Curriculum Units by Fellows of the National Initiative
2019 Volume V: Perimeter, Area, Volume, and All That: A Study of Measurement

Introduction

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The seminar *Area, Perimeter, Volume and All That* was intended to be a general survey of the issues of measurement, but in fact most of the Fellows of the seminar were interested in area, perimeter and volume, so we focused on those topics, and most of the units deal with them.

Several of the Fellows were concerned that, in standard instruction, students do not grasp that area and perimeter are quite distinct quantities, and that some students think that shapes with the same area will have the same perimeter. To counter this sort of misconception, we discussed two strategies.

First, one can study situations in which one of area and perimeter is fixed, and see how the other behaves. For example, rectangles with sides of length 1 and 9 inches, or 2 and 8 inches, or 7 and 3 inches, or 6 and 4 inches, or 5 and 5 inches, will all have a perimeter of 20 inches. However, their areas will be 9, 16, 21, 24, or 25 square inches, respectively. The 5 by 5 square has more than two and a half times as much area as the 1 by 9 rectangle. With larger perimeters, the variation in area will be even more dramatic.

Second, one can go beyond rectangles and contemplate more complex shapes. To do this, we worked with polyominoes – shapes made by putting unit squares together, subject to a couple of simple rules. The number of possibilities increases remarkably rapidly with the number of squares used. You can have a lot of fun with polyominoes, but to cut to the chase: you can create polyominoes of area, say, 8 (aka *octominoes*), with perimeters that vary from 12 to 18. A large number of these octominoes have the same perimeter as a rectangle of area 20. Thus, polyominoes can build student intuition about the possible complexity of shapes, and at the same time, help them refine their understandings of area and perimeter.

Polyominoes turned out to be an attractive topic for many of the Fellows, and were incorporated into most of the units, even being featured in the titles of five units, ranging from kindergarten to middle school.

The units also incorporated the individual perspectives of the Fellows. The units of Jamie Griffin and Dennis Williams integrated gardening with geometry. The units of Marnita Chischilly and Andrea Thomas investigate Native American culture (architecture and farming, respectively). Joseph Parrett's unit uses polyominoes to create novel counting challenges for his kindergarten students. Ricardo Moreno uses Chicago architecture to motivate volume calculations. Tina Berry will have students create sculptures based on polyomino ideas. Kathleen Gormley will utilize the addition and multiplication tables to engage in some systematics, as sketched above. Lianne Aubert Sanfeliz will have her students measure schoolyard structures, and have her students create sensory pathways in the school halls. Trisha Williams seeks to give her second graders a leg up through an early study of area and perimeter.

The remaining two units have somewhat different themes. Valerie Schwarz investigates the common special classes of quadrilaterals, and shows that they are described well in terms of symmetry. Tierra Ingram seeks to adapt the interpretation of multiplication in terms of rectangles to develop the box model for multiplication, and to extend this to multiplication of polynomials. As a whole, the units display well the diversity and creativity of the Fellows.

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