Curriculum Units by Fellows of the National Initiative
2019 Volume V: Perimeter, Area, Volume, and All That: A Study of Measurement

# Area, Surface Area and Volume: From Misconceptions to Skyscrapers

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## Measurement

In my unit, I want my students to understand how architects and engineers use measurement in finding the area, surface area and volume of structures they will compose and construct. When determining any given space, we use measurement to describe its size. After learning this unit, students will know architects and engineers and the use of volume of a box. They will know the amount of space, and that the product of the side lengths in the 3 directions (length, width, height) tells what any box or rectangular prism can hold. Students will understand to use different ideas like form, space, and quantity when conceptualizing a new structure.

Students will work with two dimensional arrays and three dimensional rectangular prisms or boxes. Area is introduced with manipulative hands on activities such as working with foam square units in composing and decomposing rectangular arrays. Surface area of rectangles is understood by forming arrays of columns and rows of unit squares. The focus on volume is centered on how unit cubes occupy solid prisms as layers and will conclude by investigating how formulas are used.

As most elementary math teachers can attest, teaching measurement and hoping for student comprehension, particular with area and volume, can be challenging and frustrating. The mathematical strand of measurement has tended to become the “black sheep” of the math family. Often neglected and overlooked in its teaching, its profound impact on the overall understanding of elementary math can leave students with gaping holes in grasping the entirety of Common Core Mathematics curriculum, with some students unable to overcome the deficits. From "my" experience, measurement can be overlooked as a unit but as we continue, we can determine how measurement impacts all mathematics curriculum.

A simple rectangular array is basic in appearance, a polygon we have drawn as students since early childhood (see figure 1). Well, rectangles maybe, but the array structure is something that many students need to be shown explicitly. A student may determine the area of the rectangle in multiple ways: by counting each individual square unit to get a total of six square units; by decomposing the array into three groups of two and adding each of the three groups (2 + 2 + 2 = 6); others will count the length of square units (3) and count the width of square units (2) then solve by using the formula to find the area (A=L • W).

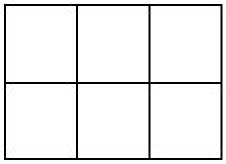

Figure 1. Rectangular Array

In my experience as a math teacher of over ten years measurement curriculum is taught to students quickly without the opportunity to explore. The scenario unfolds as such: state testing has been completed in late spring towards the end of the school year, and you as a teacher have completed teaching the units found in the front of the curriculum such as algebraic thinking, base ten number systems, fractions, and geometry, etc. You're feeling pretty good about the school year coming to an end, when you realize you still have 3-4 weeks left of school to teach. As a teacher, I am committed to a strong finish to the school year but I anticipate half-completed assignments and my student's attention wavering away towards their summer vacation.

A talk I often hold with the grade band teachers I work with will focus on how my middle school math students come ill-prepared in basic foundational skills at the beginning of a school year. Often these conversations lead to full out rants on how some of my students still have not mastered the multiplication table, how the base ten number system is too complex, and their struggle in understanding the difference between a numerator and a denominator.

A student's misconceptions of new mathematical knowledge can often be overwhelming for them. This lack of a general, foundational understanding of math skills becomes increasingly challenging to a student as he/she moves from primary, intermediate, middle, and high school math. As students move through one grade to the next, the curriculum becomes more complex and rigorous. Through informal and formal assessments, I can identify a student's lack of understanding of measurement and how this contributes to mathematical misconceptions.

Education research attests to the difficulties of the measurement concepts that students face in the classroom. "Units are difficult for elementary school students and these difficulties seem to stem from trouble understanding area and volume. Specifically, elementary school students have trouble with arrays. Researchers have found that students have trouble visualizing and using the unit structure of an array" (Battista & Clements, 1998).

Students need to be provided extra support to re-learn basic measurement concepts. "Rather, they add the lengths together to get the area, misappropriating length units for other spatial measures seems indicative of trouble with dimensionality, e.g., length is one-dimensional but area is two-dimensional." (Dorko and Speer).

The intention of this unit is to have my students understand the structure of rectangles as rectangular arrays: an arrangement of unit squares into rows and columns. My students will know the characteristics of a rectangle and how it can be formed by defining rows and columns of squares. I want to clarify to my students that area is the two-dimensional space contained within a region.

When the figures are rectangles whose sides are whole numbers, and in related cases, this area can be measured by the unit squares that cover the space. My focus will be on students with misunderstandings about rectangular arrays and prisms. Rectangles do not need to have whole number side lengths and unit squares only. I will focus on both whole number side lengths and unit squares to introduce learning goals of measurement of rectangular arrays. Students will learn that area applies to all types of general polygons, and other figures, such as circles as well.

## Measurement misconceptions

In my unit, I will address student misconceptions for the understanding of an area and volume of rectangular arrays.

The deception of adding and multiplying unit squares can be a big misconception to students. When my math students are solving measurement activities in the classroom, one of the biggest misinterpretations they have concerns the appropriate uses of the operations of addition and multiplication.

Customarily when students are determining an area of a particular array, they assume the area will be greater than the perimeter because multiplying rather than adding gives you a greater amount, (see figure 2).

In some arrays, perimeter will be greater than an area (see figure 3). More importantly, students need to understand that it does not make sense to compare length and area, because they are measured in very different units: inches versus square inches, or feet versus square feet, etc. Also, the numbers involved depend heavily on which units are used. For example, a square with side length of 2 feet will have a perimeter of 8 feet, and an area of 4 square feet, or a perimeter of 96 inches, and an area of 576 square inches.

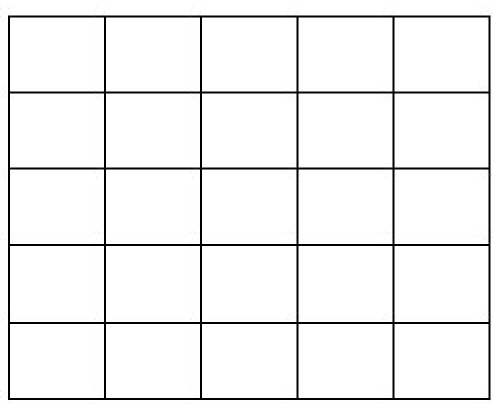

Figure 2 Finding the area of a rectangle, using an array and multiplication

(A= l • w = 5 • 4 = 20)

And the perimeter using addition

(P= S + S + S + S = 5 + 4 + 5 + 4 = 18)

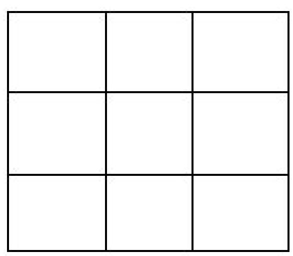

Figure 3 An array with numerically greater perimeter than area (A= L • W = 3 • 3 = 9)

And the perimeter using addition (P= S + S + S + S = 3 + 3 + 3 + 3 = 12)

My students need to be careful only to add numbers that refer to the same unit. A common mistake is when a student tries to add centimeters and inches. “If there are three feet in a yard, then there are three square feet in a square yard. The mistaken conclusion is that two dimensional unit conversions are equivalent to linear unit conversions” (Carle).

For some students, the challenge is in spatial bias and equality assumptions. Students understand bigger is larger when interpreting arrays visually. When a student is comparing two rectangles, they will assume that the one that “looks bigger” of the two must have a greater area and perimeter because students have misunderstandings with spatial relations. The first rectangular array might be in the shape of a traditional square A = L •W = 3 •3 = 9 compared to one, which looks longer and stretched out but with the same area A = L •W = 1 •9 = 9. Because it looks stretched out, the second array will look smaller than the first one.

My students also struggle with the understanding that arrays with the same area also might have different perimeters. For example, if a (4 by 4) array (see figure 4) has an area of 16 square units, and a perimeter of 16 linear units, some students will assume an array with the same area will always have the same perimeter of 16. Students will be asked to determine the perimeter of a different array with the same area, an array of (2 by 8) with the same area of 16. The students will discover that the perimeter of this second array is 20 linear units, (see figure 5). “The number representing area must be greater than the number representing perimeter, since space is more, as a student once told me. The students fail to understand the importance and value of the units attached to the number,” (Carle).

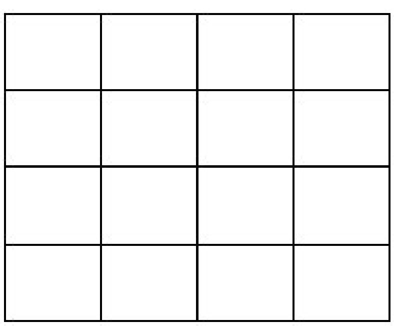

Figure 4: Array of 4 • 4 = 16 and P = 16

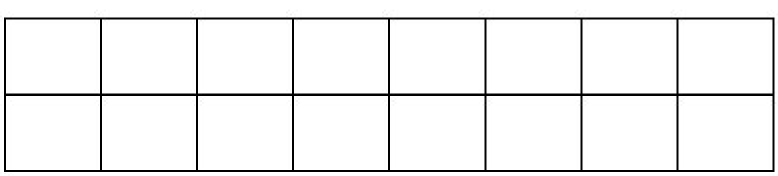

Figure 5: Array of 2 • 8 with A = 16 and P = 20

Students continue to struggle with these misunderstandings because of their lack of strong foundation skills and lack of practice with the composing and decomposing of rectangular arrays. Some of my students look upon math as a thing detached from their everyday lives, lacking importance and value to better understand real life situations all of us face each day. In the real world, they continue stumbling through school accompanied by these misconceptions.

## Context

Richard Edwards Elementary is a Chicago Public School on the southwest side of Chicago. At the start of the upcoming school year in early September, I will be teaching for the first time at Edwards Elementary School. After a decade of teaching at other schools in the city, I will have a position at a higher performing school, one centered on a more rigorous curriculum. Edwards middle school classrooms incorporate the International Baccalaureate (I.B.) program. It is classified as a Magnet Cluster (International Baccalaureate Middle Years/Dual Language/Fine and Performing Arts) with one of the largest student populations in CPS with almost 1,500 students. The overwhelming majority of the students are Hispanic (96%), and between 1% and 2% each of White and Black. Nine out of ten of the students are classified as low income, slightly over half are limited English proficient and 15% are classified as Diverse Learners. I will plan to implement this unit with approximately ninety 6th grade students. My daily schedule will have three separate classrooms of approximately thirty students, ninety minutes each. In preparing, this unit, I take into account that my new school will focus on International Baccalaureate (IB) curriculum. I will be immersed within a strong academic environment, in which social and emotional characteristics are promoted in all our students. Many of our IB students will likely perform well academically when compared to other students.

## Measurement rationale

The aim of this unit is to strengthen the area in my math teaching which I consider the weakest: measurement. Ever since I have been teaching math in the classroom, my units in measurement were not thoroughly organized or strongly assembled. Among the list of core concepts to teach in a given school year, the measurement curriculum ranked towards the bottom of math units. My biggest objective is for this unit on measurement to represent a well-organized, researched and thoughtful curriculum to serve as a model for future units involving math. My desire is for this unit to be the example I look up to when writing math curriculum.

The unit I am writing will be on understanding the array structure of rectangles and the array structures of rectangular boxes and, understanding that the surface of boxes can be cut apart, and unfolded to make an arrangement, or net, of rectangles in the plane. By reversing this procedure, these geometric nets can be folded up to make the surface of geometric rectangular prisms.

I will teach my students to recognize various characteristics of objects as measurable. I am aware that some content might be basic and foundational at first, for example counting square units and visualizing rows and columns of arrays, but this will allow me to address the many concerns surrounding student perplexity and remedying of previous student inaccuracies in dealing with area and volume.

These are some of the key understandings I want my students to absorb:

1. Measurement involves a selected attribute of an object (length, area, mass, volume, capacity) and a comparison of the object being measured against a unit of the same attribute.
2. The larger the unit of measure, the fewer units it takes to measure the object.
3. In understanding the rectangular array, determine the correct number of unit squares required to fill in

the space being measured.
4. During the activity students will clarify and incorporate key operations such as compare, combine, and replicate.

If a rectangle is subdivided into square units, and the squares are decomposed and rearranged, the area will not be changed. Students will choose a unit of measurement (a square unit) which is appropriate and conducive to the rectangle being measured.

Students will begin to understand measurement by organizing arrays into units and sets of rows and columns. My students will begin to visualize the area as represented by the total number of rows and columns within a space inside an array, (see figure 6).

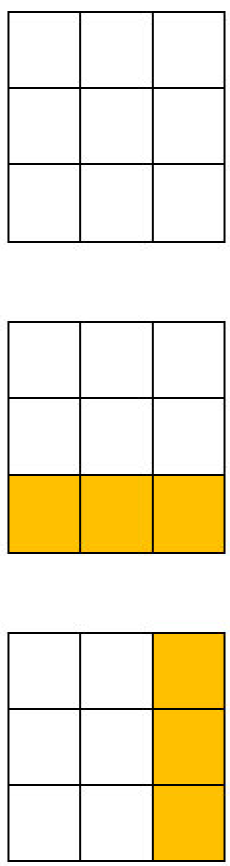

Figure 6: Visualizing rows and columns in the same rectangular array

Students will understand that if the complete entirety of the surface area of the shape is covered with the chosen units, then the total area can be determined by counting the number of units used.

I will segue from the sum of the units being used to the mathematical relationship of multiplication to determine the same area. Once these understandings are clear, I will be able to introduce the elements of algebraic representation of values and variables found in formulas.

I will be focusing on the Common Core State Standards with an emphasis on measurement involving area and volume. This curriculum will be aimed at solving real-world and mathematical problems. The primary understandings are the array structure of rectangles, and the analogous structure of boxes (rectangular prisms). The first standard I will be utilizing will be CCSS.MATH.CONTENT.6.G.A.1 with an emphasis on how to “find the area of ….polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.” (CCSS)

The second standard I will be utilizing will be CCSS.MATH.CONTENT.6.G.A.2 with an emphasis on how to “find the volume of a right rectangular prism ….packing it with unit cubes of the appropriate unit edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = b h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.” (CCSS)

“Values are often meaningless without units and there are often multiple options for the unit in which something is measured (e.g., meters, yards), creating potential ambiguities if quantities are reported without units. For example, knowing what units the answer must be in can help one determine which quantities to combine to obtain the answer,” (Dorko and Speer).

I will have my students construct a two-dimensional rectangle and a three dimensional rectangular prism while applying the idea and the visual of measurement. Students will choose units when constructing and de-constructing both area and volume of rectangles and prisms. I hope that we can give them the perspective that rectangles are simple and nice shapes in the huge zoo of planar figures. This is the reason that there being such nice formula for the area of a rectangle.

I am planning to have my students make boxes by making nets for them and then folding them up into boxes. This is halfway between activities that I hope will lead my students to understanding.

I will have my students understand that a rectangle with whole number side lengths can be sub-divided into a regular array of unit squares. I expect that this is not an idea which comes naturally to every student. I will follow this by having them understand that the area means the number of unit squares that can fit inside a rectangular array.

I will explain to my students that a rectangle can be regarded as an array of unit squares and the number of squares in a row of this array is the length of the base (in linear units), while the number of rows (or equivalently, the number of squares in a column) is the length of the side (in linear units). Therefore, the total number of squares are the length (= number in a row) times the width (= number of rows). To wit, the area of a rectangle is the length times the width times the area of the square, which we define to be 1. One condition for these activities in this unit is that we will be working with only whole numbers.

By the end of this unit, I will have my students feeling confident and clear in how to use and choose appropriate units of measurement when applied to length, area, and volume to determine quantities that are important in our everyday life. Examples of these can be how students will determine the area of floor and classroom space; the volume of various size containers.

The first goal is to understand that a rectangle can be regarded as an array of squares. Not every rectangle can be sub-divided into squares: there must be some relationship between the length and the width. They should be whole numbers. Students will understand the differences in objects by comparing them and asking direct questions of longer or shorter; greater capacity/less capacity/ weighs more/weighs less.

The next goal is to understand that a box (a rectangular prism) can be partitioned into a three dimensional array of unit cubes. (Again, we need the side lengths to be whole numbers). Students can practice by being presented with a variety of carefully selected boxes to fill with unit squares. The key for some is the activity being hands on and concrete.

Students will familiarize themselves with and master measuring tools such as rulers; measurement systems; and formulas to understand the area, surface area and volume.

The underlying factors faced by students today may lie with an absence of attention to detail and meaningful attempts of real life application to what measurement represents. Of the challenges teachers face in today's classroom, measurement can often times be found atop the list. But as measurement is now being taught across curriculums, teachers look to solutions as varied as exposing students to as many different practical experiences; understanding how to choose units; and how to collaborate and work with peers in teaching measurement across grade level and content areas. (Thompson and Preston, 2004).

In my classroom, I use various types of teaching strategies throughout my math block which will include the following:

- Visualization: a main focus to this unit is to move away from the textbook mentality of teaching and into a visual teaching method of hands on practices, where students are encouraged to leave their seats and move about the classroom. Visualization provides my students the opportunity to make practical connections to the real world. My students will use foam squares to manipulate square units into arrays. They will organize several rectangular arrays, representing the sides of a box, into a net, which they will draw on paper; and will fold paper nets into solid forms of a geometric rectangular prism.
- Cooperative Learning: One of my favorite teaching strategies is cooperative learning. This provides learners of all levels to work collectively on building and decomposing rectangular arrays and rectangular prisms while working collaboratively with each other on the understandings of measurements with the goal of their small group succeeding together.
- Differentiation: The goal of differentiation is to meet the needs of learners while challenging students who have stronger math understandings with rigorous tasks. While at the same time supporting students with lower capacities of comprehension with an increase in individual help and support.

By utilizing technology in the classroom the students will see that there is a stark contrast between the classrooms of today, and the classrooms we as teachers attended as children. The main difference is accessible technology to all students. From laptops to smart boards, technology has moved the classroom big time. The use of technology is key in supporting differentiated instruction, allowing freedom and innovation to bloom and challenging our students with more rigorous tasks. The following are but a couple of examples of websites that students can use to strengthen comprehension in area and volume.

"It is essential that teachers and students have regular access to technologies that support and advance mathematical sense making, reasoning, problem solving, and communication. Effective teachers optimize the potential of technology to develop students' understanding, stimulate their interest, and increase their proficiency in mathematics. When teachers use technology strategically, they can provide greater access to mathematics for all students," (NCTM).

https://www.geogebra.org/m/pCv2EvwD

http://www.shodor.org/interactivate/activities/SurfaceAreaAndVolume/

The structure of the unit will include the following key points I will be teaching on measurement:

1. Understanding the concept of an array of any sort of figure arranged in a rectangle
2. Understanding that area of rectangles can be found by decomposing the rectangle into an array of unit squares, and counting the squares, or, more quickly, by multiplying the length and the width.
3. Visualizing groups of rectangles into nets
4. Geometric form of rectangular prisms into the shapes of buildings

## Concept of a rectangular array

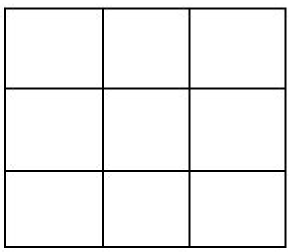

Figure 7: (3 • 3) Rectangular Array

I will work with my students in understanding rectangles and how to decompose or sub-divide these rectangles into arrays of squares using columns and rows, (see figure 7). I will work with my students in understanding that a specific region has a two dimensional space inside it. After this idea is mastered, I will work with my students to arrange arrays into nets to build and fold into boxes. They will then study the volume of the boxes in terms of 3 dimensional arrays of unit cubes.

A rectangular prism solid (a box) is what we will be using to understand volume. We will be using a unit cube as a standard example of a unit of volume. I will work with my students in understanding that the space within this solid is called volume. We can determine volume by counting the rows and columns of cubes or cubic units required to fill the prism. There are three dimensions to account for here: they will make layers that are rectangular arrays of unit cubes, and stack several of these layers of cubes into the boxes. I will have students use standard units of measurement to strengthen skills such as comparing, putting into order, and how to measure area and volume.

Before launching the lessons, I will engage students in a discussion about real-world geometry with a classroom poster/anchor chart. I will show how geometric shapes can be found in everyday life. I will ask students where they have seen these shapes in their daily lives. I will write on the anchor chart basic shapes and solids for area and volume we will be using for this unit, (see figure 8).

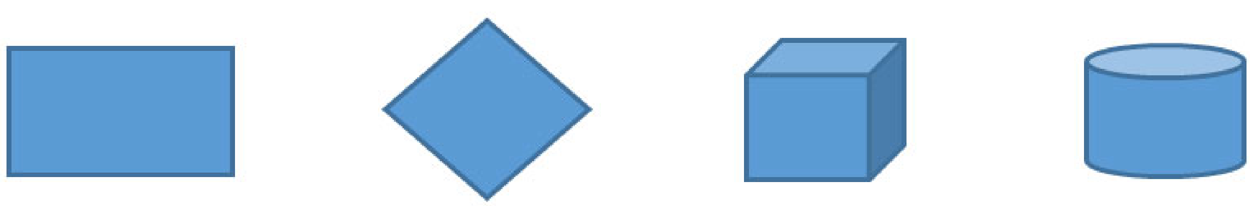

Figure 8: Basic shapes and solids

When my students work with a rectangular prism, I will use the activity of covering the surface area of a series of boxes representing rectangular prisms. Students will use wrapping paper to cover all the sides of the box. The students will understand when wrapped that that each side of the box when added together constitutes the surface area of a box. I will show them examples of boxes, such as cereal boxes or packing boxes, so that they can see how, when the box is decomposed that the box was actually formed by taking a flat piece of cardboard (with a few extra flaps) and folding it up into a rectangular prism or box. The amount of wrapping required the cover the box will be the surface area.

## Measurement Nets

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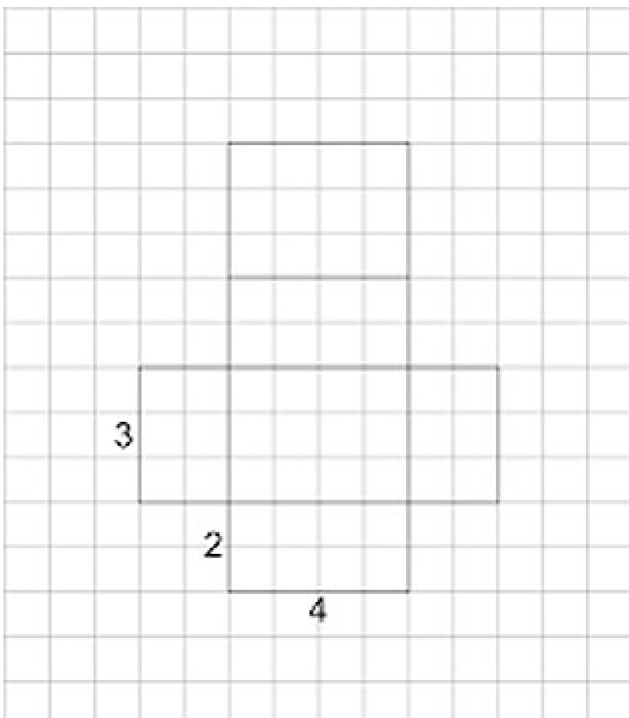


Figure 9: Arrays forming geometric nets

The interesting thing about folding nets is that it helps us to transition from two-dimensional rectangles (see figure 9) into a three-dimensional geometric solid. It’s a way for developing the idea that boxes can be made in this way. It is also an activity that helps develop comprehension through hands on activities and manipulative units. Cardboard boxes often come folded flat, as sort of doubled nets, and you make the box by unfolding and taping edges together to make the 3D shape. Rectangles can have any side length, any width but we are going to have a condition using only whole number lengths of sides of arrays.

## Geometric form of rectangular prisms

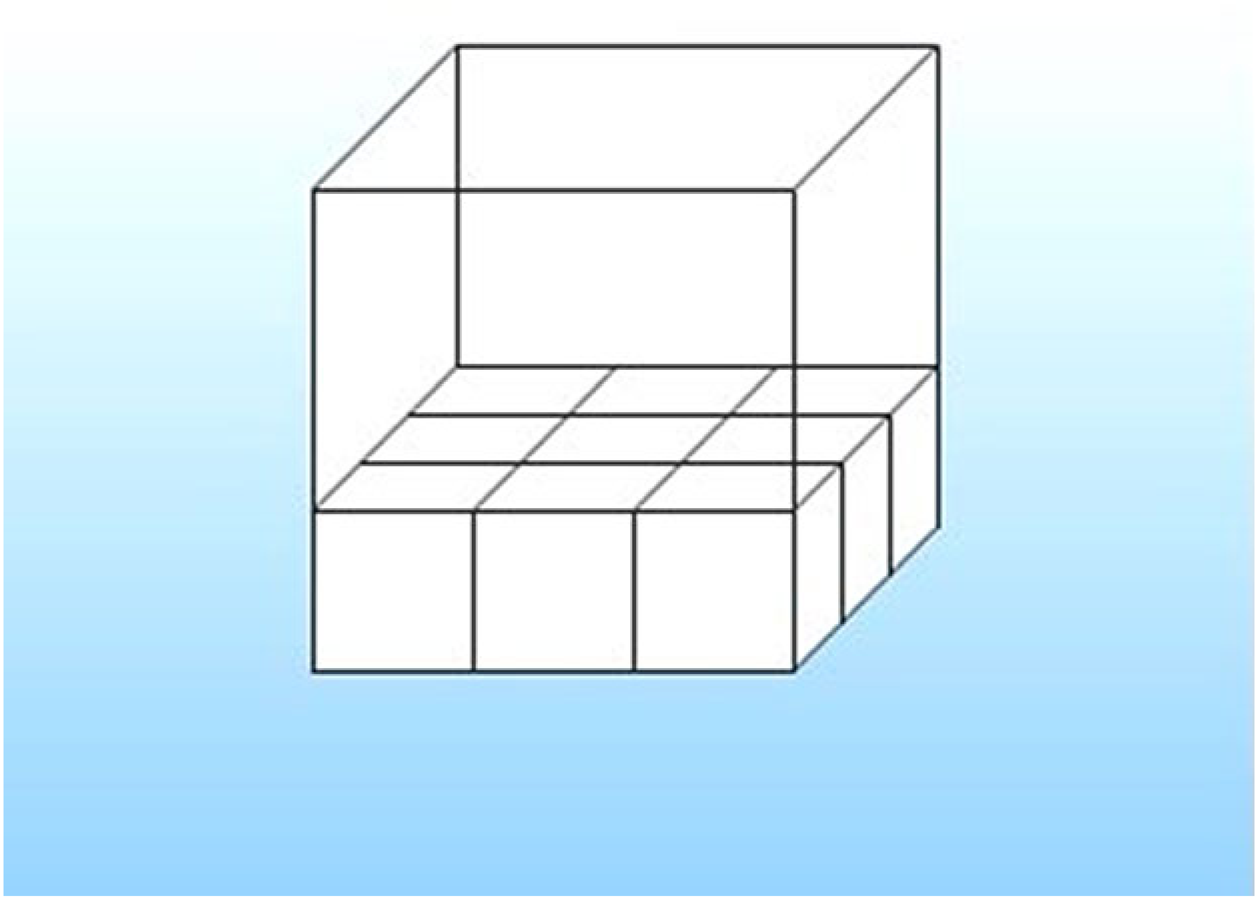

Figure 10: Rectangular prisms with layers of cubes

My students will understand that the space inside a solid shape like a rectangular prism is called the volume. I will have my students examine examples of real life geometric solids, so they can understand the volume: cereal boxes (rectangular prisms), soda cans (cylinders or circular prisms), ice cream cone (cone). I will compare and contrast the surface areas and volumes of these solids so we can begin to understand the different aspects of formulas: length, width and height. Students will learn the volume of rectangular prisms by counting the number of cubes needed to fill the prism, starting with a rectangular layer on the bottom, and working up, layer by layer, to the number of layers needed to fill to the top of the prism (see figure 10). This visual representation of volume using layers of rectangular arrays of counting blocks will help the students as they begin to use formulas for volume $V = L \bullet W \bullet H$. I will emphasize the analogy with the area of a rectangle. The process is similar, but now there is a third dimension to worry about.

## Activities

Before starting the unit, I will give pre-assessments to my students. This will allow me to measure comprehension as well as how to identify and address the misconceptions and weaknesses of my students. By utilizing pre-assessments I will have the opportunity to determine my students' level of understanding of measurement. At the unit's completion I will use post-assessments to determine the growth of student learning. Both assessments will be similar in the use of questions, material, and prompts from the entire unit. Students will be asked to respond in written form, using a sequential method of description. On a daily basis, I will use a variety of brief activities to gauge understanding and growth. At the beginning of each day, I will have students reflect and write a brief journal entry on previous lessons. At the end of the class, I will administer a short exit ticket. My students will answer an explicit prompt, so I can check for understanding. Throughout the unit on measurement, I will include time for student engagement in the practices described in the Common Core State Standards. Students will make approximate measurements, then form calculations. Next, students will use tools appropriately, i.e., how to read and use a ruler correctly. Finally, students will attend to precision in using and understanding measurement.

I will show how the following activities are related to each other with the goal of understanding measurement. An interesting feature is that these activities will be inter-mingled with the goals so that the actual participation will help promote a clearer understanding of our set of goals. Students will start the unit on determining the area of various shapes. Students will use Thinking Maps including Circle Maps and Multi-Flow maps to organize and define key vocabulary words used in the unit. Students will draw the shapes of rectangular arrays on chart paper. Students will then write key facts and details of measurement on the side of the drawing.

I will begin by developing the definition of area on a poster/anchor chart: the number of unit areas that it takes to fill a space. I will make sure that students also understand that a unit area need not be a square. For example, a right triangle with legs of length 1 and 2 is half of a 1 by 2 rectangle, so it has an area 1. But no unit square will fit inside it. Then, I will announce that in this unit, we will only study figures that can be decomposed into unit squares. I will demonstrate to my students the formula for an area of a rectangle: A (area) = (l • w). For my students to understand the composition and decomposition of rectangular arrays, I will develop this idea by having my students use grid paper to create different arrays using colored foam unit squares.

Students will also understand that arrays with specified area can have different combinations of length and width. For example, an activity would be for students to rearrange sixteen chairs five different ways (see figure 11). These different shapes all have the same area, but have different perimeters. I will give my students a number (18, 36, 200), and challenge them to find all the rectangles with whole number side lengths and the given area, and to find the perimeter of each.

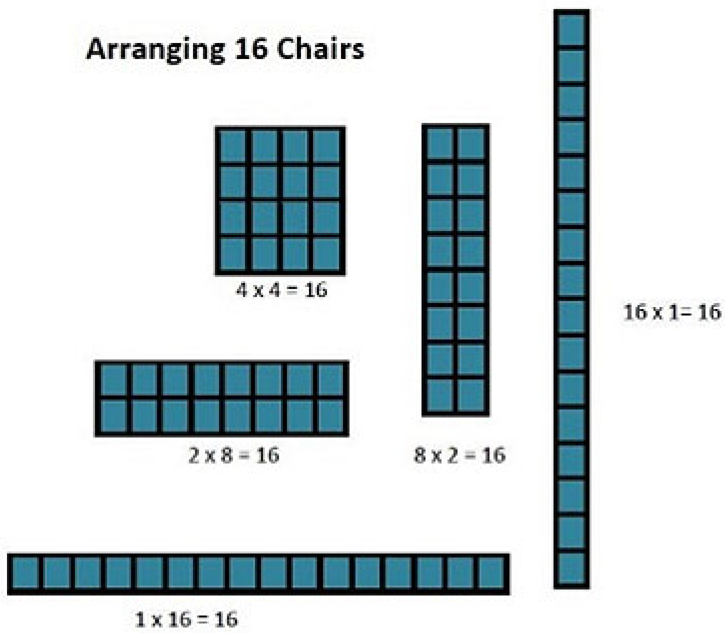


Figure 11: Arrays with the same product

I will explain to my students that the surface area of 3D objects is measured in square units and is the sum of the usual plane areas of all the 3D object's surfaces. I will demonstrate this by asking students what the surface area be of the shape they have drawn. Students will draw rectangular arrays on grid paper which can be formed into nets for cubes. I will explain to my students that now that they've mastered measuring the surface area of 3D shapes, they can move on to measuring volume, which is the amount of space inside a 3D shape, measured in cubic units. I can refer to the poster which provides essential formulas.

Towards the conclusion of the unit, I will work with my students in determining the surface area and volume of prisms and boxes that represent original structures of buildings by using formulas for surface area and volume (figure 12).

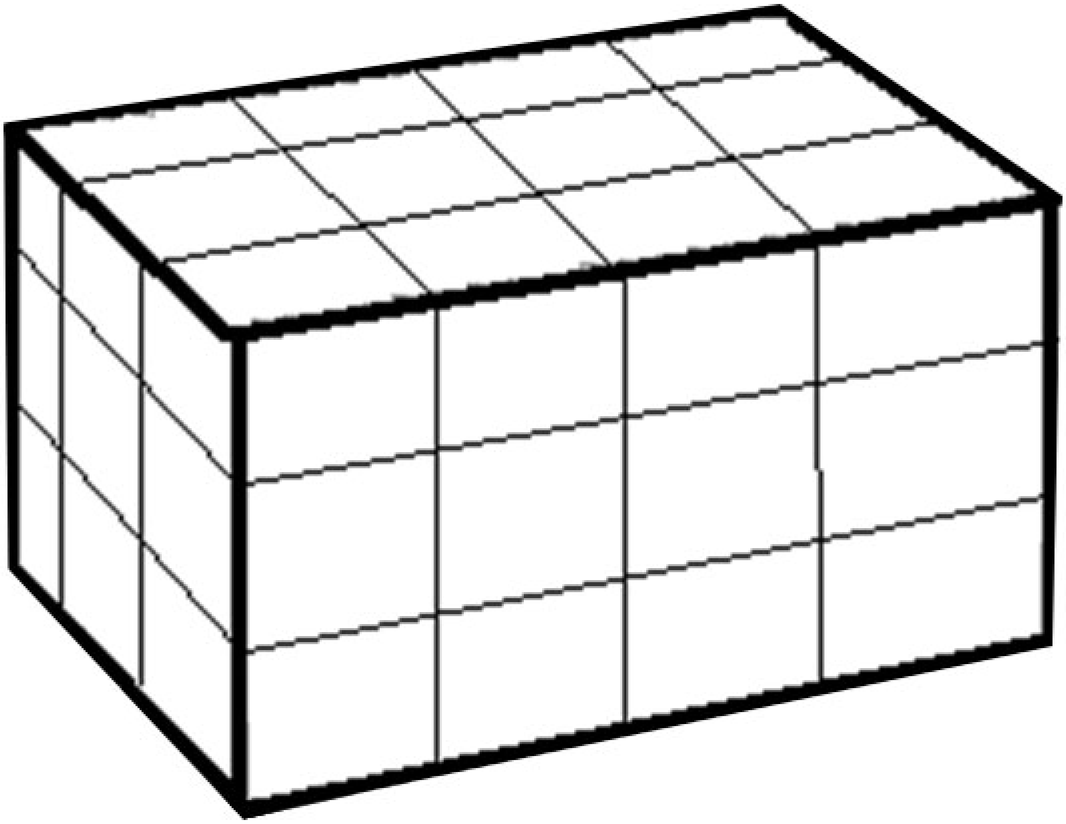

Figure 12: The formula for the surface area:

SA= 2LW+2LH+2WH

The formula for volume:

V= L•W•H

My students will create different types of boxes/prisms: long, large short and tall. To challenge my students, we conclude the unit in complexity and rigor with the activities. Using cardboard in the form of nets folded into long rectangular prisms, I will develop my student's understanding of area, surface area and volume. My students will begin with building a replica model of the Aon Center in downtown Chicago. Using cardboard and sectioned into nets, students will fold the nets into a long rectangular prism, (see figure 13). The Aon Center has the shape of a long rectangular prism. Next, I will have students work with a variety of different prisms in height to build a replica of the Willis Tower. These prisms will be by organized side by side of each other to form the model of the building.

The students will use their understanding of area, surface area and volume to measure rectangular arrays into nets which will then be folded into a series of rectangular prisms. They will then be tasked to build their own original structure/building from rectangular prisms. The students will research number of floors, surface area and volume of both buildings. I will have students use formulas to determine this data. By using area and volume, I will have my students apply understanding of rectangular arrays, surface area, nets and rectangular prisms to real life applications in the form of the building we have in our city of Chicago.

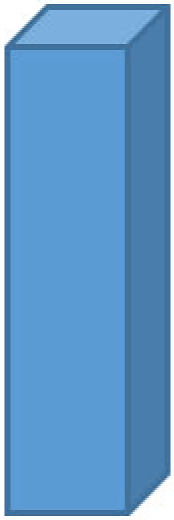

Figure 13: Understanding a basic rectangular prism and comparing the prism with similar real world applications such as the Aon Center in Chicago

The summative task is to design a building for an architectural company. Students will understand that architects and engineers take into consideration the amount of limited resources needed to compose original structures. The volume amount of the rectangular prisms will be given as a condition, and the students will come up with an original design of a building. Students will determine the surface area of their original design and be responsible for drawing a net of their rectangular prisms. Students will conduct a presentation to the class showing their structures and explain the details of area, surface area and volume.

## Appendix on implementing district standards

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This unit will be focused on how to solve real-world and mathematical problems involving area, surface area, and volume. The district #299 of the Chicago Public Schools follow the Common Core State Standards when composing and implementing curriculum.

One of the main goals of this unit is the composition of rectangles into square units.

- MATH.CONTENT.6.G.A.1 Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in solving real-world and mathematical problems. The unit will focus on the volume of rectangular prisms and how students will use measuring tools in the form of unit squares, unit cubes and formulas.
- MATH.CONTENT.6.G.A.2 Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths, and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas $V = l\ w\ h$ and $V = b\ h$ to find volumes of right rectangular prisms with fractional edge lengths in solving real-world and mathematical problems. Student's use of nets into rectangular prisms is aimed at addressing the real-world applications of engineers and building structures.

- MATH.CONTENT.6.G.A.4 Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in solving real-world and mathematical problems.

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