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Measures and Counts and Shapes, Oh...Polyomino?

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Introduction

The Unique Nature of Kindergarten

Kindergarten is unique, at least with regard to our curricular standards as specified in the Common Core State Standards (CCSS). When considering the kindergarten requirements of the CCSS, it is readily apparent that there are not many standards compared to the other grades. Point of fact: there are only 9 math standards for kindergarten (though some of these are subdivided into more specific points of focus). This is both a challenge and an opportunity. The opportunity comes as the luxury of time. Kindergarten teachers have time to explore and address a standard through a variety of approaches. This is a very good thing, because although there are a relatively small number of standards, each one is focused on critical learning. These are absolutely critical foundational pieces that the students' later math learning must rest upon. Case in point: standard K.CC.B.5 mandates that students must "count to answer 'How many?' questions." I will give my students scores of opportunities to practice and strengthen this absolutely critical skill. The sheer amount of counting that my students do to assure success in this skill area is staggering, and this in turn highlights the challenge in having so few standards. My students become bored with "the counting jar" and similar counting activities. A kindergartener should never become bored. Not only is this an undesirable introduction to the school setting but honestly there are just too many opportunities to stretch them and help them grow beyond the requirements of the CCSS. I just have to leave the safety of 'the box'.

Demographics

My school is a place that celebrates 'outside the box'. It is Kathleen Wilbur Elementary School. Wilbur is located in New Castle, Delaware. My school hosts almost 1,200 students of the grades K to 5. These children come from wildly varying backgrounds. We have students arriving to us from mansions, mobile homes, and motels. We have a large and growing English Language Learning population, and many students with IEPs. Last year my class size ranged between 22 and 19 students, though I typically host 22 or 23 at any given time. I'm usually close to an almost even split of boys and girls. Wilbur's population is predominantly African American, with a large Hispanic population, and fair numbers of white students and students of Asian decent. We have our share of students who receive learning support, or have behavioral supports in place to aid in their success.

Goals of the Unit

The goals of this unit are to address what I see as “areas of need” for my students. The activities, described in detail in subsequent sections, will challenge students and allow them additional opportunities to practice and strengthen skills in a manner that will be hands on and highly engaging. Additionally, these activities are intended to expose students to content that they will experience later on in their academic career. This very early pre-teaching of content will happen through activities that will make these bits of content attainable and manageable for my young scholars. Someday, a teacher from an older grade will likely appreciate this effort, while my students will just be exploring, working, thinking, explaining, and enjoying the tasks.

What are these challenges that I have identified and that I am addressing through this unit? My students risk boredom when overly exposed to common math activities like the counting jar (a container of objects to be counted), grab and count (which is exactly like it sounds, using small manipulatives like connecting cubes), or other similar recommended tasks. Yet they need much repetition of the counting process to strengthen the skill. Boredom in students is undesirable, but repeated exposure to counting activities is necessary. To allow for repetition of task while keeping the students engaged, I have reached outside of my prescribed K curriculum and found content that will draw them in while having them count, and count, and count some more. Win/Win.

From my personal experience and from speaking with other teachers, it seems that a lack of number sense is evident across the grade levels. This is a highly alarming idea and not just to me. Keith Devlin of Stanford University wrote, “students who lack a strong number sense have trouble developing the foundation needed for even simple arithmetic.”[1] This is a need that should be addressed and I believe that emphasizing this issue in kindergarten provides the greatest chance for students to begin developing this important understanding of mathematics. Built into these lessons, students will be collaborating and discussing their thinking throughout the process. They will be making mathematical choices and justifying their ideas by sharing data collected through the counting and measuring of geometric attributes.

A third challenge I face is the apparent disconnect between the varying strands of our mathematics curriculum. This seems especially true of our geometry strand. In the eyes of my students, we work on math from late summer, through the fall, and into the beginning of winter. Then we cease math activities for a month and we explore shapes. At the conclusion of our geometric work, our triangles and cubes are put away and math returns until the end of kindergarten. Students retain content considerably better when it connected to other content. This ‘spider’s web’ of interconnected content is a powerful aid in remembering and recalling facts and details.

Learning Goals

The objectives of this unit revolve around raising the skill level or ability of my students in three key processes: counting, measuring, and critical thinking. The vehicle for presenting this opportunity for growth is the polyomino. I should talk a bit about polyominoes to aid in the understanding of my goals.

Polyominoes

Polyominoes are shapes that are created by assembling congruent squares. The rules of assembly are:

1. Two squares that touch do so either at a corner of each, or along a full edge of each.
2. The figure is connected, in the sense that one can move from any square to any other square by passing through middles of edges (i.e., not passing through corners).

(See Figure 1). These shapes can be small or large, depending on the number of squares that are connected together. The flexibility offered by the polyomino as a tool in my lesson means that these shapes can be easily differentiated to meet the needs of students. For example, the bulk of the class may find it challenging but accessible to work with tetrominoes (polyominoes made of four squares). The use of trominos (made of three squares) would perhaps be more accessible for struggling students. To meet the needs of advanced students or even as an extension activity for students who have successfully mastered the tetromino, the pentomino or even hexomino (shapes of 5 or 6 connected squares respectively) could be a good option.

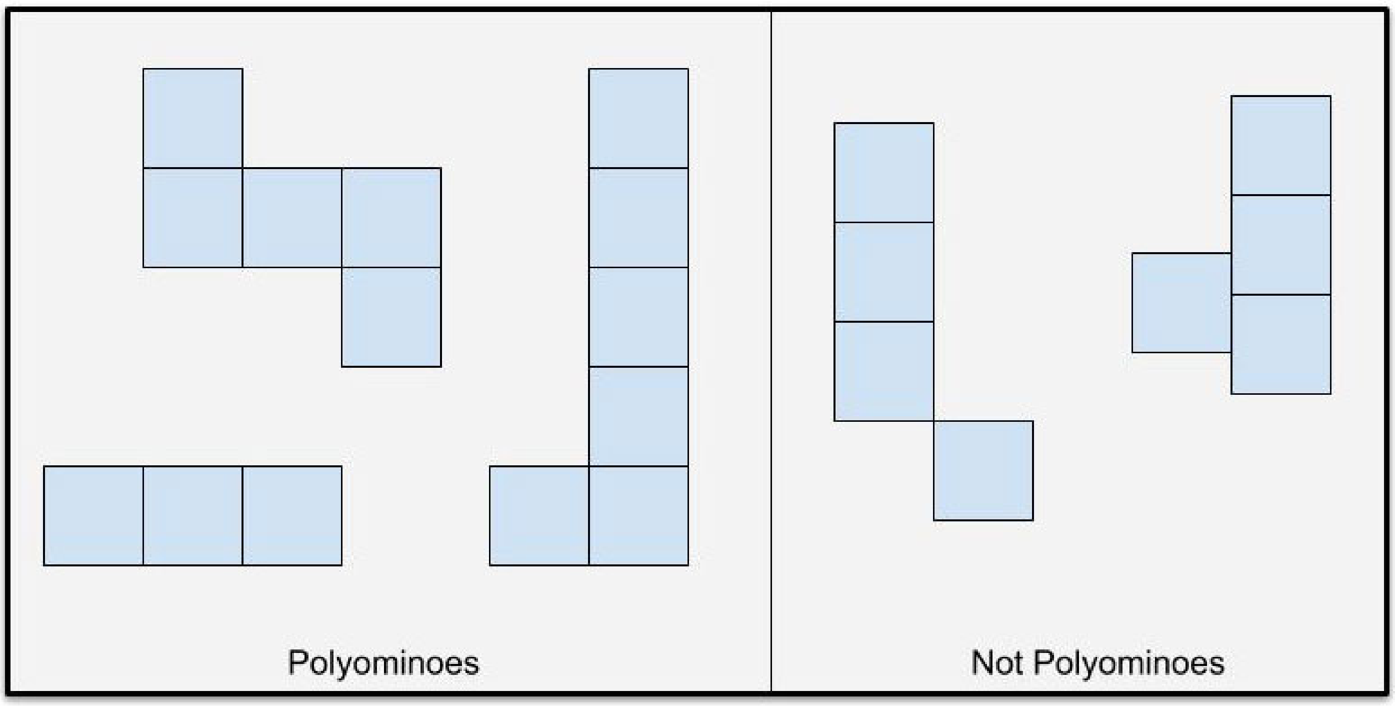

Figure 1: The left box shows polyominoes made of congruent squares sharing a common edge. In the right box are examples of non-polyominoes. Each violates one of the conditions i) or ii).

Counting

In kindergarten we count. Mostly this activity takes the form of counting physical objects (fluffy balls, cubes, plastic bears, seashells, and the like.) Counting in this form is necessary, as the act allows children to develop strategies to aid in the work; for example organizing all the bears in a straight line before beginning the count is helpful. I have come to realize that there could be another way to count that offers an added benefit. In this unit I will be asking students to count various attributes, such as the number of squares (the area), the number of exposed edges (the perimeter), the number of straight sides or the number of vertices, of polyominoes (see Figure 2). I asked myself, “Will counting the straight sides or vertices of a polyomino allow the student to practice counting?” The answer is yes. “Will the counting of straight sides or vertices allow the student to develop strategies to assure that they are counting in an organized manner?” Yes, again. But, I also recognize that something else is happening. The students will be reinforcing their knowledge of shape.

Specifically, they will be reinforcing their understanding of the vocabulary every time they count the corresponding attributes. So instead of reinforcing our concept of plastic bears, we are deeply interacting with an attribute of geometric shapes. As a bonus, since my students have been mostly counting physical objects, this activity of counting attributes will not feel like more of the same. It will present the act of counting as something new and different. Reinforcing geometric vocabulary and concepts plus minimizing the boredom of counting manipulatives over and over, equals a win for both educator and student.

The counting of straight sides and vertices offers an additional opportunity for learning. Polygons are closed shapes. Polygons are constructed of some number of line segments. These line segments share endpoints with the adjacent straight sides of the polygon. Because of the sharing of endpoints and the closed nature of polygons, the number of straight sides will always be equal to the number of vertices. As we record our data regarding these attributes, I will guide my students to this discovery. It will show that counting can lead to larger discoveries. This will aid my students later in the year when we are explicitly observing the attributes of shapes. Generally speaking, my students have an easy time recalling the number of straight sides to a shape, but they sometimes find it more challenging to remember the number of vertices. Knowing that the number of straight sides and vertices are always equal in polygons will help them in our geometry unit.

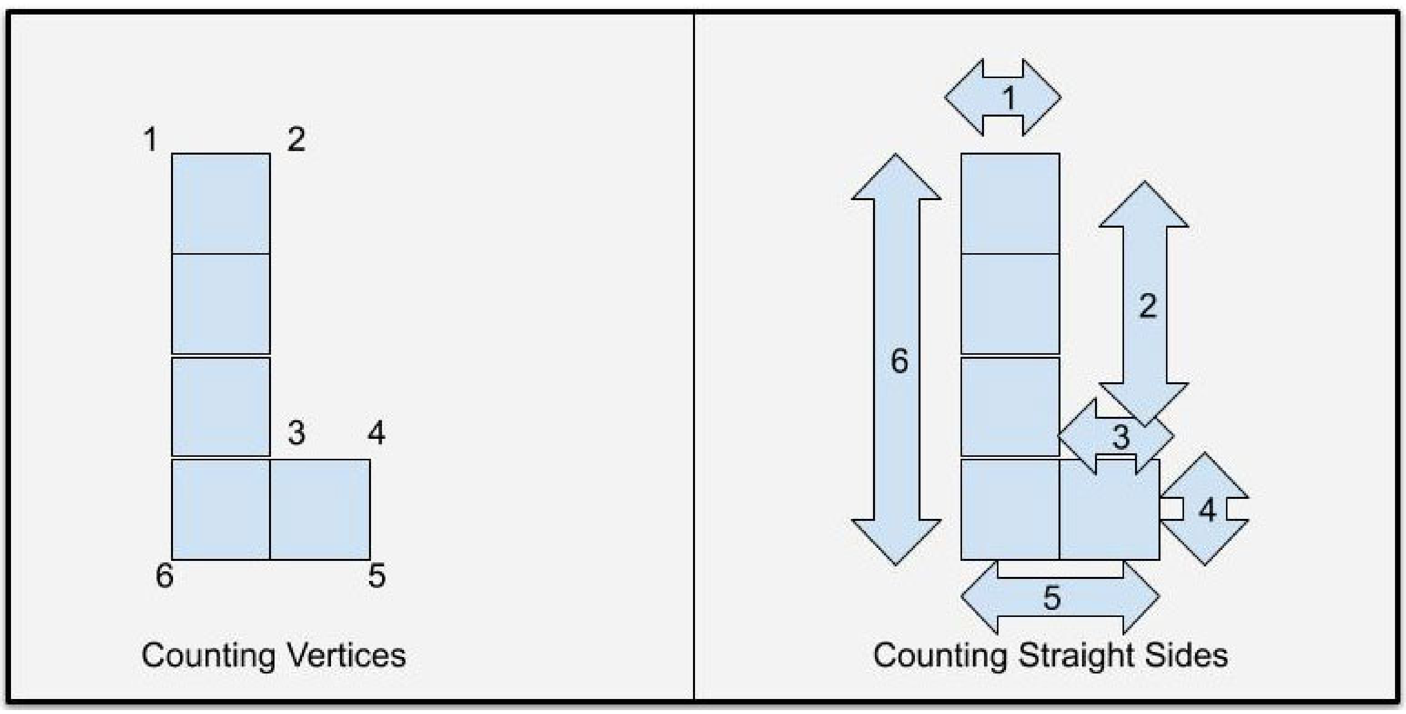

Figure 2: On the left the vertices are counted, note even interior vertices are counted. On the right, counting the total straight sides.

Measurement

We have a way of practicing measurement in kindergarten. We measure in non-standard units. Usually we will measure a physical object (like our foot, or a pencil lengthwise) with connecting cubes, or popsicle sticks, or paper clips. This sort of measurement makes sense and is certainly a valuable experience. After all, once standardized units are introduced in the later grades, kids still measure something with a physical tool (ruler, yardstick, etc.). However, in this unit my students will be measuring polyominoes. We will not be measuring length (a typical activity for grade K.) We will be exploring perimeter and area. HOLD THE PHONE! Those are definitely not mentioned in the CCSS for K. In the CCSS, measuring largely deals with examining the length or weight of a given object. Further it is required that kids compare similar attributes of two objects. Thanks to the ‘connected square’, grid-like appearance of polyominoes, it will be super easy for students to ‘see’ the perimeter by counting the edges of the squares around the outside of the shape. Area is even easier to count, as they just count up the total number of squares in their polyomino (see Figure 3). This counting of congruent squares can even lead to an observation that when measuring we are counting objects (units) of equivalent size (e.g., paper clips, connecting cubes, inches, or miles, depending on the context).

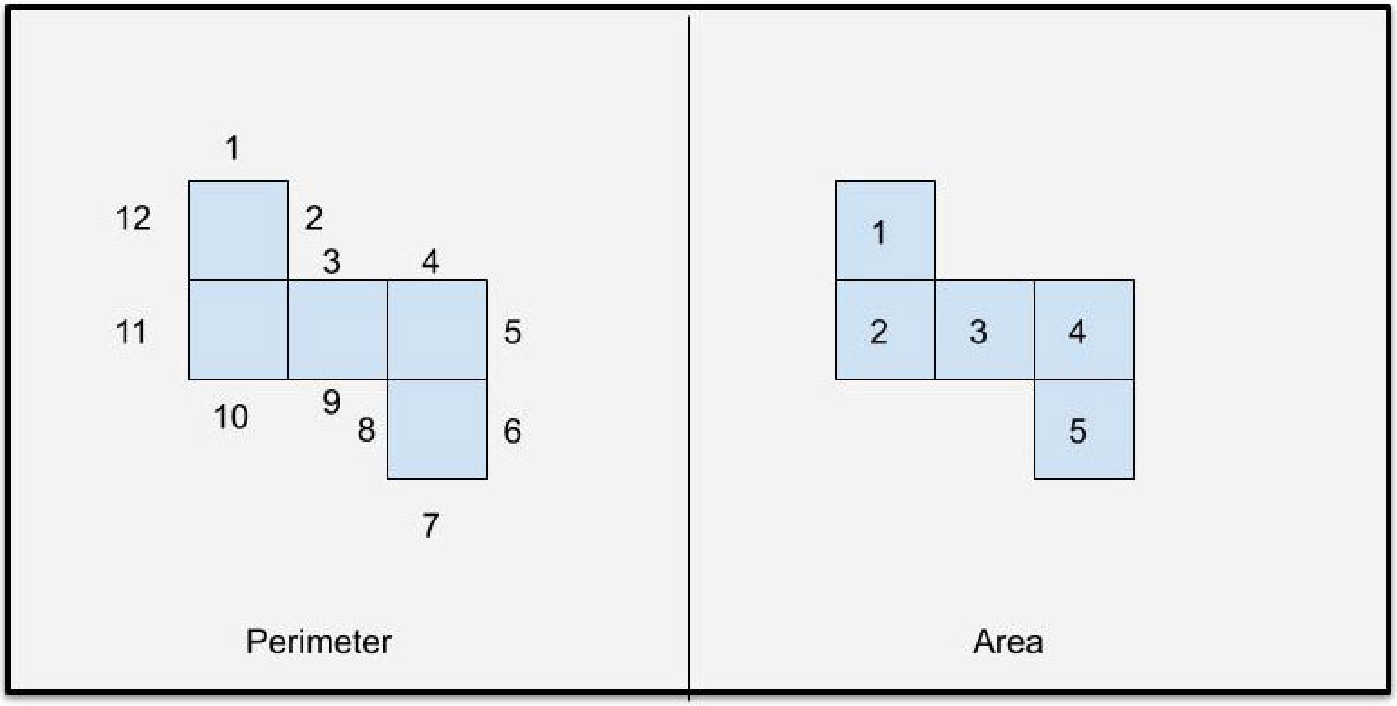

Figure 3: The left box shows the measurement of perimeter of this pentomino. To the right is the measure of area for a congruent shape.

Since the units we are measuring are not paper clips, but squares, kindergarten kids will be measuring area in, wait for it… square units. Think about that, would it be helpful for a student to enter third or fourth grade with a recollection of measuring area in square units before? I think this is a wonderful bit of knowledge that could serve these students well. Imagine, a group of children who when confronted with a concept like square units can draw upon an image of a shape constructed of congruent squares. That would boost math confidence. That would be an example of engaging with Math Practice 4 (math models) from the Common Core. “This shape is blue and that shape is red” is a typical kindergarten response to a discussion of attributes. They are right and I’d be happy with the answer, but imagine hearing this: “Those two shapes are alike because they both have an area of 4 square units”. These polyominoes can also afford my students the opportunity to make discoveries about the concepts of perimeter and area. For example, polyominoes of equal area can have differing perimeters (see Figure 4). Or, by changing the arrangement of our connected squares we can maintain area and can either raise or lower perimeter. This idea will highlight the fact that area and perimeter are two separate and independent concepts. I am excited at the prospect of leading the math talk about that.

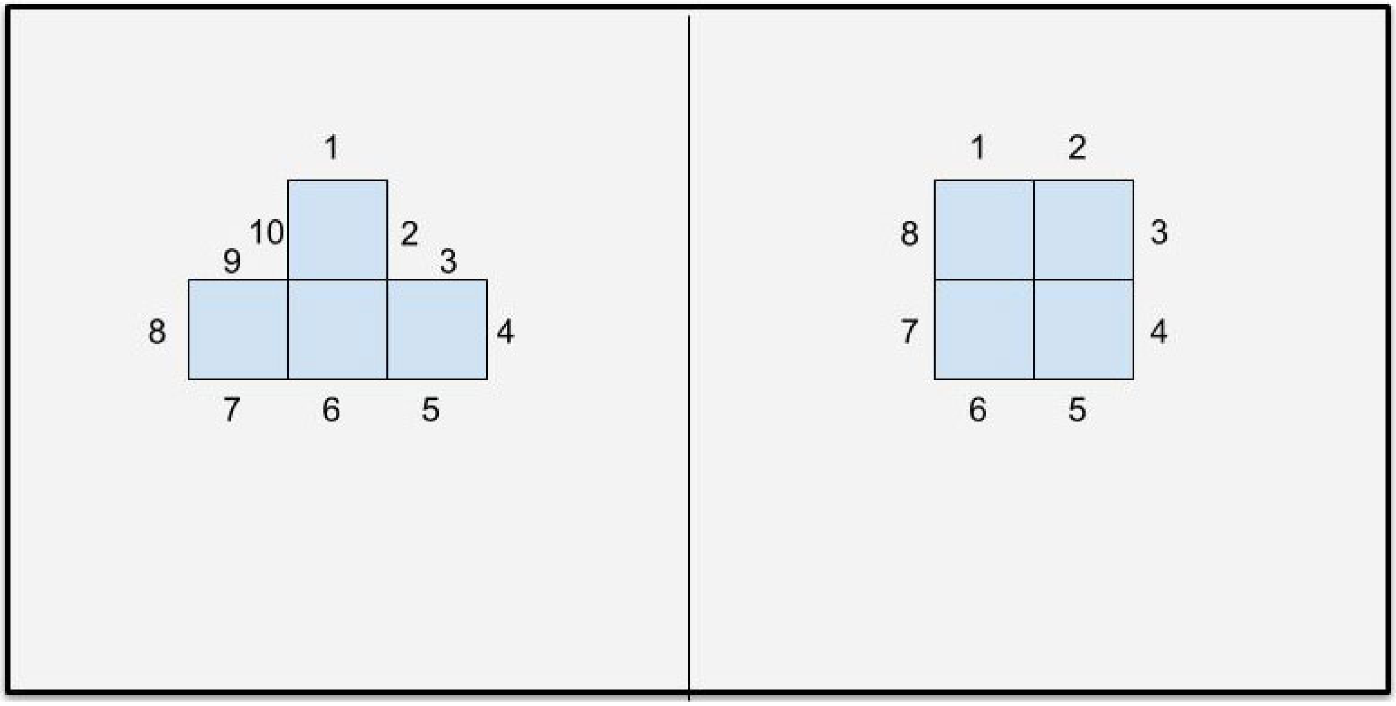

Figure 4: Both of the above tetrominoes have an area of 4 square units. The perimeters of these shapes differ.

Measuring polyominoes will increase the level of rigor in my class discussions of mathematics. The deeper the math chatter is, the greater the sense of math concepts that my students will develop.

Critical Thinking

Students need opportunities to think. This seems obvious, but too often what is required of primary students involves little thought. Unimaginative tasks yield unimaginative solutions and result in unimaginative students. A worksheet problem that asks a student to draw 4 flowers requires little consideration, even from our littlest scholars. This unit was designed to challenge young scholars. Through the activities detailed below students will be practicing skills such as those mentioned above, but they must also make models, make choices, and justify those choices using data gathered by counting and measuring and studying attributes of geometric shapes. They will have to sort shapes based on their attributes to best match a set of requirements. Put simply, they will be required to think. They will be supported throughout the process and they will succeed. They will be engaged in rigorous and relevant lessons as they create polyominoes, and address multiple CCSS in a thoughtful and purposeful way.

Other Items to Consider

Spatial Awareness

In a case study by Moss, Hawes, Naqvi, and Caswell we read: “Increased efforts are needed to meet the demand for high quality mathematics in early years classrooms. Despite the foundational role of geometry and spatial reasoning for later mathematics success, the strand receives inadequate instructional time and is limited to concepts of static geometry.”[2] I find this assertion alarming, as should all practitioners of early childhood education. Spatial reasoning/awareness is important and yet lacking in most classrooms. To me, this is unacceptable. Spatial reasoning is one of the 9 identified types of intelligence. I have witnessed that some of the least traditionally academically gifted students excel with spatial reasoning. Activities that require them to manipulate shapes in space are their opportunity to shine. There is something powerful about watching a child who struggles demonstrate high competence at a task. The opportunity for students who struggle elsewhere to share their expertise with a peer is confidence building. The tasks in this unit will directly respond to the charges above. Students working through these activities will move beyond static geometry. They will be manipulating shapes and looking closely at their attributes and parts.

Interconnectivity of the Math Strands

A goal of mathematics education is to create numerate citizens. The New Zealand Ministry of Education has stated; "Numerate students are able to apply their number understanding to a range of content, from the other strands in mathematics."[3] I have found that sequencing and pacing of the curriculum tends to isolate geometry, and to a lesser extent measuring and data, from the rest of the math strands. As previously stated, my students don't even consider geometry work as part of math. This is a missed opportunity to strengthen the impact of our unit. If we are able to make clearer and deeper connections between the strands in our lessons, then this unit will be addressing standards from multiple strands. This connectivity should aid in retention of information and the recall of all concepts involved. Moreover, connectivity does not necessarily relate to only grade level content. Linda Gojak, the president of the National Council of Teachers of Mathematics stated; "it is important to think about the connections among concepts within the grade level or courses that we teach, it is also important to reflect on the connections across grade levels."[4] The memories students will have of these lessons will resurface a few years down the road and connect our explorations and takeaways from polyominoes into a solid basis for their new lessons on area and perimeter.

New Connections and Innovations, Same Standards: A Brief Overview of the Unit

Activity One: Meeting Polyominoes

I will present the students with this challenge statement; "Given four square tiles, find and record as many different shapes (tetrominoes) as you can." I will then introduce conditions i) and ii) for the creation of these shapes.

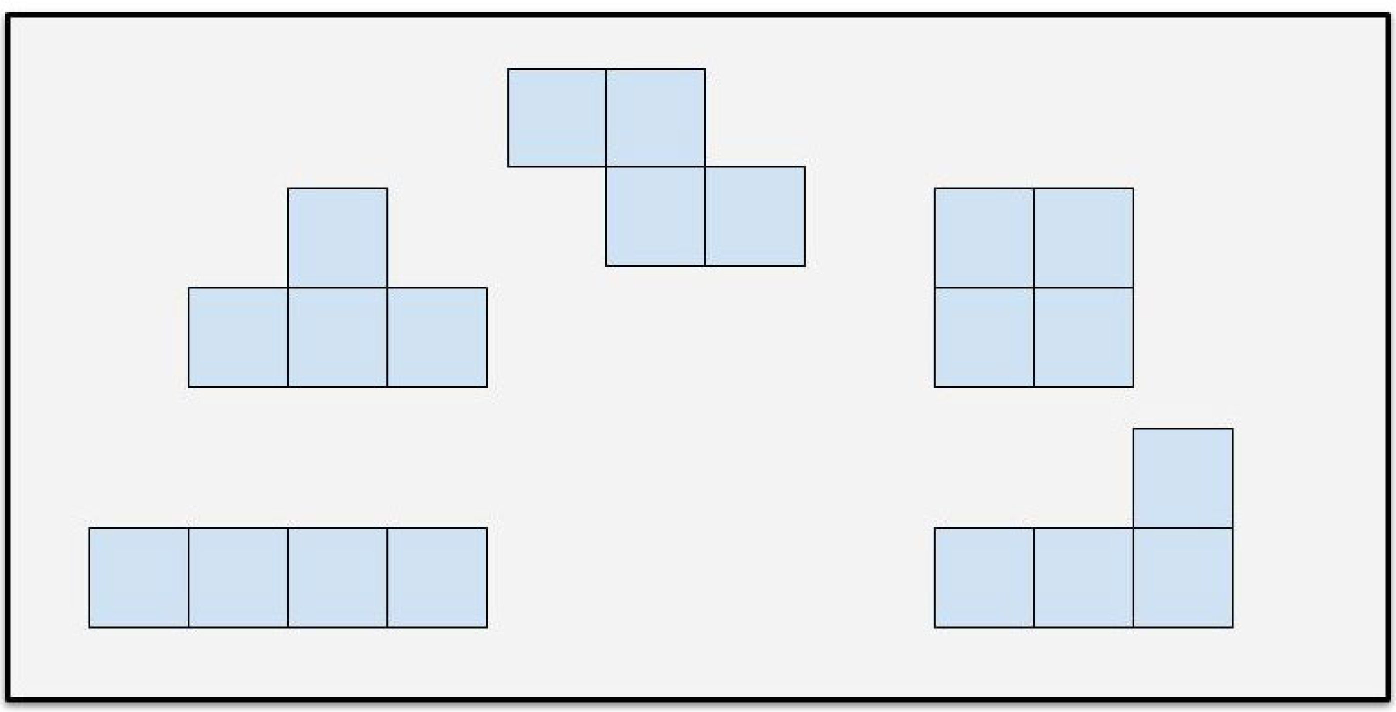

Figure 5: These are all the tetrominoes.

This activity will have many positive outcomes. One outcome is that it will address the CCSS, K.G.B.4, which basically requires students to describe similarities and differences between shapes by examining their parts and attributes. Typically, kindergarten interacts with this standard by comparing regular polygons, such as a triangle and a square. They then compare and contrast the two. They may note that both shapes have corners or that there are differences in the numbers of sides. Examining the tetrominoes will require a more critical eye for details. Further, discussing the similarities and differences between two of these polyominoes will demand a more rigorous level of explanation. Instead of 'they both have straight sides", the observation could be more like "they both have three squares connected in a row." This deeper level of discussion would be indicative of a deeper level of mathematical thinking. A second benefit of having students work through this activity is that they will be developing 'math endurance' and perseverance during math studies. Finding new and unique polyominoes can be challenging, though due to the hands-on nature of these explorations, still engaging and even fun (see Figure 5) Working through mathematical challenges and struggling through mathematical processes is the only way to truly strengthen the CCSS Math Practice 1, which is all about persevering through tasks. Finally, during this lesson, I will be introducing or reinforcing vocabulary. Common kindergarten words that will be reviewed and reinforced through this activity are attribute, similarities, and differences. These three terms are highly important as they cut across the content areas of our curriculum. Additionally, this will give me the opportunity to expose my students to the terms rotation, reflection, and translation. These really aren't kindergarten terminology, but I know my young students can manage them and my class is almost always super excited to receive "secret older kid knowledge", that I of course tell them they are not even allowed to know. The terms rotation, reflection, and translation will naturally enter our conversations as we determine whether similar shapes (say a left facing and a right facing L shaped tetromino) are indeed different shapes. The students will find that we can discover many similar shapes by simply manipulating our tetrominoes.

Activity Two: Counting

Once introduced and familiar with the tetrominoes, I will challenge my students to "examine our new shapes and to then count and record the numbers of vertices and straight sides of each tetromino." This is a central task of my unit; COUNTING. As indicated earlier, my students need to be counting everyday. The counting also has to look and feel different to stave off lack of engagement. I honestly believe that my students will not even realize they are busily counting. To them, they will just be exploring attributes off 'their' shapes. (Allowing the students struggle to find these shapes in activity one gives them a sense of ownership for these polyominoes.) But in my evil genius mind, I've got them counting a whole bunch: "Mwuhaha". Point in fact; as the school year progresses, and I continue to involve my students in counting, I can circle back to this activity by introducing pentominoes. Adding a fifth square tile will increase the level of challenge, and of course the amount of vertices and straight sides possible. Working with the pentominoes will feel familiar enough that this activity could easily be introduced into math workshop. The activity will also offer enough new challenge, that my students should not find it boring.

There are multiple positive outcomes of Activity Two. First and foremost, we will be addressing several required standards of the CCSS. As my students record their counts they will be practicing writing numerals. K.CC.A.3 requires that students can represent a number of objects (or in this case attributes of tetrominoes) with the written numeral. The skills of standard K.CC.B.4 are awakened anytime counting occurs correctly and highlights the relationship between quantities and numbers. My class will also be answering those "how many" questions (as in how many vertices are in this shape) as prescribed by CCSS K.CC.B.5. This lesson also focuses student attention on standard K.G.B.4; a geometry standard that focuses on exploring similarities and differences between shapes with a focus on the attributes and parts of those shapes. This activity allows me to link two different math strands together, connecting geometry to counting and cardinality. The lesson provides an opportunity to explore the relationship between vertices and sides of polygons. The number of sides will always equal the number of vertices. This is because a vertex acts as an endpoint of a straight side. Each adjoining side is connected at a shared endpoint. So 4 sides equals 4 vertices. This relationship could be a useful bit of knowledge to aid the students in remembering the attributes of regular polygons. In my experience, young students more easily remember the number of sides than that of vertices. Knowing they are always equal, will aid in describing the attributes of a given polygon.

Activity Three: Measurement

Activity Three presents a similar but different challenge to my students than the last activity. They will explore the tetromino shapes and then measure and record the perimeter and area of each shape. I anticipate that my students will have a challenging time differentiating perimeter and the number of straight sides. This confusion does open the door for me to point out a major difference between counting and measuring. It will be easy to show them that when counting straight sides, the lengths of those sides may (though not always) vary. When measuring the perimeter we are not counting sides, but the number of the edges of the squares that make up the shape. Unlike the lengths of the sides of a polygon that may be different, all of the edges of our squares have the same length. Measuring requires units of the same size, while length is ignored in the concept of “side”(see Figure 6). It will often turn out that shapes with quite different numbers of straight sides will have the same perimeter!

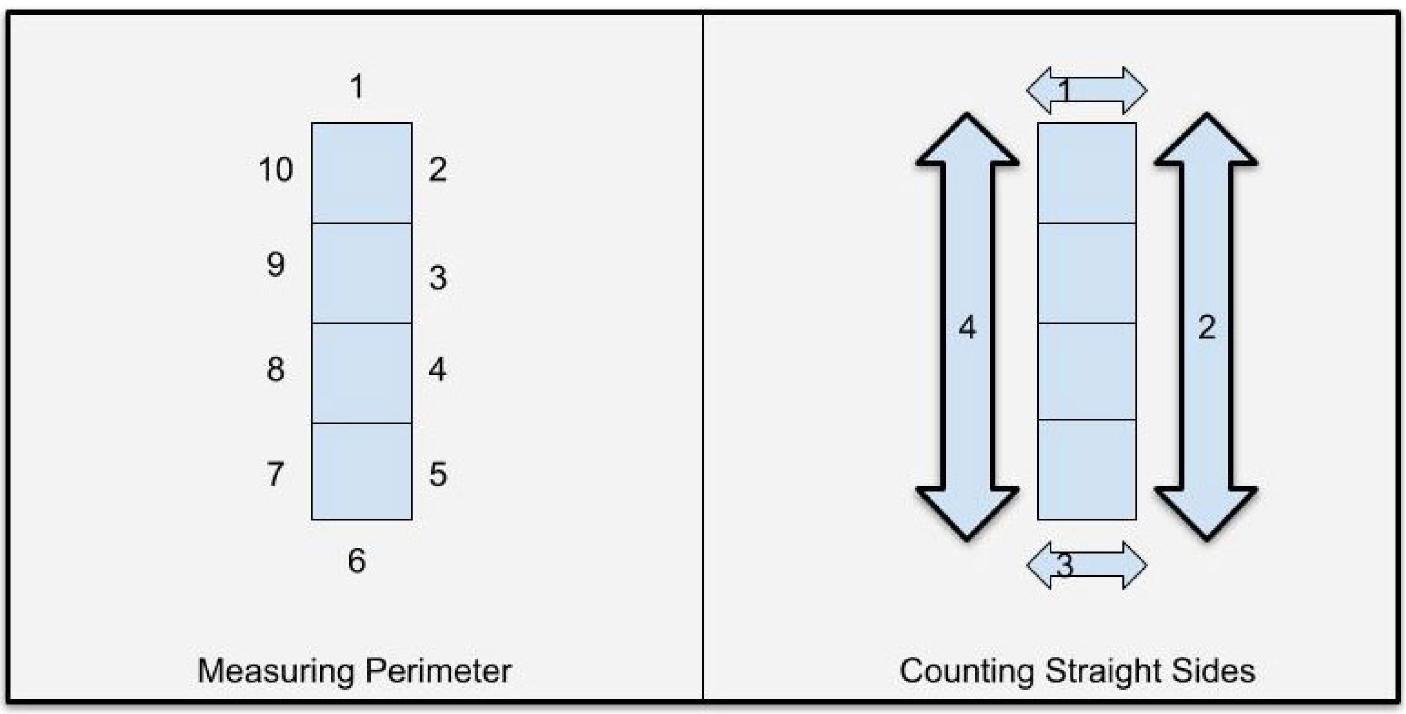

Figure 6: The differences between measuring perimeter and counting straight sides is highlighted. The difference will require explicit teaching.

As a result of this activity the class will be engaging with several Common Core standards. Firstly, standard K.MD.A.1 will be addressed. This unit has students describing measurable attributes of shapes (area, perimeter). They will also be revisiting the same Counting and Cardinality standards as the prior lesson. Much as the last lesson drew a connection between counting and cardinality and geometry, this lesson will tie the counting and cardinality and measuring and data strands together, as well as connect them both to geometry through geometric measurement. Connected strands make for easier recall and deeper understanding. As with the other activities in this unit, vocabulary will also be introduced or revisited. Terms for the students will now include perimeter, area, and measurement.

Activity Four: Critical Thinking

This final activity will begin with a story, which I include now. “I have a fence in my backyard. I also have three dogs. Bean loves walking the fence. She always looks for squirrels and wants to change directions a lot. Dory loves walking the fence too. But she wants a long stretch of fence to race along. Izzy also loves walking the fence. She would like a fence that gives her a very long walk.” With this story in mind, my students will pretend that my backyard fence is in the shape of one of our tetrominoes. They will be challenged to match a fence (shape) up to each dog that would satisfy that dog’s desires. I will further insist that students justify their pairings of fences to dogs. This lesson will grant access to several CCSS for my students. Tetrominoes will be sorted into different categories. The specific categories, while based on the dogs from the stories, will in truth be based on attributes of the shapes. Bean’s fences will have the most vertices possible. Dory’s fence will have the sides of the greatest possible length (it will be a 1 by 4 rectangle, in the case of tetrominoes.) Izzy’s fences will have the greatest perimeter. This classification of objects is related to standard K.MD.B.3. While my students are comparing objects by considering their attributes, they are also working to master standard K.MD.A.2. A major reason for this activity is to engage students in a critical thinking exercise. They are required to consider multiple shapes and determine which of the shapes has more or less of a given attribute. They must then justify their selection by explaining why it would be the best choice for Bean, Dory, or Izzy. This type of thinking will engage the student with Math Practice 3: constructing viable arguments. This final activity will also reinforce much of the vocabulary that has been in use throughout the unit. It is worth noting, that while Dory will always find one perfect shape for her, Izzy and Bean may find multiple shapes that will satisfy their wishes. There will be an opportunity for an interesting math discussion if teams of students find different (though both viable) solutions for Izzy or Bean. Questions surrounding different shapes with the same perimeter or number of vertices could provoke a valuable conversation. An interesting bonus task could be to have the students identify the only shape that would not match any of the dogs’ preferences (it’s the square-shaped tetromino). Another possible adaptation would be to change the names of the dogs (Bean, Dory, and Izzy are my actual pets and I refer to them frequently during the school year so my students have a connection to them). Without this connection it may best to change their names to better scaffold the activity. Bean could become Victoria as she loves Vertices. Izzy could become Percy, to match Perimeter. Dory could become Larry, for Long distance.

Summary of Benefits

This level of rigor is rare in kindergarten. In the typical K class the students have many opportunities to talk math and solve challenges, but the segmented appearance and hands-on nature of the polyominoes makes incredibly high-level math thinking accessible for even the youngest of students. The students will be determining perimeter and area, using standard measurement like square units, and focusing deeply on the geometric attributes of shapes. These three too often disparate strands of our mathematics curriculum are twined together making a strong rope of knowledge leading the students beyond the typical. The fact that my students will be exposed to content from upper grades in a manner that will allow them to both understand and successfully complete activities will give them a leg up when the content is revisited in later years. Meanwhile, the subtext of this unit is that while all this heavy lifting is being undertaken, the students are also practicing multiple critical kindergarten standards from multiple strands of the CCSS.

Possible Extension Activities

As stated earlier, I am always looking for ways to engage my students in things that will allow them to practice basic skills in exciting ways. This unit can be easily differentiated or extended as time and student interest allow. It is easily scaled up by using polyominoes with more than four tiles. Introduce a 5th tile and the polyomino's area goes to 5 – you get a pentomino. While there are only 5 distinct tetrominoes, there will be 12 distinct, pentomino arrangements. Adding a sixth tile and creating hexominos raises the total number of distinct shapes all the way to 35.

Another potential way to extend the unit and raise the level of challenge is to move away from polyominoes altogether and explore polyiamonds. Polyiamonds are very similar to polyominoes Polyiamonds are made not with square tiles but by connecting congruent equilateral triangles by at least 1 common edge. This substitution would keep the basic lessons the same but is would give it a new appearance and feel to the students. In this variation, I would move my discussions of area back to nonstandard units by measuring the area in triangles. Another substitution for the square tiles would be hexagonal pattern blocks. Connecting these congruent shapes would have the students creating polyhexes.[5]

Teaching Strategies

Polyominoes

The polyomino is remarkable well suited as a delivery mechanism for learning at this level. Being made of connected square tiles, they are easily countable for kindergarteners. This feature means the young students can readily access the advanced concepts of area and perimeter. We will sidestep the multiplicative aspects as they are not prepared for that, and they will instead be practicing counting repeatedly. As an additional bonus, square tiles are pretty much a standard supply found in primary classrooms. The children will be familiar with them. This familiarity should allow for a comfort level as the class struggles to find the tile arrangements.

Math Talks

"Learning math is not a process of acquiring a set of facts or procedures, but a process of becoming one who participates in a community that does mathematical work."[6] And in a community we share ideas, identify errors, and defend our choices. Math talk will take many forms in these lessons. Students will be working with a partner to complete tasks. During these tasks they will be discussing the shapes and attributes of their polyominoes. They can guide each other in the creation of these shapes. They can discuss the attributes and listen for inaccuracies as the counting and measuring is occurring. Students will also be talking math with me as they explain their solutions to my challenges, and defend why their choices are indeed the best choice. Another aspect of this community that does mathematical work is that through this collaboration and sharing we will be reinforcing that our classroom is a safe place for risk taking and sharing ideas.

Notice/Wonder

Notice and wonder is a regular strategy that I use in class. It appears in all content areas and is a powerful tool. Essentially it is just what it sounds like. I present an image, an equation, some words, or a physical object that the students are then asked to consider. All students are given the opportunity to share out to the class. They can choose to share an observation, make a connection, or even tell a brief story. If a student prefers he or she may ask a question about what they have examined. This strategy is excellent because every student can share out. I accept and sometimes record all responses without judgment or validation. Much like math talks, this is a great activity to maintain the 'safe space' atmosphere that I deeply desire in my classroom. Perhaps my favorite aspect of this strategy is that it is very open. I regularly get notices or wonders that I had never considered myself. It is unscripted and can take the class to unexpected destinations. This open aspect of the strategy also offers valuable insights into the understanding of the members of the class. It is worth keeping in mind that while observation is extremely important in mathematics, the open nature of this strategy can lead us to some challenging conversations.

Collaboration

Working together on a shared task is pretty standard fare in elementary school and my room is no exception. The benefits of collaboration are many. First, it allows kids to have a sounding board for their ideas. They can share directly to a partner, which frees me up to circulate about the classroom and listen, ask probing or guiding questions, or just take the heartbeat of the class and activity. Additionally, by regularly changing up partnerships, collaboration helps bond the students together and build trust. Trust is important to maintain that 'safe learning place' environment. Collaboration allows the students to step into the role of teacher for a time. I have always found that if a person can teach a thing to another then they really have an understanding of that thing. Collaboration in these activities will be crucial as two or three sets of eyes are much better at finding and interacting with the polyominoes.

Hands-On Learning

Kindergarten is the land of the manipulative. All students, but especially the young ones, learn much through a physical connection to content. Don't look at a picture of a tree, go outside and hug one. When introducing new concepts the more concrete the better. Polyominoes are physical manipulatives that the students will be using as they are introduced to some rather abstract geometrical and measurement concepts. They will also be configuring arrangements of square tiles to find differing polyominoes. It will be invaluable to be able to work through rotations, translations, and reflections of our shapes by actually picking them up, turning them, sliding them, and flipping them. Kindergarten students are better able to understand a thing when they can make it, pick it up, manipulate it, and recreate it if necessary.

Activities

Activity One - Meeting Polyominoes

In preparation for the activity, I will pair my students with a partner (or two if I have an odd number of students) for collaborative work. I will be making a concerted effort to form partnerships based on mathematical skill level. I aim to pair high performing students with middle range students and struggling students with middle range students as well. I would like to avoid pairing struggling students with high performing students, as this combination may not be conducive to equitable roles during math talks. Once teams are made, I will reintroduce my students to the square tiles. They will be familiar with these tools from the beginning of the school year when they were available to them for free explorations. Each team will be given only 4 square tiles and some graph paper. I will then introduce the term tetromino (a shape made of 4 connected tiles) and explain how to make them according to conditions i) and ii) (see above). During this explanation, I will be modeling these conditions on the board. I will then pose this challenge: “connect your 4 tiles to find as many different shapes as you are able.” I will then require them to record each new shape that they find, by coloring boxes on the graph paper.

At this point, the students will begin collaborating on the task. I will be circulating throughout the room, observing the work. During this time, I plan on asking questions, guiding explorations, or encouraging/praising effort dependent upon the progress of my teams. I anticipate that these teams will be able to find some if not all of the tetromino shapes. I also expect my students to include multiple copies of similar shapes on their paper. I imagine that many will not recognize rotated or reflected shapes as “being the same” at this point.

At the close of the task, the class will gather on the carpet and prepare to share out and engage in math talk. I will begin by asking teams to share shapes that they found. As I record these shapes on the board, I will invite other teams to add to their papers if they are missing one. When a shape is mentioned that is congruent to one already listed, I will record it on the board but then open a notice and wonder session about the two congruent shapes. This will be a great time to introduce the terms rotation, translation, and reflection, as well as the term congruent. On the board I will show how different looking shapes can actually be similar. I’ll then invite the students to search their papers for similar shapes that have undergone a rotation or reflection. Students will continue to share out their ideas until we no longer have new shapes to add. If all unique tetrominoes are accounted for, there will be a total of 5. If class ends and we have not found all 5, I would revisit the activity the next day with the challenge of finding the missing shapes.

Activity Two - Counting

I will begin the activity by introducing the term “attribute”. The students will have likely heard the term before, as I use it in science class frequently. I will talk about how the number of straight sides of a shape is an attribute of that shape. I will then model the counting of sides of an assortment of shapes (I’ll draw some irregular polygons.) Students will also be given the opportunity to count the sides of shapes. When I’m confident that the kids understand the procedure, I will introduce the terms “vertex” and “vertices”. I will then model counting all the vertices of an irregular polygon. I will make certain that I include some polygons with interior vertices. Typically my kids can easily count the exterior “corners” but they infrequently encounter the interior vertices in our work with regular polygons. The students will have a chance to count vertices at this time also.

Each student will receive their graph paper containing shapes (from activity 1), 4 square tiles, and a new polyomino attribute-recording sheet. They will return to tables to collaborate with their partner as they count and record the numbers of straight sides and vertices for each tetromino. As students work, I will circulate from team to team asking them to count aloud. This will allow me to assess their understanding of the activity, the term vertices, and their proficiency with basic counting. A quick glance at their papers will also afford me the opportunity to assess their ability to write numerals. I would expect a fair amount of numeral reversals in their writing. Reversals are very typical of kindergarten students.

As the teams complete their job, they return to the carpet prepared to share their data. As students report their findings, I will fill in the data on the board. Should any disagreements arise we will of course discuss our ideas and come to an agreement as to the correct response. Teams that may have mistakes or incomplete portions of their paper will be encouraged to correct or complete any answers that require it.

“Notice and wonder” will provide the class with an opportunity to engage in math talk. The focusing question of this strategy is: “what do we notice about the number of straight sides and the number of vertices.” Does anyone notice anything or wonder anything about what our data shows? After examining all 5 or our tetrominoes, we should note that the number of sides and vertices are always the same. If the class is unable to make this connection on their own, I will guide them there. This fact is a take away I intend my students to leave the activity understanding. I will conclude the activity by collecting papers for safe keeping until activity 3.

Activity Three - Measuring

The activity begins with a review of the term “attribute”. I will give the students the opportunity to share attributes with which they are familiar. I anticipate them answering with attributes like color or size. I would also expect them to respond with number of straight sides and the number of vertices. If the class does not list these two as attributes I will be certain to get them included. I will then introduce the term “perimeter” as a new type of attribute. As a means of introduction, I will share a picture of a fenced-in yard. This image will help me define perimeter for the students. At this point I will explain the difference between counting sides and measuring perimeter. I will demonstrate the difference by finding both the perimeter and the number of sides for a polyomino made of 3 tiles (in which the amount of these attributes is different) on the board. After modeling the finding of perimeter, I will ask the class if they notice anything different between counting and measuring. I do not expect the students to find this difference without help. At their level, I would like the kids to understand that measuring is also a form of counting, but that it is the counting of units of the same size. I can point this out on the board. In the counting of “sides” activity, the lengths of the sides are not the same. Some sides may be long and other could be short. This is different than when measuring. After this distinction is made, I will introduce the term area and demonstrate how we measure it. In this unit, the measuring of area is simply counting the (squares) units used to create our polyominoes. When the students are working with tetrominoes, the area will always be 4, as tetrominoes are always made of 4 square tiles. I will explain that when we report the area of a polyomino, that we will state the area as a number of square units, since we are counting units that are square.

I will then return 4 square tiles and the graph paper of tetrominoes to the teams and have them begin measuring and recording the perimeters and areas of each shape. As the students work through this stage of the activity, I will circulate about asking students to find the perimeters for me. By observing this I can again check on understanding of perimeter as well as assess their skill at counting. I will follow up by asking them what the areas are of the different tetrominoes. If they reply with 4, I will remind them that they need to include the units, and that the proper answer is 4 square units.

As the class finishes the task, they will return to the carpet to report our measurements. I will record the class’ measurements on the board and encourage all teams to correct or complete any mistakes or incomplete sections of their paper.

Activity Four- Critical Thinking

A review of the tetrominoes and their attributes will start this activity. I will then relate the story of my three dogs (found above in Content.) Following this story, I will provide the teams with a recording sheet and the 4 square tiles (should they wish to use them) for the challenge. The challenge consists of the teams matching tetromino shaped yards to the needs of the specific dog.

The teams will compare the needs of the dogs to the attributes of the tetrominoes. Students will discuss the matching of tetrominoes to dogs and be prepared to justify these pairings by using data about the attributes of each shape. While these choices are being made, I will circulate about and listen in to the conversations. If teams are not considering the attributes of our shapes during their decisions, I will guide them in the direction to do so.

As a final step to the activity and an assessment of my students' learning, I will have them state their choices and explain their thinking. There are many ways to have the students do this. For example, they could report to the teacher one team at a time. Another option would be to have the class record their thoughts with an I-pad. I will have my students use an App called Seesaw. With the Seesaw app, the students can record a video of themselves sharing their ideas about the final challenge. They will then upload this video to my computer and I will be able to view their responses at a time of my choosing.

Appendix

Counting and Cardinality

K.CC.A.3 – Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Students will be engaging with this standard in both activities 2 and 3, as they record the numbers of attributes or measurable attributes for the tetrominoes. I expect some reversals of the numerals but will be diligent about having the students correct this common error.

K.CC.B.4 – Understand the relationship between numbers and quantities; connect counting to cardinality.

Student understanding of this standard is strengthened each time they engage in counting. They will be practicing this skill in both activities 2 and 3. I will be able to informally assess students understanding as I float from team to team and require them to count aloud so they I may observe.

K.CC.B.5 – Count to answer "how many?" questions about as many as 10 things in a scattered configuration; given a number from 1 – 20, count that many objects.

Students will be answering "how many?" questions with regards to the number of the varying tetromino attributes. They will primarily focus on this standard in activities 2 and 3.

Measurement and Data

K.MD.A.1 – Describe measurable attributes of objects, such as length or weight, describe several measurable attributes of a single object.

Students will be focused on measurable attributes in activity 3 as they measure the perimeter and area of tetrominoes.

K.MD.A.2 – Directly compare two objects with a measurable attribute in common to see which object has “more of”/”less of” the attribute, and describe the difference.

Students will be comparing measurable attributes of tetrominoes as they determine which tetromino shaped yards would best match each dog in activity 4.

K.MD.B.3 – Classify objects into given categories; count the numbers of objects in each category and sort the categories by count.

This standard will be addressed in activity 4 as students sort tetrominoes to categories based on the dogs.

Geometry

K.G.B.4 – Analyze and Compare 2 and 3-D shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts, and other attributes.

The 2-D component of this standard is addressed in each of the four activities of this unit.

Math Practices

MP1 – Make sense of problems and persevere in solving them.

Activity 1 requires great perseverance as the students search for the different tetrominoes. Some students may also need perseverance to get through the counting and measuring found in activities 2 and 3.

MP3 – Construct viable arguments and critique the reasoning of others.

As students justify their choices in activity 4 they will be addressing this math practice.

MP4 – Model with mathematics

As students use tetrominoes to model a fenced in yard in activity 4, they will be addressing this math practice.

Resources

Beckmann, Sybilla, and Sybilla Beckmann. *Mathematics for Elementary Teachers with Activity Manual*. 4th ed. Boston: Pearson Addison Wesley, 2011.

Good explanation of geometrical concepts as well as thoughtful activities.

"Book 9: Teaching Number through Measurement, Geometry, Algebra and Statistics." NZMaths. https://nzmaths.co.nz/sites/default/files/Numeracy/2008numPDFs/NumBk9.pdf.

Article focuses on activities about connections between numeracy and other strands of the math curriculum.

Campbell, Matthew, and Johnna Bolyard. "Why Students Need More 'Math Talk'." The Conversation. October 29, 2018. Accessed July 14, 2019. http://theconversation.com/.

The article offers an interesting examination of the importance of offering students a chance to engage in mathematical discourse.

Devlin, Keith. "Number Sense: The Most Important Mathematical Concept in 21st Century K-12 Education." Huffpost.com. January 1, 2017. https://www.huffpost.com/entry/number-sense-the-most-important-mathematical-concept_b_58695887e4b068764965c2e0.

This is an interesting source about lack of number sense and strategies to teach the skills and concepts that help build number sense.

Gojak, Linda M. "Making Mathematical Connections." National Council of Teachers of Mathematics. October 03, 2013. https://www.nctm.org/.

An article that relates the importance of making connections between pieces of content and offers tips for doing so.

Help Your Kids with Math: A Unique Step-by-step Visual Guide. NY, NY: DK Publishing, 2010.

Good visuals to use with students.

Heuvel-Panhuizen, Marja Van Den, and Kees Buys. "Young Children Learn Measurement and Geometry." Sensepublishers.com. Accessed June 15, 2019. https://www.sensepublishers.com/media/1295-young-children-learn-measurement-and-geometry.pdf.

This is an interesting article full of ideas for possible lessons.

Knapp, Brian, and Duncan McCrane. *Math Matters! Size*. Danbury, CT: Grolier Educational, 1999.

Includes good examples for use with students.

Long, Lynette. *Painless Geometry*. B.E.S. Publishing, 2018.

This is a good refresher for geometrical concepts.

Moss, Joan, Zachary Hawes, Sarah Naqvi, and Beverly Caswell. "Adapting Japanese Lesson Study to Enhance the Teaching and Learning of Geometry and Spatial Reasoning in Early Years Classrooms: A Case Study." *Zdm Mathematics Education* 47, no. 3 (2015): 377-90. doi:10.1007/s11858-015-0679-2.

The article shines a light on the failings of American early childhood classroom with respects to the developing of geometry and spatial reasoning skills. It does go on to recommend 4 suggestions of ways to begin rectifying this situation. Insightful.

Rangel-Mondragon, Jaime. "Polyominoes and Related Families." *Mathematica Journal* 9, no. 3 (2005).

This article offers in depth information about polyominoes. I found the information on the related families to be useful when considering extensions to my activities.

Endnotes

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1. Devlin, Keith. "Number Sense: The Most Important Mathematical Concept in 21st Century K-12 Education."
2. Moss, Joan. "Adapting Japanese Lesson Study to Enhance the Teaching and Learning of Geometry and Spatial Reasoning in Early Years Classrooms: A Case Study."

3. NZMaths. "Book 9: Teaching Number through Measurement, Geometry, Algebra and Statistics."
4. Gojak, Linda M. "Making Mathematical Connections."
5. Rangel-Mondragon, Jaime. "Polyominoes and Related Families."
6. Campbell, Matthew & Bolyard, Johnna. "Why Students Need More 'Math Talk'."