100. Some students are not even able to count to 20 without experiencing difficulty. When using nonstandard units of measure, the numbers can be contained. Some students need practice with numbers less than ten, other students can count much bigger numbers. I will be able to differentiate tasks based on the needs of my students. Eventually, when we use rulers all students will get repeated practice with counting up to twelve since there are twelve inches in a foot (the length of a ruler). Overall, measurement can greatly help my students become better at counting by implementing new strategies for them to use.

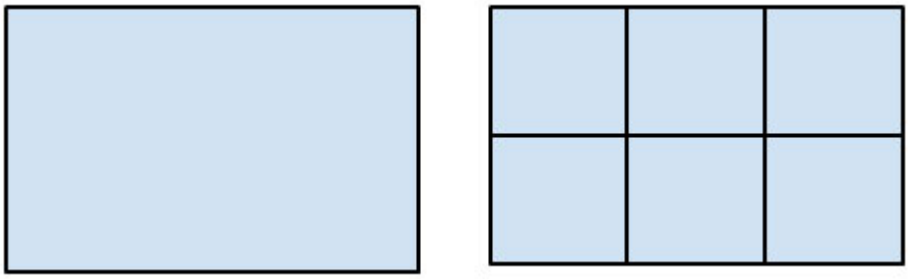
Using a ruler is not the expectation in first grade but will add relevance for my first grade students. "This kind of work with lengths can strengthen the learning of arithmetic by reinforcing symbolic work and work with unstructured collections of objects. Equally important, it should get children used to the idea that measurement is a natural domain for application of number ideas"[11] By using a manipulative such as a ruler, students will begin to see the connection between counting and measurement. They will need repeated practice with using a ruler. However, over time they will begin to see that the length of the object is the number of inches that they count. Hopefully, they will begin to realize that they will not need to count all the inches as they get fluent in using the ruler. Beyond this connection, they will see that measurement is relevant in their lives, as we use inches to measure in real life. Most importantly for my first graders, they will see that the counting sequence of whole numbers is critically important for the real life application of measuring in standard units.

Array Structure of a Rectangle

I wanted to add an aspect of geometry to my unit since geometry can be clearly connected to arithmetic through measurement [12] . Geometry, arithmetic, and measurement all play a role at the first grade mathematics level but sometimes feel disjointed from one another. Our shape and measurement units often feel out of place in the rest of our mathematics curriculum. I often struggle to create connections to bring these concepts together for my students. Using the array structure of a rectangle can help my students see the relationship between geometry, measurement, and arithmetic.

"An arrangement of objects, pictures, or numbers in columns and rows is called an array. Arrays are useful representations of multiplication concepts."[13] An array is a mathematical drawing that plots out a space, commonly using a grid structure. Arrays can be drawn in many ways. I will be drawing them as rectangles composed of squares. The reason for this is that squares and rectangles are simple shapes and it is easy to find their measurements. The larger rectangle can be decomposed into smaller squares. This relates to measurement since students can count the number of squares that make up the rectangle. This can be done in the same way they count how many smaller non-standard units make up a larger object. For example, in Figure 1, the rectangle can be shown as a rectangle or it can be decomposed into six squares.

Figure 1:

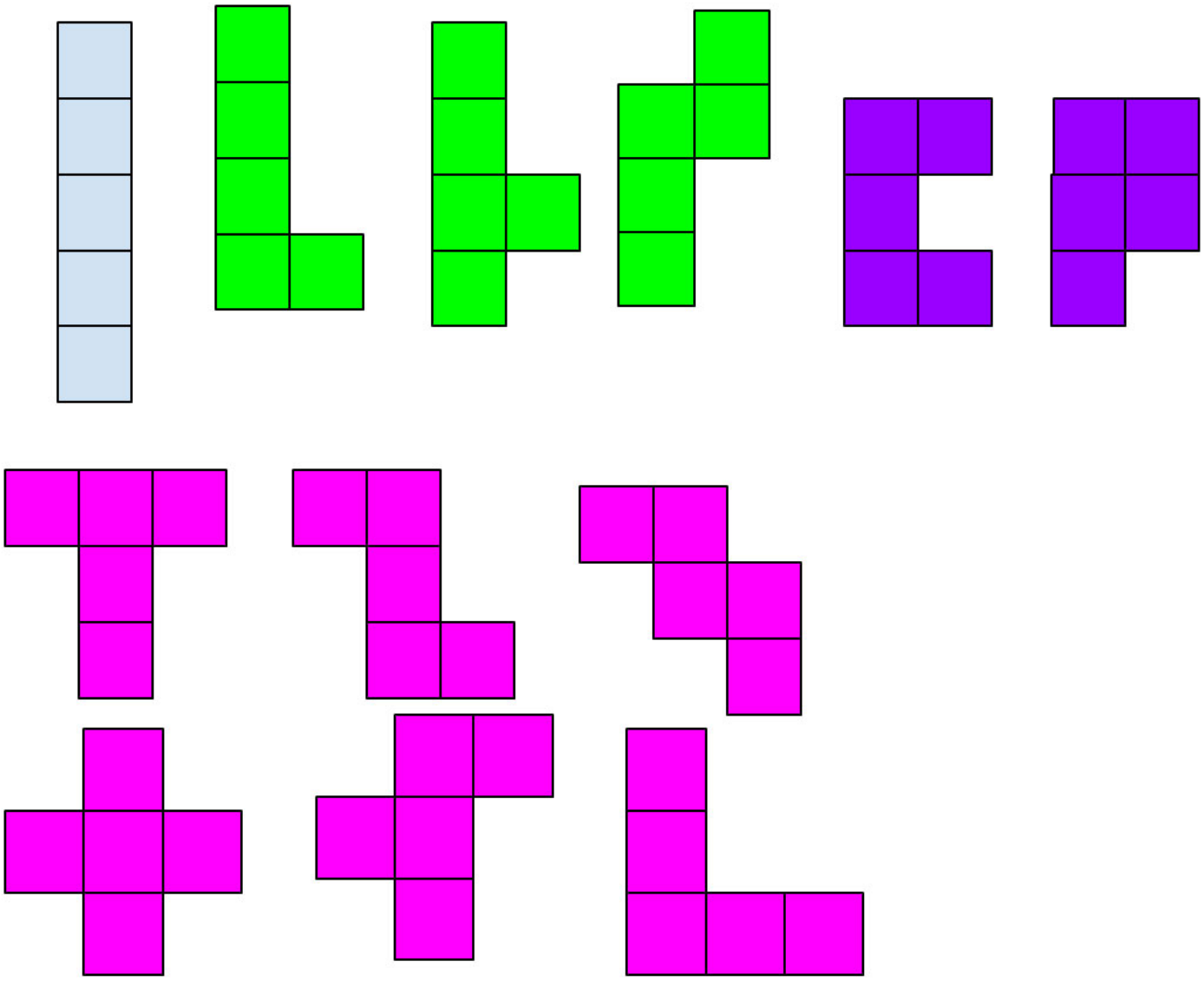


In order for students to see the array structure of rectangles they will need to use square tile manipulatives. Using these square manipulatives students will be able to construct rectangles on a plane. This is the first step for students to see that they used squares to compose a rectangle. Once students have mastered composing rectangles with squares, they can explore polyominoes. Polyominoes are "plane geometric figure formed by joining one or more equal squares edge to edge."[14] Polyominoes can be created with any number of squares. In order to keep my students within boundaries that make sense for their young age, I will have them focus on using five squares to create pentominoes.

Pentominoes and Enclosing Rectangles

A pentomino is created when five square tiles are joined with edges touching to create a geometric figure. There are twelve ways to create pentominoes, as shown in Figure 2.

Figure 2: Figure 2 shows the 12 different pentominoes. They are color coordinated based on their enclosing rectangle (the smallest rectangle they will fit inside of). The blue rectangle is its own 5×1 enclosing rectangle. The green pentominoes fit in a 2×4 rectangle. The purple pentominoes fit in a 2×3 rectangle. And the pink pentominoes fit in a 3×3 rectangle, but no smaller one.



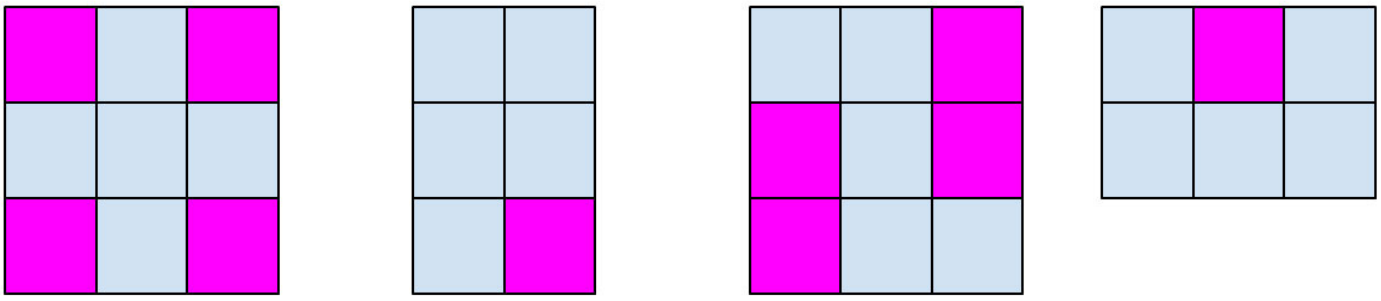
Pentominoes mostly have a perimeter of 12, except one of the 2×3 shapes which has a perimeter of 10. Pentominoes always have an area of five tiles. However, the area of the enclosing rectangle depends on the shape. From the above description, we see that the enclosing rectangle can have an area of 5, 6, 8, or 9, depending on the shape of the pentomino.

Enclosing Rectangles

In order to relate pentominoes back to the array structure of the rectangle, I will introduce my students to the idea of an enclosing rectangle. As mentioned above an enclosing rectangle refers to the smallest rectangle that would cover the entire shape of the pentomino. In order to create such a rectangle, one must "fill-in" the missing square tiles to create the overall rectangle. This new rectangle would cover the original pentomino figure. The relationship between the pentominoes and their enclosing rectangle further establish that square tiles can be used to compose a full rectangle. Or the reversal of this, that a full rectangle can be decomposed with square tiles. Enclosing rectangles can be seen in Figure 3. For example the "plus sign" shape has an enclosing rectangle of 3 high and 3 wide. However, the "P" shape next to it has an enclosing rectangle of 3 high and only 2 wide. While they are both pentominoes, they have different enclosing rectangles. The "Z" shape and the "C" shape provide further examples of 3×3 and 2×3.

In Figure 3, the original pentominoes are shown using blue squares. The enclosing rectangle is shown by completing the rectangle with magenta squares. By completing the rectangles students can see that the rectangles have been composed by squares and can eventually use this to determine area and perimeter.

Figure 3:

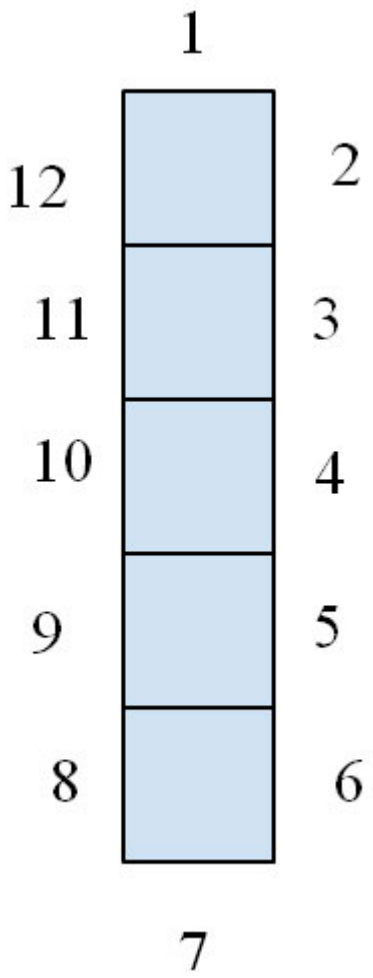


Area and Perimeter

After conceptualizing that rectangles can be decomposed into squares, and after I introduce them to the concepts of area and perimeter, and how to measure them, my students will use what they have learned about the array structure of a rectangle to determine area and perimeter. They can measure both area and perimeter of polyominoes they are presented with, and also of their enclosing rectangles. Students will first need to be introduced to what these concepts are, as well as how they are measured.

"All measurements were converted to linear distances. So it makes sense to start by discussing that most basic, most fundamental of measures: distance."[15] Perimeter is a type of linear measurement, so I think it makes sense to introduce it first. Students are already learning how to measure length with non-standard units of measure and by using a ruler. When you are measuring distance, you are really measuring how far away the two end points are from each other. First grade students in New Haven get an entire math unit dedicated to working on the skill of measuring the length of different objects; by the time I start this curriculum unit they will have repeated practice in measuring the length of different objects. For this reason, students should be able to easily adapt the skill of measuring length to the skill of measuring perimeter. Instead of just measuring how long something is they will be determining how long the entire outer edge of the object is. Instead of saying a 5×1 pentomino, as shown in Figure 4, is five square tiles long, they should be able to say that this pentomino has two edges length of 5 and two edges length of 1, for a total perimeter of twelve. They will determine this by counting how many edges make up the outside of the figure. Instead of just counting sides of square to find "length" they will be counting to find the length around the object.

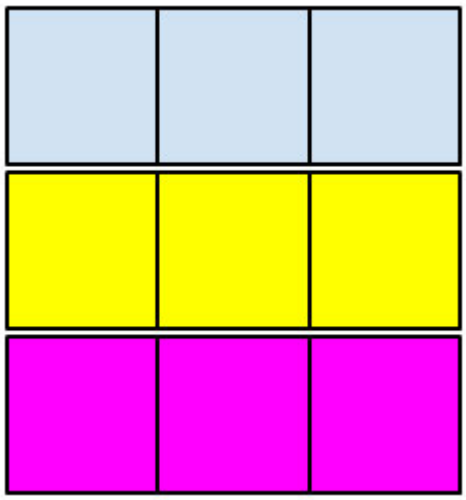
Figure 4: Instead of counting how long this design is in terms of square tiles, we are counting the surrounding perimeter by using the edges. There are 12 outside edges in this pentomino therefore it has a perimeter of 12.



I anticipate that area will be a more complex concept for my young students to grasp. “Our brains need help when thinking about areas because so much of our early learning about numbers and measurements is linear.”[16] Perimeter will likely make more sense to students since it is linear and that is what they are well practiced in. “There are no tools in the set of school mathematical instruments to measure area: areas are almost always the result of calculations, and are seldom measured directly.”[17] While there are many manipulatives to help students calculate linear measurements, there are none that specifically measure just area. Area must be calculated with the use of an operation, or by counting. Area of rectangles is typically taught using multiplication. For this reason, it is not often taught in the lower grades.

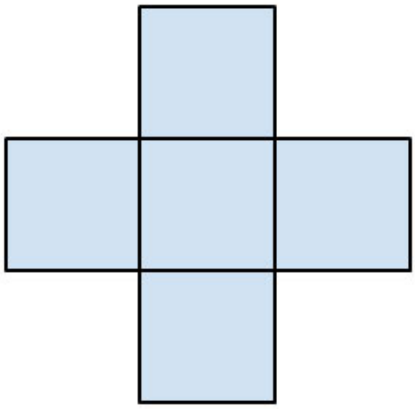
However, the area of polyominoes is usually best found simply by counting. So I can introduce the idea of area to my students using polyominoes. Then we will graduate to finding the area of rectangles. I plan to teach my students about area of rectangles in terms of repeated addition. This will eventually become multiplication in the higher grades. For example, if students have a 3×3 rectangle decomposed into squares they would have three rows of three and three columns of three. They could either count the total number of squares used to create this figure or add 3+3+3 representing the three rows of three to get an area of nine square tiles as shown in Figure 5.

Figure 5: Students can clearly see that there are three groups of three squares. They would simply count all the squares that compose the shape, or if they are more advanced in their math skills they could add three blue squares plus three yellow squares plus three purples squares to get a total of nine squares.

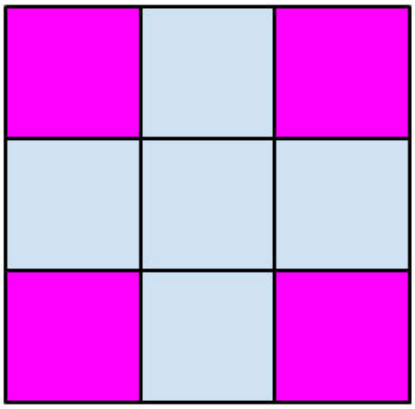


Area and perimeter can be determined for pentominoes and their enclosing rectangles similarly. Students can count the edges of the pentomino figure to determine the perimeter and can count the total number of square tiles used to create it to determine the area of the figure. They would determine the area and perimeter of the enclosing rectangle when they filled in the gaps in the array structure to compose and complete rectangle and then count the edges for perimeter and total number of tiles used to determine area (as shown in Figure 6). They may be surprised to find that, although the area of the enclosing rectangle is always larger than the area of the pentomino (except for the straight bar, when they are equal, since this shape is a rectangle), the perimeter of the enclosing rectangle is never larger, and can be smaller. Students will need to explore the pentominoes to discover this.

Figure 6: A pentomino was created by using five square tiles. Its enclosing rectangle was created by completing the shape and adding squares to complete the shape to a full rectangle (which in this case is a square). The areas and perimeters have been determined for both shapes. The blue represents the original pentomino and the purple shows the enclosing rectangle.



This pentomino has an area of 5 squares and a perimeter of 12 edges.



This rectangle has an area of 9 squares and a perimeter of 12 edges.

While these concepts are not required at the first grade level, I am confident that my students will be able to reach them with support and using proper strategies. I also expect that if they begin this work in first grade they will be less likely to have misconceptions when they reach testing grades. In doing this work I believe I will help my students see how geometry and measurement are truly a part of mathematics and set them up to be successful in future years.

Measuring with Manipulatives and Addressing Misconceptions

Using manipulatives is a critical math strategy in the primary grades. There are many types of manipulatives that can be used for different mathematical concepts.

“Manipulative materials, such as geoboards, pattern blocks, chip trading boards, counters, algebra tiles, attribute pieces, fraction bars, and Cuisenaire rods, have been employed to teach children and adolescents a variety of math concepts. These manipulative materials have been used to teach students counting, place value, word problem solving, basic computation, numeration, and equation solving skills.”[18] .

Different manipulatives serve different purposes. This unit will focus on using square tiles, rulers, and eventually pre-made plastic polyominoes. We would start off by using a non-standard unit such as a square tile (with sides that are one inch long). My students would eventually move on to using a ruler. Towards the end of the unit when they are familiar with the different pentomino shapes, they would get to explore using the plastic pre-made versions of them. They will use these instead of constructing them on their own out of

square tiles.

Square Tiles

My students will begin this curricular unit measuring with square tiles. They will be measuring area while exploring pentominoes. And, they will be measuring length with the one inch edges of the squares when determining perimeter. These concepts will be new for my students and I will need to give explicit directions when explaining the different activities. I would not want my students to get confused over area and perimeter because I was not clear in my directions. There are some important factors to consider when choosing which type of square tiles to use. I decided to use square tiles that have an area of one square inch. This means that each side is one inch long. While working in groups, my students will be assigned a specific color to work with. Eventually, when they move from measuring to constructing pentominoes they will need tiles of two different colors. In order for my students to see the polyominoes and encompassing rectangles they will need to construct the polyominoes in one color and fill in the remaining area with a different color. This will help them to visualize the pentomino shape and the rectangle that encloses it.

Misconceptions while using Square Tiles

When working with square tiles (or any other manipulative) for measurement I will have to address what gaps and edges are. When students are measuring both length using square tiles, they must make sure there are no gaps between the tiles. The edges are the sides of the square, one side must directly line up with and touch the side of the next square. It is important for the tiles to be lined up edge to edge is so that the students are not including any extra spaces. If there are gaps between the tiles, they will be measuring the space between the tiles as well as the length of the object. Students must ensure there are no gaps so that they can be sure they are only measuring the actual length of the object.

By placing the tiles directly next to each other, edge to edge, they are showing that they understand that measuring length is really measuring distance. They are measuring the distance from the beginning of an object to the end of an object. When they count the tiles to see how long the object is they are showing that they understand that counting numbers are related to measurement as they are counting how far away the end point is from the start point.

Rulers

I know that if I want my students to be successful in this unit, they will need to learn how to use a ruler properly. Unfortunately, when many students first hold a ruler, they automatically want to use it as a sword. Using a standardized unit of measure is critically important when measuring but is not expected at the first grade level. Some specific criteria my students will need to learn in order to use a ruler is: that they lie flat, they must be flush with the edge of the object being measured, that they count on by ones to twelve, and that they must make sure that the end of the object is lined up with the zero on the ruler, since the numbers on the ruler are telling the distance from zero. Since my students would be unfamiliar with rulers, I feel sure that they would be excited to use them. Due to their excitement I will have to be very careful and clear when explaining how to use a ruler as a tool for counting and measurement. I will start by having them line up the one inch squares against the ruler, starting at the zero on the ruler. My students and I will observe that the number at the end of the line just tells the number of squares that have been placed.

Misconceptions while using a Ruler

This unit will be my students first exposure to using a ruler. They will need to get explicit instructions on how to use a ruler properly. There are certain misconceptions that will need to be addressed. Students will need to learn that rulers must lie flat, with the numbers facing the person who is doing the measuring. They must also be made aware that it must be lined up with the object being measured. These concepts must be taught in order for students to use rulers appropriately.

Sometimes the zero appears at the edge of the ruler, and other times the zero is embedded a little in from the edge. Students must be able to identify where the zero is on their rulers. They will need to learn that regardless of where the zero is, this is where they must start measuring from. Some students may wish to start at one instead of zero. It will be important for them to realize that since they are measuring length (which is distance) that they are determining how far away from zero the object extends. It is critical for them to understand that a number on a ruler is telling them how many inches it is away from zero. It is describing distance in multiples of one inch. If they begin at one instead they are measuring the distance from one and therefore will not calculate the proper number of units. Ensuring that students know where to start measuring from is critical to their success in this unit and as they continue to use measurement in their lives. For this reason I will tell my students that when we measure with a ruler, we start at the zero on the ruler. The activity where we line up the squares against the ruler will help them with this.

Finally, while measurement is clearly correlated to counting, students may become familiar with measuring with a ruler and not applying counting strategies at all. This would demonstrate that they have truly mastered how to effectively use a ruler as a measurement tool. At first students may wish to count all the inches between zero and the far end of the object since they are familiar with one-to-one correspondence when counting. As students become more familiar with using a ruler, they will not need to count all the inch marks. However, as they become more accustomed to using a ruler they will be able to identify the length as the number that it is lined up with. When they are doing this, they are determining the distance from zero to the edge of the object. Therefore, they are finding the length in terms of how far away one edge is from the other in inches.

Plastic Pentominoes

My curriculum unit will require my students to use pentominoes when planning out walkways and a plot for our garden. I realized that when they are planning these different elements it might be challenging to continuously recreate them using square tiles. Pre-made plastic pentominoes already exist as a manipulative. I was able to order all twelve pentominoes in six different colors for a total of 72 pentominoes. The different colors make it easy to see where one pentomino ends and the next begins. I knew that this manipulative would help my students visualize aspects of our garden.

Since the pentominoes are pre-made, students would not need to worry about accidentally touching one tile and changing the whole design, an issue they might experience when using seperate square tiles. By using these manipulatives that are already in the pentomino shapes students will be able to move, flip, and rotate the pentominoes to see the different ways that they fit together. These manipulatives will be a great asset for them to see how the pentominoes can be used to create rectangles with different areas, but are still composed of squares, further emphasizing the array structure.

By using this tool and exploring the concepts of area and perimeter, my students will be able to relate counting to geometry and arithmetic. Even though the pentominoes are already in their twelve different

designs, you can clearly see that they are still composed of squares. I will check that my students can also see this. Due to this, students will be able to count all the edges to determine the perimeter and count all the squares inside the shape to determine area. These will be a great resource to help further the understanding of these above grade level concepts. As these manipulatives have not been used before they will breathe life into the curriculum, and engage my students in a new way.

Gardening

Gardening is something that I do with my class every year and it always engages them. My students love the hands-on aspect of planting. They love the reward of watching the seed they planted grow into a plant. I want to incorporate gardening into this curriculum to engage my students in measuring.

When planning and planting a garden, measuring is often used. My students will need to determine the areas and perimeters of their garden plots. We will be using rectangles to create garden plots and squares to create walkways. Both are simple shapes which will help us to organize our garden space. This will further develop the idea that we commonly use rectangles in our everyday lives. Decomposing our plots into squares reiterates the array structure of the rectangle. Finally, since these plots will be outside and must be measured out, my students will need to use a ruler to measure out the spaces. They will see how important it is to use a tool of measurement and a standardized unit. These gardening activities encompass all the learning goals in my curriculum unit.

Cooperative Learning

"To achieve success in learning mathematics, students should be given the opportunity to communicate mathematically, reason mathematically, and develop self-confidence to solve mathematics problems. One of the ways this can be done is through cooperative learning."[19] In order to help my students feel more successful in their math block, they will work together to solve problems. Students will communicate regularly with their peers and share their ideas with one another. "Explaining your work is what, in mathematics, we call reasoning, and reasoning is central to the discipline of mathematics."[20] Due to the nature of my school, students must reason during our Socratic seminars that focus on mathematics. Reasoning is a key component to my students' success in these seminars in addition to cooperative learning.

Students will also be working in groups during this curriculum. Group work helps students to think through what they are doing with the help of their peers. "There were lots of ways to start the problem and success on the problem would take good communication among team members, with students listening to each other and giving each other a chance to think through their work."[21] By working with other students, children will feel secure in sharing their ideas. This can help relieve math anxiety as they will be working with a group of their equals. Students will have a chance to share their ideas freely with others. They are likely to be more open in a small group than they would sharing in front of the class. By working in groups students are given the opportunity to share their ideas and learn from each other.

"They need to produce arguments that convince other mathematicians by carefully reasoning their way from one idea to another, using logical connections. Mathematics is a very social subject, as proof comes about when mathematicians can convince other mathematicians of logical connections."[22] Using mathematical reasoning is a skill that my students will need to develop over time. "Math Talks" is a strategy that will help my students develop these crucial skills.

Math Talks

"This opened up mathematical pathways and allowed students who had not understood to both gain understanding and ask questions, adding to the understanding of the original student."[23] Math talks allow students to learn from each other. They must share their reasoning with one another in logical ways. By sharing their reasoning students can learn from one another and get the opportunity to ask each other questions. "Students love to give their different strategies and are usually completely engaged and fascinated by the different methods that emerge."[24] Not every student learns in the same way, by sharing their ideas and reasoning students can learn strategies from their peers. This will put my students in the "teacher" role as well. Math talks give students the opportunity to act as the math expert. This will help them to develop confidence in math and feel proud of their work.

Spatial Awareness

All the learning goals and strategies used throughout this unit provide opportunities for students to strengthen their spatial awareness. "Spatial awareness is the ability to be aware of oneself in space. It is an organized knowledge of objects in relation to oneself in that given space. Spatial awareness also involves understanding the relationship of these objects when there is a change of position."[25] By using different types of manipulatives such as square tiles, rulers, and pentominoes my students will get to explore the given space in terms of objects. Furthermore, when the students are gardening they will be able to comprehend the spatial relationship between the walkways and the plants that will be surrounding them. Finally, when exploring the pentominoes students will get to experience how the space changes if a pentomino changes positions, is flipped, or rotated.

Many students explore spatial awareness in dance classes or at sports practice. Due to the nature of my students neighborhoods and living situations, not many of them participate in these types of extracurricular activities. All the activities in my unit give my students the opportunity to explore our classroom, our courtyard, manipulatives, and themselves in terms of spatial awareness. Since they may not be developing this awareness anywhere else, I am particularly pleased they will be able to develop it in my curriculum.

Classroom Activities

Activity One: Measuring With Square Tiles

Materials: Multiple color square tiles, classroom objects

Students will measure different classroom objects using the edges of square tiles. They will make sure there are no overlaps, no gaps, and the tiles are edge to edge. After showing mastery in measuring different classroom objects with the tiles, they will get time for exploratory play.

Working in groups, the students will create “paths” using the square tiles. They will use as many tiles as possible to see who can create the “pathway” that takes up the most space. The pathway that takes up the most space has the greatest area. Pathways are created by having tiles touch edge to edge. This activity is an introduction to polyominoes since students are creating them when they connect the tiles by edges.

Activity Two: Using a Ruler

Materials: Rulers, classroom objects, poster paper, markers

In this lesson students will be introduced to using rulers. The teacher will gather the students on the carpet and model how to use a ruler. Certain misconceptions must be addressed. The first misconception would be where to start measuring. Some rulers have zero listed at the very edge or the ruler. Others, have the zero further in. It is important to reiterate to students that they start measuring at the zero. This will be justified later on in the activities, by having the students line up one inch square tiles along the ruler, so that they can see that the number at the end of the row of tiles tells the number of tiles, which is the length of the row. Secondly, students must be taught that in order to measure the ruler must lie flat. The reason for this being that you are counting the linear distance between zero and the end of the object. After these misconceptions are addressed, the students may help the teacher to make a poster which explains how to use a ruler. They can reference this poster throughout the unit.

After making the poster, students will be allowed to measure with the rulers. They can use the same objects they used when using the square tiles to measure length. The students will line up the tiles against the ruler. Students will be able to observe that the length of the line of tiles is the same as how many tiles are being used. This will allow them to explore if the length changes based on which measurement tool is used. Since the square tiles are one inch in length, the objects will have the same length in inches as tiles used. This can lead to a hearty discussion where students discover that the tiles are one inch long on each side.

Activity Three: Intro to Pentominoes and Enclosing Rectangles

Materials: two different color square tiles, poster paper, markers

In this lesson I will first tell my students we will be creating figures called pentominoes. I will explain that pentominoes use five tiles and that there are twelve pentominoes in all. I will challenge my students to create pentominoes using the square tiles. The overall goal of this lesson is that the students find all twelve. They will be informed that the pentominoes must have edges touching, and two adjacent tiles must match along a whole edge of each.

Students will work in groups to find the different pentominoes. After a certain period (I will probably use 15 to 20 minutes) we will come together as a group for students to share their different figures. We will add their figures to the poster as they share. At this point, students will be made aware that two shapes that can be rotated or flipped into each other count as only one pentomino. If the students are missing any of the twelve we will work as a class to find the missing pentominoes.

Once we have found all twelve pentominoes, and they have been listed on the poster, I will introduce the concept of enclosing rectangles. Using a different color square tile I will fill in the gaps in the pentomino. When all the gaps have been closed, I will have a rectangle composed of two different color square tiles. I will ask my students to create enclosing rectangles as well. Once my students have created enclosing rectangles, I will ask them what they notice about the new shape. This new question will lead to a discussion. The goal of this discussion is for students to come to the conclusion that rectangles can be composed with squares.

Activity Four: Pentomino Walkways

Materials: graph paper, colored pencils

In this lesson students will use graph paper and colored pencils to design the pentomino walkways for our garden. The enclosing rectangles will be filled in with plants which is why the colored pencils are necessary.

Working in groups students will design different walkways for our garden. They must use pentominoes as the "shape" for their walkways. They will be told that the edges must touch. Instead of using tiles, students will use the squares on the graph paper as their basic shape. They must use five and edges must touch. These walkway designs will be used when we create the walkways in our real garden.

Activity Five: Tetris Garden

Materials, a 6 ft X 10 ft plot of land, twelve different types of plants to represent each pentomino, plastic pentominoes for planning use, square tiles, graph paper, colored pencils

Students will first plan out their "tetris plot". They can either use plastic pentominoes, square tiles, or draw the designs using the graph paper and colored pencils. Students will work in groups to determine how all twelve pentominoes fit together in this plot. There is only one way for all twelve of these pentominoes to fit together in this rectangular shape, therefore, all groups will eventually come up with the same design. The groups will share their designs with the whole class using math terminology. If students are struggling to figure out how the pentominoes all fit together, I will provide them with certain hints. One hint would be giving them the outline of the enclosing rectangle which all the plastic pentominoes fit inside. Another would be giving them a piece of graph paper with some of the pentominoes already drawn. I will have several versions of this paper available. They be scaffolded to different levels. Depending on how much help the group needs they can get a paper with either 1, 2, or 3 pentominoes drawn on already. These hints would allow the students to explore the way the pentominoes fit together with extra support. If they continue to struggle we can work together as a whole group.

After the designs have been drawn up, the plot will be divided into the pentomino shapes. Each shape will have a designated plant and color that represents it. When the garden in finished, you will be able to clearly see each shape due to this.

Appendix

The Common Core Standard that my curricular unit aligns to is:

CCSS.MATH.CONTENT.1.MD.A.2

Express the length of an object as a whole number of length units, by laying multiple copies of a shorter object (the length unit) end to end; understand that the length measurement of an object is the number of same-size length units that span it with no gaps or overlaps. Limit to contexts where the object being measured is spanned by a whole number of length units with no gaps or overlaps.

My curricular unit aligns to this standard as my students will be using different manipulatives to measure bigger objects. My students will be using square tiles and rulers to measure different classroom objects appropriately. Beyond this, when we are gardening my students will be using square tiles, plastic pentominoes, and graph paper to design our garden and walkways. Students will need to be able to measure without gaps and overlaps in order for these garden features to be correct. They will also need to express the length of the gardens and walkways in terms of whole number units related to the manipulatives we have used to create them.

Annotated List of Resources

Boaler, Jo. *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching:* This book focuses on helping students develop a positive math mindset. It explores how many teachers have math anxiety themselves, and provides ideas on how to overcome this. Finally, this book provides many activities to inspire students in math.

Cass, Mike. "Effects of Manipulative Instruction on Solving Area and Perimeter Problems by Students with Learning Disabilities": This journal article explores the benefits of manipulatives used by students who need differentiation.

Eckersley, Sian. "Spatial Awareness." Occupational Therapy for Children, occupationaltherapyforchildren.over-blog.com/article-spatial-awareness-108726104.html.: This online blog discusses what spatial awareness is and why it is important for children to develop this type of awareness.

Elliott, Andrew C. A. *Is That a Big Number?* Oxford University Press, 2018.: This book explores mathematics in a way I have never thought about before. Elliott offers explanations to mathematical ideas in new ways and made them much easier for me to understand.

"Find Textbook Support Materials." *Houghton Mifflin Education Place*, www.eduplace.com/. This website offers curricular resources and lesson plan ideas for teachers.

Howe, Roger. "Three Pillars of First Grade Mathematics, and Beyond." *Mathematics Curriculum in School Education Advances in Mathematics Education*, 2014, pp. 183–207., doi:10.1007/978-94-007-7560-2_10.: This short essay explains the three components that students must learn in first grade and how they are interconnected.

"Mathematics Standards." *Mathematics Standards | Common Core State Standards Initiative*, www.corestandards.org/Math/. The

Common Core standards can be read on this website.

“Polyomino.” Wikipedia, Wikimedia Foundation, 16 Aug. 2019, en.wikipedia.org/wiki/Polyomino.: This wikipedia page explains what a polyomino is.

“Ross Woodward.” *Ross Woodward*, 18 June 2019, rosswoodwardschool.org/.: This is the website for my school which shares valuable information about our approaches.

Zakaria. “The Effects of Cooperative Learning on Students' Mathematics Achievement and Attitude towards Mathematics.” *Journal of Social Sciences*, vol. 6, no. 2, 2010, pp. 272–275., doi:10.3844/jssp.2010.272.275.This journal article explores why cooperative learning is critical in mathematics.

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