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Curriculum Units by Fellows of the National Initiative
2019 Volume V: Perimeter, Area, Volume, and All That: A Study of Measurement

Geometric, Classroom Object Calculations

Curriculum Unit 19.05.12, published September 2019
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Introduction

As a public school elementary teacher in the United States of America, I find that many students experience difficulty attaining an acceptable level of achievement in Mathematics. I encounter many challenges helping students become proficient within the subject of Math. Although many students perform better overall in Mathematics; as compared to Reading, there are numerous gaps within the educational system which may be contributing factors for low test scores in many schools within the school district that I work in.

I am developing a curriculum unit in order to provide students with a solid foundation in the area of elementary level geometric formula calculations. My educational goals are to provide my students with multiple opportunities for geometric exposure so that they gain a thorough understanding of the concepts of perimeter, area, and volume. Additionally I intend to ensure that students are able to utilize all skills acquired by way of the instruction provided, so that they will ultimately be able to apply those skills to perform various tasks. Upon completion of this unit, my students will be able to differentiate between the various formulas used when calculating the perimeter, area, or volume, as well as calculate the dimensions of one, two, and three dimensional figures, routinely and efficiently.

Rationale

The second grade standard, VA SOL 2.13 which pertains to Measurement and Geometry, does not include the concepts of perimeter, area, and volume. VA SOL 2.13 indicates that the students should identify, describe, compare, and contrast various solid figures, and that is the full extent of the student's exposure and involvement with geometry in the second grade. This can prove to be problematic as each teacher is left to determine the amount of geometric detail and depth they will provide their students with. This leads to fewer students being exposed to relevant skills; hence, students will be introduced to the concepts of perimeter and area in third grade, and during that same year they will be assessed on the concepts. Volume is not introduced until the fifth grade. The time frame for implementing these concepts seems to be much too late and may interfere with the progress of many students. Almost simultaneously when students are presented

with the concepts, they are required to not only solve problems related to perimeter, area, and volume, but also, students may be asked to perform at Blooms Taxonomy's, higher end analysis level regarding geometric concepts on standardized tests. By the time that the student enters fifth grade, he or she will have less time to practice skills covering the concept prior to assessment time. Students who are exposed earlier have a greater opportunity for success.

The Math state standardized exam for third, fourth, and fifth grade students includes approximately 10% of geometrically comprised questions. However, my efforts are an attempt to scaffold, by laying a foundation for future successive, more complex geometry concepts as each student progresses to higher grade levels. Other countries address their pedagogical approach in a varied manner. When considering the delay of the presentation of the geometric concepts, some may view our planning and scheduling for geometry as slightly haphazard. Although the mathematic prowess of many students has improved overall, some students still suffer mathematically. Bolstering geometry as well as basic skills provides for a solid foundation that ultimately enables all students to achieve mathematic growth.

Geometry plays a role in our daily lives, as it is all around us.

> "Geometry can be found in the structure of the solar system, in the geological formations, in rocks and crystals, in plants, and flowers, as well as in animals, and it is a part of our synthetic universe (art, architecture, cars, etc.); Geometric explorations can develop problem-solving skills; Geometry plays a key role in the study of other areas of mathematics; Geometry can also be fun."[1]

While geometry can be fun, it is time that our students be immersed with geometric concepts from preschool through grade 12. A change is necessary. That change would include new methods geared towards including as many foundational geometry concepts as possible, in pre-school as well as all grade levels. Accordingly, it has been concluded that,

> "by engaging in the processes of conceptual understanding, procedural fluency, adaptive reasoning, productive disposition, and strategic competence (areas addressed when working through tasks in geometry), students learn mathematics by *doing* mathematics".[2]

My current focus is to prepare the students that I am working with as well as many of the other students that are in my school building, and so I will share this unit with my colleagues. I will promote the idea that geometry must be taught in all classrooms at my school more consistently, while requesting that we all add depth to our Geometry lessons(mostly in the lower grades as more rigorous efforts are already being carried out in grades 3-5). No longer should anyone believe that children should wait until they join a testing grade to be immersed in higher level geometric based tasks.

Content Objectives

My student's lessons will be based on the objectives listed below:

VA. SOL 3.8

The student will estimate and a) measure the distance around a polygon in order to determine its perimeter using U.S. Customary and metric units; and b) count the number of square units needed to cover a given surface in order to determine its area.

VA. SOL 5.8

The student will a) solve practical problems that involve perimeter, area, and volume in standard units of measure; and b) differentiate among perimeter, area, and volume and identify whether the application of the concept of perimeter, area, or volume is appropriate for a given situation.

Modelled Sample Geometric Methods

Initially, students will receive extensive, direct instruction on all three concepts (perimeter, area , and volume, beginning with the formula for one dimensional figures for a day or two and progressing to the other concepts in the same manner) to include mini lessons, teacher modelling, related story books and or song/video use that also exemplify methods of using the concept, hands on exploration with the use of manipulatives, exploration of digital game play, and old fashioned board game play, as well as geo-talks between classmates, as well as worksheets as practice assessments. Students will be given numerous figures of varying sizes in order to carry over all concepts and procedures learned. The tasks indicated here will be mentioned in detail later in this unit.

The following is a sample lead in (mini lesson) and exercises (see figures A-C) for day five of the unit. The lesson will be implemented in order to activate prior knowledge of the geometric concepts of perimeter and area that were taught earlier in the week. The activity will begin with perimeter and I will instruct my students as follows:

> "When thinking about perimeter, think about the following: perimeter is a, 'linear' measurement. The perimeter is the total of the lengths of the sides of a figure. Think of starting at one corner of the figure and walking all the way around it until you get back to your starting point. The distance walked is the perimeter of the figure".[3]

An anchor chart that I will create in the classroom with the students will feature a man or woman walking around the outside of a figure with the word perimeter in bold print.

> "The students will have a permanent picture in their minds and will remember to focus on the outer boundary of the figure when performing calculations pertaining to perimeter".[4]

The students will be presented with the following mini exercises**:**

I will initially review the vocabulary to re-familiarize the students with the terms and meanings, as something similar to the exercises below will be provided to the students on or around day

five.

Vocabulary: perimeter, area, volume, array, dimensions

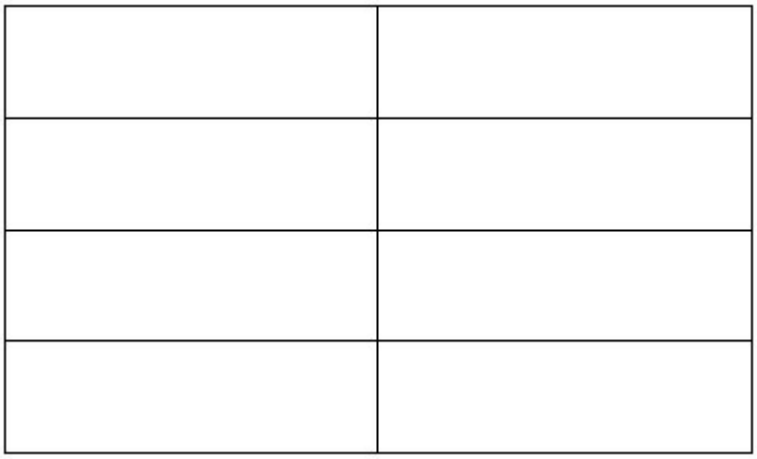


Figure A. (Use the box to find the perimeter) box constructed from word

a) What is the perimeter of the box (figure A)? *The students should be able to determine fairly quickly that the perimeter is 12 units. Discussions would require the students to reiterate why they know that 12 units is the correct answer, which will include thoughts about the formula for perimeter and how to apply those concepts to finding the perimeter of the above box. I would include an additional exercise that changes the dimensions of the box and direct the students to solve for perimeter.*

Upon my satisfaction (as there are numerous perimeter exercises to choose from, for example, geo talks, journal entries, online digital game play) with the oral responses and written evidence that my students provide with perimeter, I will move on to the next part of the activity. It will focus on area and I will instruct my students as follows:

> "When thinking about area, think about the following: area is a 'square' measurement. The area is the surface of the object. Area answers questions such as, how much paint do you need to cover a surface, or how much grass seed do you need to seed a lawn?".[5]

An anchor chart will be created with the students in the classroom that features a fenced- in section with the words, area, appearing in big, bold letters, inside or on top of the fenced- in section. I will ensure that the students understand that they are to count the units inside of the figure when solving for area.

b) How many boxes are within the larger rectangle (figure A) or what is the area? *The students should determine that based on the interior of the box, there are four rows of two boxes for an area of 8 units.*[2] *Students will most likely count one by one, however, with more practice they will soon become more familiar with multiplying, although most area exercises will be scaled back to smaller dimensions due to their skill levels. Students will be asked to identify what formula they solved for. The students should respond by saying 'area'.*

Next, within a few days of implementing the above exercises and additional exploration, the students will be presented with polyominoes. There are twelve groups of polyominoes, however, specifically for this unit study, we will use pentominoes. Students will be asked to identify the perimeter as well as the area of each figure of each pentomino (see figure B). A detailed discussion covering pentominoes will be addressed later in this unit study, however, the exercises below will follow a detailed presentation of pentominoes within the classroom.

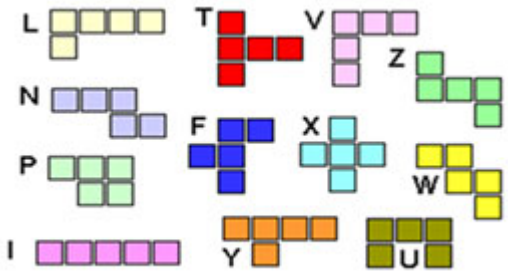


Figure B. (Use the above figure to find the perimeter and area of each pentomino)

The students should walk away from this exercise with the understanding that despite the fact that each group contains five, hence the name- pentomino, the rearrangement will cause the area to hold constant to five units^2 , however, the perimeter will not remain constant and the students can determine which pentomino arrangement will yield a different perimeter from the others.

After viewing the various pentominoes during whole group, the students will work with a partner to explore the various transformations (rotations, reflections and translations) of the pentominoes as well as the area and the perimeter. They will be directed to journal in their notebooks about the exercise concerning what they learned and provide information about pentominoes and the various transformations as indicated above in order to detail all that they have learned about pentominoes.

Upon my satisfaction with the oral responses and written evidence that my students provide with area, I will move on to the next part of the activity. It will focus on volume. I will instruct my students as follows:

“When thinking about volume, think about the following: Volume is a ‘cubic’ measurement. Volume is the number of cubic units needed to fill a given solid”.[6] Volume answers a question like, how much space is occupied inside of a solid, 3 dimensional object. An anchor chart will be created in the classroom with the students that features cubes of varying sizes. The cubes will be broken up into three rows, highlighting the columns as well. The cubes will display the dimensions of length, width, and height. The chart will remind the students to count the amount of cubes in each of the rows which will also account for the columns. The chart will indicate that students should perform repeated addition in order to obtain the volume. Emphasis will be placed on the idea that the total number of cubes that are accounted for match up with the total number acquired after counting the amount per row. This is demonstrating how to identify the amounts for length, width, and height. Although my students will view a pictorial version of a three dimensional object, they will also be presented with a hands- on three dimensional object.

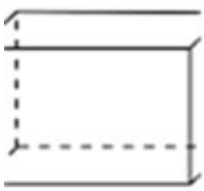


Figure C. (3 dimensional cube)

Students will use cubic square blocks. The blocks will be used to derive the volume of the 3-D object in figure C., with similarities to the one above. Students may also encounter an exercise such as, the one below. When completing a technology assignment (Figure **D**), such as the exercise below, it will follow the same procedure in determining the volume that my students will us to find the volume for figure C.

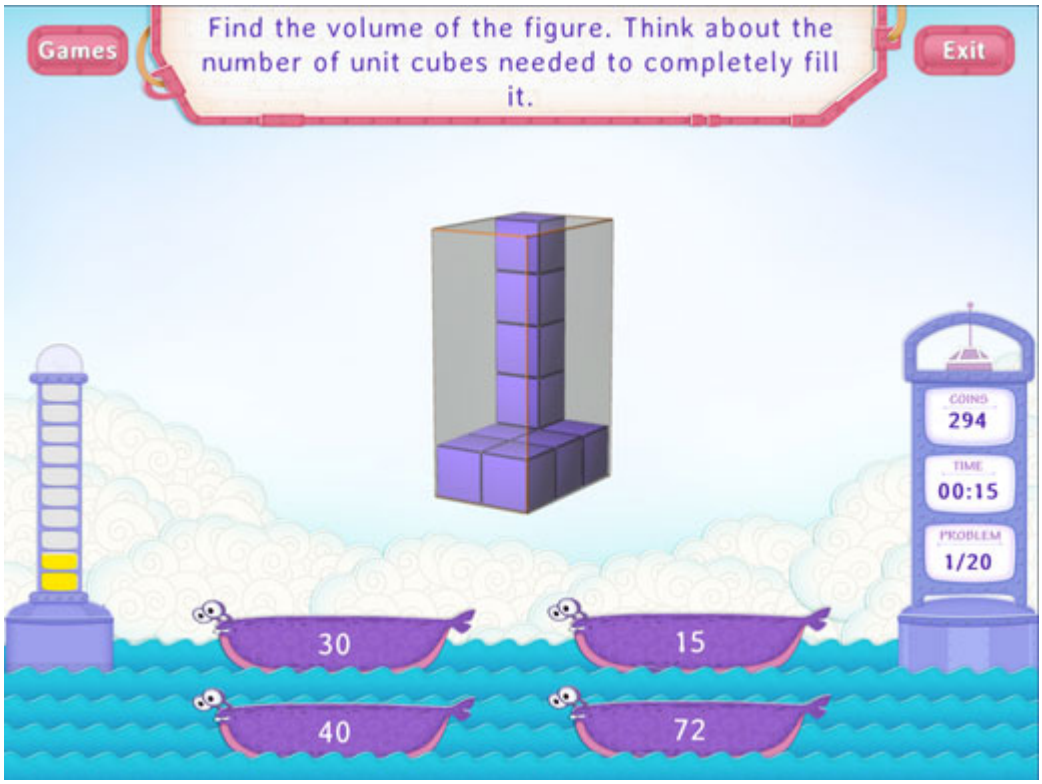


Figure D. (A technology exercise)

My students will be expected to recall that they are to use repeated addition and add the amounts of the rows and columns which will be: 6+6+6+6+6 and yields a volume of 30 units.[3]

e)Describe how to find the volume of the cube.

By the time that students are asked to complete this task, they would have had 1-1 ½ weeks to become familiar with volume. The students would be expected to provide me with as much information as they can recall concerning volume, such as identifying that using cubed blocks to identify the length, width, and height is the way to proceed since volume is three dimensional. Students should be able to recall that they will count the amount of blocks that are in each row, and perform repeated addition to identify the volume in cubic units.

Pre-requisites

Benjamin Banneker once said, “Presumption should never make us neglect that which appears easy to us, nor despair, make us lose courage at the sight of difficulties”.[7]

During a Geometry seminar that I attended, the seminar leader, Roger Howe indicated that “students must have a firm grasp on the use of a ruler as that is a prerequisite for better insight with geometric concepts”.[8] Students should practice using the ruler and starting at the line marked zero. The zero mark should be lined up with the edge of the object to be measured. The ruler can be used to determine the measurement of the object by determining the distance from the edge of the object where the zero mark is to the other end of the object. The students ability to use exactness when measuring an object from the zero mark matched up to the edge of the object is a necessary step in that many students have the misconception that when using the ruler, it is unimportant as to where you line the ruler up! Students will be unable to use the ruler to count the sides of an object if they aren’t familiar with ruler guidelines.

Students should be aware of what an array is, as it is an indicator that there is a group of things that have the same size and shape or equal rows and equal columns is another way to put it. I want students to be aware of what they are doing, so as to proceed in an orderly fashion thereby minimizing confusion and/or mishaps.

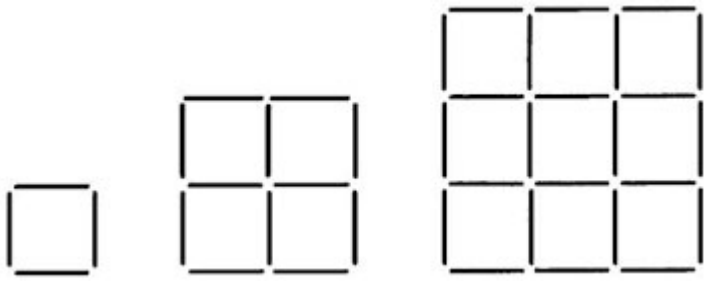


Figure E. (one, two, and three dimensional arrays)

In addition to the term ‘array’, I will also ensure that students have a solid understanding of other vocabulary that they will be exposed to during the unit study. When students become familiar with the number of sides of the various geometric objects as well as their proper names (Plane figures: square-four sides, rectangle-four sides, hexagon-six sides, octagon-eight sides, Solid figures: cube- six sides, rectangular prism-six sides) it is very helpful to them as they complete various exercises. There is the chance that fewer mistakes will be made if the students are knowledgeable about the fact that they should be adding eight sides when finding the perimeter for an octagon as opposed to them mistaking the number of sides of the octagon for that of a pentagon-five sides, or a heptagon-seven sides, as this sort of activity will most likely be found on a worksheet that the students must complete or a as a question on an exit ticket. In this way, students will be able to talk in an informed fashion when exploring the various geometric concepts.

Students should, “use appropriate tools strategically and become familiar with a variety of tools, and they should learn to choose which ones are most appropriate for a given situation”.[9] Taking care of the above mentioned preliminary matters is a firm way to provide for a smooth flowing unit study. Good ruler use, knowledge of geometric terminology, being able to identify the shapes and their number of sides, as well as becoming familiar with pertinent tools (for example, geoboards)that can be used for this study, are all important initial steps which are vital to the success of the unit study.

Positivity allows for more positivity and so I believe it is wise to add a geometric exercise to the spiral review portion of my daily lesson plan.

> "Research conducted revealed that geometry-related literature that was included in daily activities in an elementary classroom actually improved the achievement and attitudes towards geometry of the students involved in the study".[9]

Consequently, if you can embody positivity concerning something that you may not be thrilled about, then chances are high that your optimistic viewpoint may prove to be true and things could actually work out for the best! A similar statement of sentiment was once uttered by the late, great, Benjamin Banneker. Mr. Banneker was an African American who was primarily self- taught and he is notably known as an astronomer, mathematician, a publisher of an almanac, creator of the first clock in America, in addition to other significant accomplishments. This section is preceded by the thoughts of Mr. Benjamin Banneker. Much like he alluded to, it is true that, once a child decides to apply himself/herself, then it is at that point that difficulties won't permanently impede his/her ability.

Many students experience negative thoughts during their elementary school years. To counteract some of the negative thoughts that my students may go through, it is significantly important that the students are made aware of their roots and lineage in relation to the pyramids, especially since their current unit of study is tied to the pyramids of Africa. Many of them are unaware that their distant relatives, the Egyptians of Africa were very astute with respect to geometry as they are the architects and mathematicians who used the formula for volume in the construction of the pyramids many centuries ago.

> "In observation of the astounding architectural accomplishments of the Egyptians, which surely required them to know the quantity of blocks necessary for pyramid construction, it should not come as a surprise to anyone to realize that they were a people with skills and proficiency at calculating volume".[10]

This information alone could possibly encourage and motivate a youngster to take the next step in their geometric study and persevere when facing mathematical challenges.

Curtailing Misconceptions

Similar to the need for preliminary skills that students must have to perform grade level tasks, I will be on the lookout to help them to avoid pitfalls commonly called misconceptions.

> "Consider the following situations involving perimeter, area, volume and children's misconceptions. What can teachers do to help them to understand correctly? Let's explore a few common misconceptions. Misconception: Andi says that the area you can make with sixteen tiles (figure F or some similar square) is always bigger than the corresponding perimeter, because

working out area uses multiplying, but working out perimeter is just adding lengths".[11]

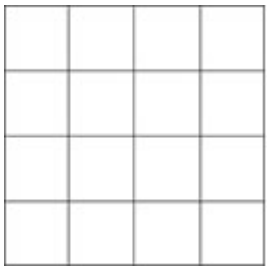
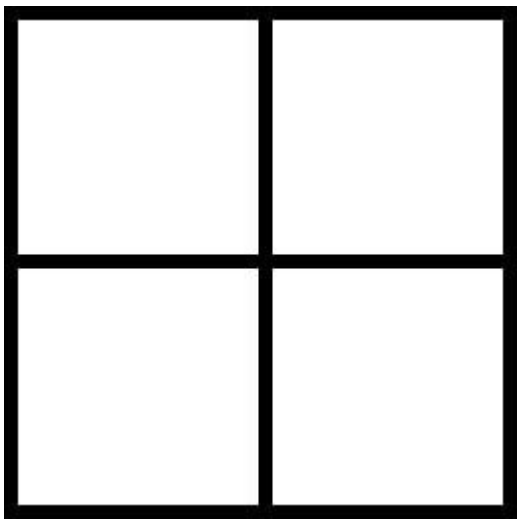


Figure F. 4 x 4 square

Perimeter- count to add sides, Area-count the inside boxes of the large box P=16 units, A= 16 square units

Figure G. 2 x 2 Rectangle



Perimeter= Count to add all sides

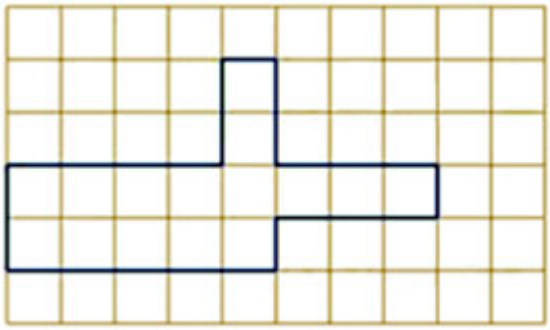
Area-count the inside boxes of the rectangle

P= 8 units, A=4 square units

"Intervention: The teacher must provide the intervention, by trying different rectangles to find some suitable counter-examples".[12] The units of area and perimeter are different and therefore they should not be compared and to do so is a mistake. The above, square A is 4x4 square units. It shows that while the area is equal to sixteen square units, the perimeter is also sixteen units. Andi would be encouraged to work through finding the perimeter and the area in order to make sense of his error, as he stated that the area would always be greater than the perimeter because of the multiplication involved with area. Additionally, the students will be asked to construct the following rectangles: 1x16 of which the perimeter of 34 units is larger than the area of 16 units^2 and 2x8 of which the perimeter of 20 units is larger than the area of 16 units.^{2} Rectangle **G** is 2x2. It shows that while the area is four square units, the perimeter is actually greater than the area which is once again contradictory to Andi's assumption.

> "Another method for this issue is to point out that the standard formula for the perimeter of rectangles includes multiplication. Andi may not understand the algebraic formula P = 2 × (L + W), but may be helped to see that there is a doubling of the sum of length and width; obviously doubling is like multiplication — in fact, it is multiplying by two. Since this shows that both perimeter and area involve multiplying, maybe the initial argument is not as solid as Andi thinks".[13]

Andi should be led to conclude that it doesn't make sense to compare length and area. There is also substantial reasoning behind ensuring that Andi is aware of the fact that length and width must be multiplied twice or at least counted twice and has to do with the fact that one should be aware that the opposite sides of the figure are alike. This lessens the frequency of errors, as I have observed students miscalculate when they aren't focused, for example, P=4+5+7+8, as opposed to the actual dimensions P=2(4+4) + (7+7). The former calculation is impossible since opposite sides of the figure will always be the same. Students also think in the following ways:



H. (lopped rectangle)

> "Misconception: Collen says that the area in this "lopped" rectangle is eighteen square cm, (see figure H). Intervention: Draw a sketch plan, or even a precise centimeter-square grid; or use a geoboard (a rectangular array of nails, using rubber bands to loop around corners of figures). A geoboard is an invaluable aid for teaching area and perimeter, because it allows easy construction of geometric figures, and easy counting of units of length and units of area. Mark the centimeter scale along the top portion of the lopped rectangle (figure H), as well as the bottom portion of the lopped rectangle, and draw the square grid pattern over the figure. Count the number of unit squares of area".[14]

I will be sure to always reiterate to my students that when we find the area of a figure, we are focusing on the inside portion of the enclosed area of the figure.

With the routine review and practice of various geometric exercises, the misconceptions will most likely occur less frequently. I am aware that practice makes perfect therefore continuity of these skills is essential. Students will be able to reason and problem solve without getting stuck as routinely as they once did.

As a result of my research, I found much of what I learned to be very helpful in preparing lessons for my students. I plan to help my students begin to think about transferring the skills of the measurement of

perimeter, area, and volume acquired within the lessons that I implement to higher level thinking tasks. By transferring those skills, the students will be able to perform the calculations within the upcoming assigned project. My intentions are for the students to not only be able to transfer the skill of calculating perimeter, area, and volume of a given object, but they should also be able to generalize and make application of those same skills to perform other higher level thinking exercises, (i.e. geometric formula measurement of triangles, trapezoids, and even cylinders, etc.) as they move on to higher grade levels.

Students will eventually (at this time, the students are not ready to perform calculations as they are too complex) come to the realization that perimeter, area, and even volume concepts are experienced during real life situations, as students will eventually be faced with situations where they will have to help their parents/relatives/friends identify the measurements of: walls that require painting (area), or floors that will undergo the installation of carpeting (area), or shingles that are to be replaced on the roof (area), or being faced with identifying which belongings that they own will fit into a moving truck when relocation is a reality (volume), and upon arriving to their new residence there may be an interest in purchasing a fence for the yard (perimeter). The above mentioned concepts are tasks geared toward older children and the idea is for the exercises and activities within this unit to be used to prepare the student to eventually be able to complete those higher level, more complex tasks. The more students are provided with practice that entails geometry (much like the other academic topics that are addressed in school which act to prepare the student for real life), the more comfortable they will be at handling various tasks in their life of which natural math will play a part.

Strategies

In order to approach the geometric concepts of perimeter, area, and volume, I will be certain to provide my students with the following:

Manipulatives

Students must use and become familiar with plane figure and solid figure manipulatives, as well as tiles (activities using manipulatives and tiles are identified in various locations within this unit paper), and geoboards (see figure H) to create objects for calculating perimeter, area, and volume. My students will create (paper creation –see figure C1- or manipulatives, or other resource already mentioned) a shape and subsequently they will identify facts about the shape and provide its perimeter, area, or volume, as well as answer questions in which there are several answer choices. By providing several choices students will become involved in reasoning in order to identify the answer. “Educational research indicates mathematical achievement increases when manipulatives are put to good use”.[15]

Anchor charts and Journal writing

Anchor charts are valuable to students. A best practice, is to create the chart during the mini lesson as the students watch. I will write the concept and the important, detailed information as well as examples as I teach the lesson. I will then hang the chart in the classroom in order for the students to refer back to it as needed. “Anchor charts build a culture of literacy in the classroom by making thinking—both the teacher’s and students’—visible”.[16] Similarly, there are also benefits from participating in the journal writing assignment.

This is an opportunity for the student to write down in his/her own words and thoughts that he/she recalls about the concepts. This strategy is essential.

> "Assumptions about mathematics communication via writing include: (a) students possess writing skill, (b) students can transfer this writing skill to mathematics writing, and (c) mathematics writing is representative of a mathematics knowledge".[17]

Other forms of visual aids

Rulers are good tools to use since the student can refer to the numbers on the tool and the numbers will correspond to various sections on the object so it is visually helpful to the students. "It has been concluded that there is considerable evidence of a relationship between the Approximate Number System (in this case the ruler) and symbolic math that is meaningful".[18]

Other tools & Methods

Use of online game play to explore and solidify understanding during play is beneficial. In terms of intellectual and expressive motivations, researchers found that,

> "Children enjoy video games because of the challenges involved in achieving mastery, the opportunities to express one's creativity, and the make-believe and discovery moments that happen during game play".[19]

Since students are interested in achieving mastery, when using video games, the students will still have the same agenda (to achieve mastery), however, now within the school setting, the games are academically based and used to support the lesson. Based on the determination of the student, the goal of mastery will be attempted and possibly achieved and consistent with that feat. The student will simultaneously achieve academic gains as well. The use of the online games will subtly push the student to master the concepts that they have been taught while using the game. This is a winning combination! The technology game that is similar to the above mentioned can be seen in figure D (this game explores volume), and other technology games are: perimeter shape digital game practice, figure I and splash math area digital game practice, figure J.

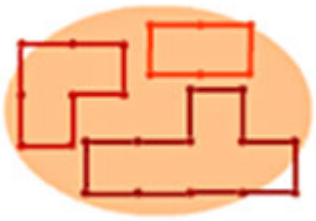


Figure I. (technology enhanced perimeter game)

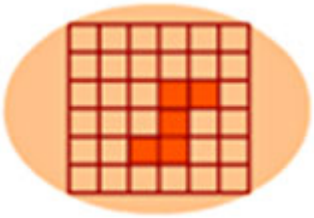


Figure J. (technology enhanced area game)

Use of online story books and songs to introduce the concepts also works well to capture the students' attention.

> "A basic reason for using stories is to utilize them as a means of entertainment and having fun since they inject lots of amusement and interest. Another goal is to instruct, for example, stories are meant to give moral lessons, whereas the good people are rewarded and those that are bad are punished".[20]

Peer tutoring is an effective method. Students will also participate in whole group discussions wherein they will be required to participate in deductive reasoning. During deductive reasoning, the students are responsible for arriving at a conclusion based on the information that they are given. Discussions that take place whether with the teacher or peers is also recommended so that the students can practice defining various concepts. When you are able to verbally discuss facts about a given topic, then you are proving that you know and understand the topic. During the whole group discussions, students will be given a few choices and asked to arrive at the best answer based on their abilities to refer back to previous discussions, concepts, and game concept- focused activities that they recall, as well as their own journal notes.

It is common knowledge that children often confuse perimeter and area. Teachers must ensure that they,

> "Give students many opportunities to solve problems in meaningful contexts. We must also give children strategies for making sense of the mathematics in problem-solving situations to help them connect the situations with the appropriate computational processes. This teaching strategy requires the appropriate use of representations that support such connections for children in meaningful problem contexts. Children also need opportunities to use both concrete and pictorial representations that support this sense-making process and to share their thinking strategies with peers".[21]

Educational leaders commonly advocate that children should not just learn one method to arrive at an answer, but quite the opposite, data exists that supports the fact that students should be given a plethora of tools to use (use of rulers, manipulatives, and opportunities to count independently). When it is necessary, they can pull which tool they want to use from their resources according to what they need.

> "Chinese students have advantageous knowledge on how to think and solve the same problem from different perspectives, which will be helpful for them to switch to different strategies once they get stuck".[22]

Often times what matters is that you have a resource to reach for when working through math activities, as opposed to the necessity of the resource that you access, being identified as innovative or one of popular use. At times, good old- fashioned rote and/or memorization techniques that are still useful and more importantly, are handy for the student tend to work wonders. This is what is best!

Noteworthy Popular Ideas to Support Concepts

Roger Howe discussed using polyominoes with the students. I initially mentioned polyominoes in the content objectives section of this unit paper. Polyominoes are plane geometric figures formed by joining one or more equal squares edge to edge. My students will use pentominoes during this unit study. Pentominoes are types of polyominoes. There are twelve types of pentominoes. The pentomino is a geometric shape that is composed of five congruent squares, connected along their edges. Students will be required to problem solve using the pentomino manipulatives as they form arrays. This method encourages the students to use a combination of five manipulatives at a time and to then identify the dimensions of the enclosing rectangle(the rectangular object's borders). My students will explore the activities referenced in the content objectives section (identify the various combinations of the pentominoes, identify the dimensions of the pentomino, locating perimeter and area of the pentomino). Geometry is known as a pathway that leads to deductive reasoning and thought processes, so exploration with pentominoes will prove to be beneficial for students as this all works to aid the student in the way of mathematical intellectual training. Students will be required after fourth grade to begin to use geometry in more of a fixed, routine fashion. So while lessons are often implemented as a formality, it is evident that geometry has far reaching implications that appear to be beneficial for all students. My students will explore the various combinations of pentominoes (see figure **B**).

The exploration of the pentominoes is an avenue towards the practice, as well as the enhancement of, their geometric thought and their geometric skills, and it also lends itself to setting the stage for more complex activities to come.

Classroom Activities

This part of the unit will occur in the beginning (introduction of the concepts and the Marilyn Burns story only) of the implementation of the unit. Upon introducing the concepts, I will model how to solve for perimeter, area, and volume respectively. During that time, I will make use of storybooks to help with the facilitation and implementation of the lessons. One of the stories that will be used is *Spaghetti and Meatballs for All! A Mathematical Story by Marilyn Burns.* Since perimeter and area can often be slightly complicated concepts for students, it is important to immerse them with varied resources in order to help each student with comprehending, which will allow for a smooth transfer of the skills acquired to other tasks.

The Marilyn Burns story features a couple holding a dinner to include other family members and friends. The seating arrangement is manipulated numerous times and the reader becomes involved with the concepts of area and perimeter, for example, initially, there were eight tables arranged for the guests of which the perimeter of each individual table is four which would seat four people at each of the eight tables for a total of

thirty-two available seats. Soon, in haste, the tables were re-arranged so that all eight tables were pushed together into two rows of four. With the re-arrangement of the tables into two rows of four, fewer people would be accommodated as opposed to the way that the tables were originally arranged. The students will use manipulatives to determine the most efficient seating arrangement as they listen to and enjoy the story!

1. The Geometric Analysis of Spaghetti and Meatballs for All! A Mathematical Story

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As I read the story, Spaghetti and Meatballs for All! A Mathematical Story by Marilyn Burns to my students, we will take note of the mathematical situations that occur. I will encourage the students to tell me how Mrs. Comfort should prepare for her guests regarding the number of tables that she will need for thirty-two people. At this point I will reiterate the concepts of perimeter and area so that I can inquire of my students about which concept they believe would apply (area or perimeter) in order to appropriately seat the guests. Students will be able to discuss the question with their classmates to allow for problem solving and reasoning. Moving forward with the story, the students will be asked to continue to carefully consider what will happen each time guests arrive and the arrangement of the tables are moved. Mrs. Comfort and her family re-arranges the tables numerous times.

> "Later on in the book, after six additional guests arrive, the same eight tables were then re-arranged into a one by eight rectangle after re-arranging two arrays of four. Renting eight, small, square tables and seating four people at each table, wasn't the only way Mrs. Comfort could have organized the tables for the thirty-two member family reunion".[23]

Students will be asked to,

> "Cut out squares of cardboard or use small square tile manipulatives so that they can construct the different ways the guests in the story arranged the tables. Students will be asked to experiment so that they can see that Mrs. Comfort ordered the fewest tables possible".[24]

Basically, students should construct the various combinations of the squares to simulate thirty-two available seats.

2. Geometrical Exiting, Before Lunch

During this activity, each student will be given an index card before exiting the classroom for lunch. They have time between exiting the classroom and arriving to the cafeteria to provide an oral response for the exercise on their index card. The exercises will include some of, but won't be limited to the following:

a. What is the area of a rectangle with three rows of two cubed blocks in each row?
b. Define the term perimeter.
c. Each pentomino transformation (rotations, reflections and translations) holds the perimeter and area constant. True or False and explain your answer.
d. What is an array?
e. What is the best hands on way to find the volume of a box?
f. True or False. The area of an object requires a cubic unit for the answer.
g. True of False. Pentominoes are five congruent squares, connected along their edges.

I will gather additional questions that are relevant to the concepts in this unit in order to have one question for each student in my class. These tickets can be traded among students during the next Geometrical Exiting exercise.

3. Geometric Classroom Objects

The culminating activity will require the students to transfer the skills acquired from all of the previous activities. Students will be assigned to complete one of the four activities as they will be grouped with their classmates into one of six groups.

A. Students will create their own polygon similar to the one in the picture(A1) using wood. Students will subsequently find the perimeter and the area of the figure using customary units and/or metric units. They will also be responsible for listing the characteristics of the figure (for example, the students should be able to identify the actual type of polygon, as it is an irregular hexagon and they should also identify that all hexagons have six sides and six angles just as this object does).

Figure A1 (polygon to be created by students)

B. Similar to the picture (B1), students will create various quadrilaterals and affix them to the window pane in the classroom. Subsequently, students will find the perimeter and area of the quadrilaterals using customary units and/or metric units. Students will also be responsible for listing the characteristics (for example the number of sides and angles and any other details about the figure).

Figure B1 (window pane quadrilaterals)

C. Two groups of students will construct two different pseudo community buildings similar to the community block in the picture (**C1**). Students will identify the perimeter and area of their pseudo-community buildings on their block of which they will eventually use the buildings to play with using their paper dolls. Students can determine the area of the floor space in the various buildings as well as the wall space in order to paint the walls or affix wall paper. Students will also identify the perimeter of various rooms in order to adorn the room with border on the walls and baseboard trim which will run along the bottom of the wall against the floor. Finally, for this particular activity, students will determine the volume of rooms that they select to add taller than normal cardboard, wardrobe armoires. Additionally, identifying the volume of select rooms will help students to determine how many pieces of cardboard furniture they can add to the various rooms. Students will also be responsible for listing characteristics (for example, tell if the figure is 1, 2, or 3 dimensional and any other details of the figures) of the various figures that they measure.

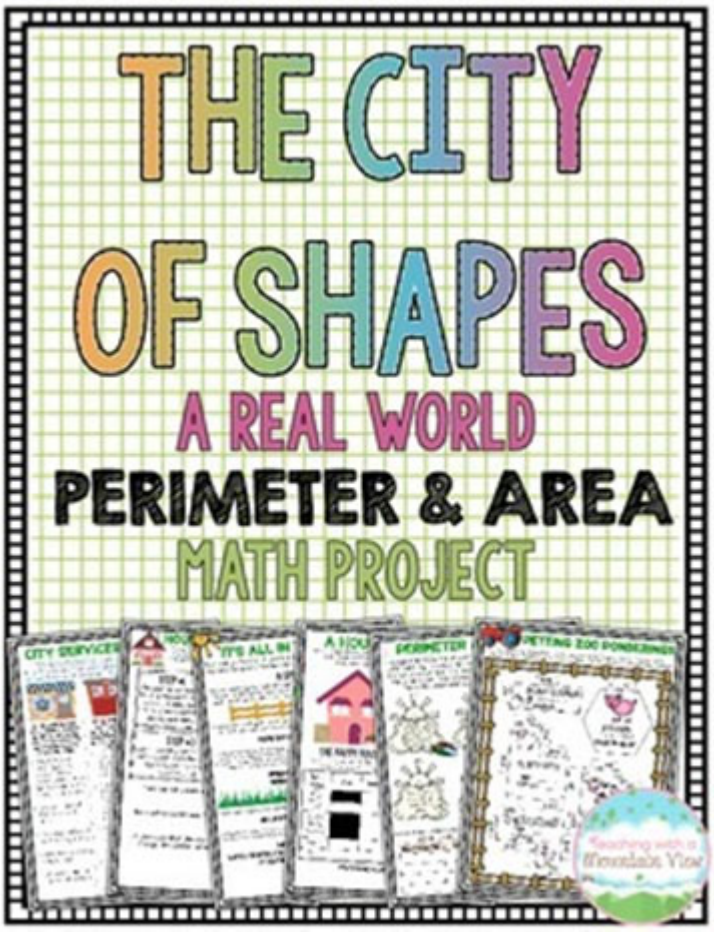

Figure C1 (Pseudo Community Block)

D. Two groups of students will use actual 3 dimensional boxes (similar to the one in picture D1) within the classroom and determine the volume using small cubes that will fit in the box and are 1 inch sq. unit cubes for standard customary measurement. Students will identify the layers of cubes that it takes to fill the box and the dimensions. The students will then use repeated addition to obtain the volume. Students will also list the characteristics (the various descriptions of the cube, for example, 6 sided figure, the names and details of its attributes-face, edge, vertex) of the object.

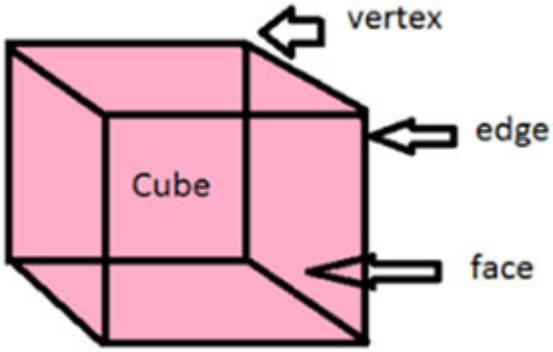

Figure D1 (three dimensional box, comparable to the box students will use)

The project results will be photographed, written about, and/or prepared in such a way so that the results can be featured outside of our classroom on our bulletin board!

Resources

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Appendix: Implementing District Standards

This curriculum unit has been designed to provide the reader with instructions on how to implement Geometric Foundational Lessons for second through fifth grade students. The lessons reference and adhere to the 2016 Standards of Learning specifically written for the state of Virginia.

The Unit outlines and identifies Geometry with a specific concentration on perimeter, area, and volume. The unit details the execution of the unit process thereof. Students will receive exposure to the concepts in the form of short stories, songs, direct instruction, as well as whole and small group focused exploration, and hands on activities such as the use of pentominoes. At the core of each activity, one will find practice that meets the requirements of the Virginia Standards of Learning. Students are required to use rulers and other customary units of measure. Each activity requires the student to differentiate between the three concepts. There are numerous tasks that ask the student to find the perimeter, area, and volume which includes thorough discussions and reasoning as to why one formula is chosen as compared to another.

Later on in the unit, students take the skills they have acquired during the unit to then apply and transfer their knowledge. At that point they will use classroom objects and/or create objects for additional exploration of which they are asked to once again find the perimeter, area, and volume.

All tasks that have been mentioned, address, meet, and adhere to all of the Standards of Learning for Virginia. The unit's instructional methods and activities that I have employed for perimeter, area, and volume will prove to be interesting to my students, yet require their tenacity, not only in order to fulfill the Virginia Standard of Learning requirements, but also to meet my expectations.