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Using a Length Model of Fractions to Compare Fractions

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Introduction

I was talking to a parent one time, and he expressed the view that fractions should be taught in terms of length. I asked him what he meant by that? He explained that his son was failing math because he still did not know how to add and subtract fractions. He said he was frustrated because as a construction worker, he must know how to add and subtract fractions quickly to maintain the tempo of the construction site. Otherwise, precious time will be lost, and the construction will fall behind. He explained that if you are slow on a construction site, you will get fired. He wanted to know why the school was not using it and allowing him to use real world applications, like using a measuring tape, when instructing his son about fractions. At the time, our school was using the Eureka Math and Saxon Math programs. The teachers were told to use the programs with fidelity, meaning that we had to teach either program the way it introduced standards, and use their strategies, and follow their exact scope and sequence. I honestly thought that students should know all the different strategies introduced in Eureka Math to be successful. Also, I really did not understand that fractions could use a number line like a measuring tape.

On the vast Navajo reservation in northern Arizona is the small town of Kayenta. The town is run by two governments: Kayenta Chapter House and Kayenta Township. The Chapter House is a political subdivision of the Navajo Tribal Government. The Chapter House does not make its own rules and laws for the community but enforces and answers the decisions of the Tribal Government. The township was established in 1986, is the only municipal-style government on the Navajo Reservation. It is managed by five elected board members. The board has the power to create and enforce rules and laws that affect only the immediate boundaries of Kayenta. According to the 2020 U.S. Census, the population of Kayenta is 5, 634 people and the largest ethnicity group (93.9%) represented is American Indian. Most of the people are employed work in Health Care/ Social Assistance, Educational Services, and Public Administration.

Kayenta is known as the gateway to Monument Valley, which is a Navajo Tribal Park known for towering sandstone buttes and crimson mesas. Visitors come through Kayenta to get to the Tribal Park. The town has several amenities to support the tourism industry including three hotels, restaurants, a grocery store, and a shopping center. At the junction of Highway 160 and Highway 163 are several sit-down and fast -food restaurants. Fast-food restaurants include Sonics, Burger King, Subway, and McDonalds. What makes Burger

King unique is the fact that it houses a small museum dedicated to the Navajo Code Talkers of World War II. Other dining includes Golden Rice Bowl, Pizza Edge, Amigo Café, and Blue Coffee Pot. Amigo Café was recently highlighted in the show Restaurant Impossible. Other amenities include a Recreation Center, a small airstrip, and many churches.

Kayenta has one of the few public schools serving students on the Navajo Reservation, which is comparable in size to West Virginia. Many of the smaller schools are either Bureau of Indian Education school or Contract S, which are both funded by the federal government. Kayenta is the only community within a 50-mile radius that has public schools, including a high school. Kayenta Unified School District (KUSD) buses in students from the surrounding communities including Dennehotso, Chilchinbeto, Rough Rock, Black Mesa, Forest Lake, Shonto, Inscription Houses, and parts of Monument Valley. The day begins at 6 am for most of the students, who ride the buses from these communities to make it to school on time. KUSD is comprised of four schools: Monument Valley High School, Kayenta Middle School, Kayenta Elementary School, and ABC Preschool. Kayenta Elementary serves about five hundred students from Kindergarten through Fourth Grade. Each grade level has 5-6 classrooms with about twenty students in each classroom. I have been looping with my students since first grade, then second grade. This summer, I was informed that I will loop with my students to third grade. The principal believes that I would help the third-grade team meet the expectations set by the department of education.

A typical school day begins when students are dropped off at the elementary school at 7 am. Breakfast is served to all schools because all students qualify for free and reduced lunch. Students who have time are often sent to the playground or the gym until classes begin. Classes begin at 7:45 am. All classes are expected to begin with circle time, which is meet and greet, and time to implement social and emotional learning standards. This is followed by English Language Arts Blocks, which include reading, writing, grammar, foundation reading, and spelling. An hour is set aside for both math and reading intervention. In addition, there is a math block. Each class gets an hour dedicated to an elective class, which gives teachers an opportunity to prepare for class. Additional classes include social studies and science. School ends for students at three o'clock in the afternoon.

Rationale

Kayenta Elementary School has been considered a failing school and has consistently been receiving a grade of F from the Arizona Department of Education. Arizona Revised Statutes § 15-241 requires Arizona Department of Education to develop an annual achievement profile for every public school using an A-F scale. The achievement profile includes student academic growth, proficiency on English language arts, math and science, the proficiency and academic growth of English Language Learners, indicators that an elementary student is ready for success in high school and that high school students are ready to succeed in a career or higher education and high school graduation rates.[1]

School Year 2022/2023 was the final year for Kayenta Elementary School to improve their letter grade. That means that if the school fails again, the State of Arizona would take corrective actions. "The school will be forced into a partnership with an Achievement District School, a Fresh Start School Operations, or be shut down, at the Arizona Achievement District Board's discretion."[2]

In addition, this year, the school adopted Beyond Textbook, which is a framework for teaching. Beyond Textbook believes in the philosophy of teaching and learning that transcends textbooks and state standards to strengthen support for teachers, facilitate teaching and learning, and improve student growth and achievement. Because of Beyond Textbook, I was given an opportunity to create a curriculum that would help students learn about fractions "beyond the range and limits" of the state standards. Often, teaching about fractions is very dull. It involves a lot of rote-memorization that has no meaning or any application. My goal is to create a curriculum unit that will instruct students about fractions in a meaningful way and have real world applications. I do not want students to forget the introductory lesson on fractions, but rather, take it to the next grade level, and build on it.

My unit will focus on introduction to fractions. The standards for fractions are first introduced in second grade. However, the fractions standards are introduced as a geometry standard that partitions a circle or a rectangle into equal shares. Students are expected to recognize the partition parts into equal shares, and describe the shares as halves, thirds, and fourths. The lessons are memorization of the parts. The problem with rote memorization is that the students often forget what they learned, especially since it does not really have any application or any meaning. I would like to tie in Navajo culture to teach fractions, so it is more applicable and meaningful for the students.

I will be creating a curriculum unit for my third-grade students using a Length Model of Fractions. This involves the number line. First, I will get my students to understand that a number line is constructed in terms of length. I will create a number line by taping down sentence strips on the carpet. One sentence strip will represent one unit length. Once the whole unit length is established, students will be given shorter sentence strips that represent multiples of halves, thirds, fourths, sixths, and eighths. The students will use these unit lengths and mark them on the whole sentence strip with whole unit length. Then the unit will focus on comparing fractions using a number line. The students will compare the fractions by laying the sentence strips side by side. Students will see that some divisions of the whole are smaller than others. I will get my students to understand that the fractions n/d with a fixed denominator d divide each unit interval into d equal subintervals. Then I will help students see that as the denominator gets larger, the space between the number n/d on the number-line gets tighter. Also, if n is fixed, and d gets larger, the fraction n/d gets smaller. However, if the numerator gets larger, the fraction gets larger. It is important for students to grasp this opposite behavior. This is not the first intuition for many students. Before I begin, I will slowly introduce each fraction, and teach the students that there is a relationship between the numerator and denominator. I will display all fractions with a fixed denominator on the number line. Will seeing and using fractions in terms of length help students see how the size of fractions depend on the denominators? Will the students be able to differentiate between the fractions on a number line? These questions will be answered as we continue with the unit.

In addition, number lines will be utilized to show how fractions grow. Whole numbers will be used as a baseline to show growth of a number. I will ask my students questions like the following ones. What does one or "the whole" look like? What happens if you add one more? What about ½? What happens when you add another half? What about one-third? Is there a special pattern or configuration that students can observe? What happens when we graph n/d as a function of d? What shape or pattern does the line show? When comparing the growth of such fractions, students will use line graphs to show different rates of growth, but also see that the line graph shows a smooth curve. I expect that this can be a productive approach.

Cultural Relevance

Formal education for Navajo students on the reservation has been based on western culture. This contributes to native language loss. Also, Native American students struggle with cultural identity. Many educators, including the ones in Dine Institute for Navajo Nation Educators, and Navajo educators who are participating and have participated in the Yale National Initiative have taken the initiative to create culturally relevant curriculums. While writing this curriculum unit, I am mindful of the following four principles:

1. Individual students need to be respected and valued;
2. acknowledgement and understanding of diverse perspectives;
3. school curriculum should be meaningful and relevant, both personally and culturally; and
4. opportunity for all students. [3]

> The curriculum units created by educators of Navajo students have integrated Navajo disciplines and cultures in teaching academic subjects. These teachers encourage students that they can be successful, and cultural practices and values are used to teach students academic content, and as well to learn to respect their culture and other cultures. The goal is to empower Navajo students.[4]

When I started this curriculum unit, I had difficulty trying to think of a way to incorporate Navajo culture in my unit, because most things in our culture, from philosophies (education, life, etc..) to hogans, follow a circular pattern. Navajo hogans, technically in the shape of an octagon, still follow a circular pattern. The mission statement for Kayenta Unified School District states that it is the individual's responsibility to attain a successful life thinking, planning, learning working together, this is what we want for you, through guidance, we will get there. Most schools, including KUSD, have adopted a Navajo philosophy of learning and incorporated it into their mission statement. If you chart it out, it follows a circular pattern as well: to the east is thinking, to the south is planning, west is living, and north is assuring. My dilemma was trying to find a cultural representation that was linear, since I am using number lines in my unit to teach fractions. I decided to use the Navajo sash because it is linear, and because the length of a sash differs depending on what it is made for.

Navajo sashes, worn around the waist of Navajo women, is one of the woven textiles that was adopted by the Navajo people in the early 1800's. Navajo women traditionally wove them. Traditionally, there were four purposes for the sash. The first purpose is to symbolically indicate that a girl has entered womanhood, and the second purpose is to promote good posture. In addition, sashes were utilized and worn during labor and childbirth. Lastly, women wore the sash to serve as a corset wrapping around a woman's waist to shape her figure or regain her figure after childbirth. Today, sashes are worn as a belt to adorn traditional attire.

The sash traditionally uses three colors: red, white, and green. According to Jennifer Tsosie, Dine Studies Teacher at Chinle Unified School District, the red represents life and the blood that flows through your veins, the green represents Mother Earth, and all the plants that grow, and the white represent father sky and the clouds and rain it brings to Earth. Lastly, the fringes represent the rainbow.[5] Sashes have many distinctive designs. My idea is to give students the same length of sash and have them separate them into smaller units depending on fractions that I will assign to them individually. I will explain in detail how I will use the sashes with fractions in the activity section of the unit.

Content

What is a fraction? Why do we have to teach fractions? Where do we use fractions in the real world? The most common fraction used in everyday language is half or one-half. For example, you often hear people say just get half, get half of it, share, given them half and so on. When do people say half are they really thinking about the concepts of fractions? Are they really thinking of parts of whole or parts of a collections?

Surprisingly, fractions can be found and utilized in many places. Some of the most familiar places include recipes, and various forms of measurements (liquids and lengths). Most of the time a fraction is defined as some number of equal parts of a whole or parts of a collection.

A common or simple fraction is often shown with a numerator displayed above a line or slash, and a non-zero denominator shown below or after the line. The numerators and denominators are natural numbers. In mathematics, natural numbers are the 1, 2, 3, etc.… sometimes referred to as whole numbers in elementary school,[6] and known as counting numbers to primary students. Other fractions that use numerators and denominators include compound fractions (complex fractions), and mixed numerals. Compound fractions, also known as complex fractions, describe a fraction in which a numerator or denominator, or both, contain fractions. A mixed numeral, also known as a mixed number, is a whole number, and a proper fraction represented together. It is understood to represent the sum of the whole number and the fraction. (See Table 1 for examples.)

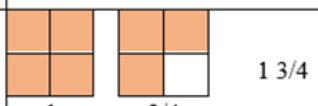
Fraction Type	Definition	Examples
Common fraction (simple fraction)	Shown with numerator above a line or before a slash, and a non-zero denominator shown below or after the line.	n/d or $\frac{n}{d}$
Natural numbers (whole numbers)	Numbers 1, 2, 3, etc., and can be used for counting. In primary school, it is referred to as counting numbers.	
Compound fractions (complex fractions)	A fraction in which the numerator or denominator, or both, contain fractions.	$\frac{2/3}{(1+3/4)}$
Mixed numerals	It is a mixed number that includes a whole number, and a proper fraction represented together.	

Table 1

Table 1 - Fraction Types

The numerator represents a number of equal parts left, and the denominator tells how many equal parts to divide the unit into which represents how many total parts make up the whole. The denominator cannot be zero because you must always have at least one part to make anything (other than zero). Zero parts cannot make a whole. For example, suppose you have ¾ of a pie. The three is the numerator that represents the number of parts present, while the four represents the parts that make-up the whole. The four tells how many (equal!) parts the pie is cut into, and the three says to take three of those parts. A common fraction may also be thought of as a quotient: ¾ can represent three divided into four equal parts. This equivalence can be

illustrated nicely with the number line. In arithmetic, a common fraction is a numerical representation of a rational number that is defined as a quotient, in which the numerator is divided by the denominator. Fractions can also be represented as decimals, percentages, or negative exponents. For example, ¼ is the same as 0.25, 2^{-2} , or 25%. Fractions can also be represented by ratios and division. Again, ¼ can represent a ratio of 1:4, and the division of one divided by four.[7]

There are so many possibilities and representations of fractions. As teachers, we try to teach fractions in a rational "sequential" order, but students need to realize that decimals, fractions, percentage, division, and ratios are all part of one system. Fraction is one of the mathematical concepts that is hard to convey to students, especially since it is no longer dealing with whole numbers, but parts. How will I prevent fractions from confusing my students? One idea that will be used in this unit is to introduce fractions in terms of distance.

A third-grade standard according to the Arizona Department of education states that: students will be able to partition shapes into parts with equal areas and express the area of each part as a unit fraction of the whole. Beyond Textbook, our current curriculum framework, states that according to the performance level indicators for proficient students, students should be able to partition shapes into b parts with equal areas, and students should express the area of each part as a unit fraction 1/b of the whole. Expectations for third grade are limited to fractions with denominators b = 2,3,4,6,8. The learning target for high proficiency students is that they can partition shapes into parts with equal areas and express the area as a unit fraction of the whole to answer questions presented in context. The relevant ideas for this unit are: shapes can be partitioned into equal parts and the parts can be represented as a fraction; one shape can be partitioned equally in diverse ways while still representing the same fraction. The shapes we should work with include quadrilaterals (rhombus rectangle, squares, isosceles trapezoids) isosceles triangles, regular hexagons, and circles.[8]

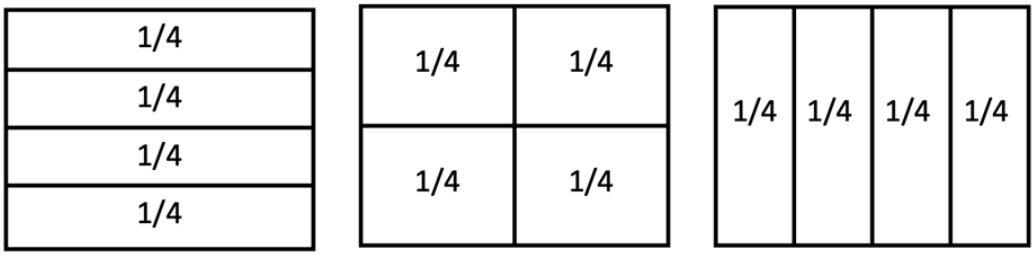


Figure 1 – Four Different ways to partition fourths.

The preferred model to use for fractions is to partition a shape into the division of equal parts. In this model students will partition a rectangle into fourths and all will represent the same fraction three diverse ways. The parts will have equal areas, and students should express the area as a unit fraction of the whole. Furthermore, the students will be required to recognize the fraction of the area of a shape that is represented by a part, identify the shapes that are divided into equal areas, shade a fraction of a shape, match given partitions with the correct fraction, and to construct a complete shape given only one of the partitioned areas of the whole shape.

Teachers within a grade level and subject then collaborate to "unwrap" the standards to determine essential ideas, key vocabulary, student-friendly language, essential questions, and performance tasks that prove mastery. "The documents produced in this process form the foundation of the core subject curriculums at each grade level and establish district-wide expectations for what teachers should teach and what students

should learn and be able to do in relation to a specific standard.[9]

Beyond Textbook reviewed state content standards to identify core sets of essential standards that established what students must learn in each content area at each grade level.[10] These are the standards that will provide students with what they need to learn by state assessment time. Additional standards are not as important as what Beyond Textbook deems as essential standards. Although the standard that states that students will "understand a fraction as a point on a number line; represent fractions on a number line diagram; and understand a fraction 1/b as a special type of a fraction can be referred to as a unit fraction (e.g. ½, ¼)", is not an essential standard according to Beyond Textbook, I believe that students still need to learn to use number lines to learn fractions. Utilizing number lines "help us understand arithmetic in terms of geometry[11]." So rather than working with symbols, number lines help us establish a more unified picture of what arithmetic is about and help students make more sense of it. However, before we begin, I should discuss what is a number line and how it would look.

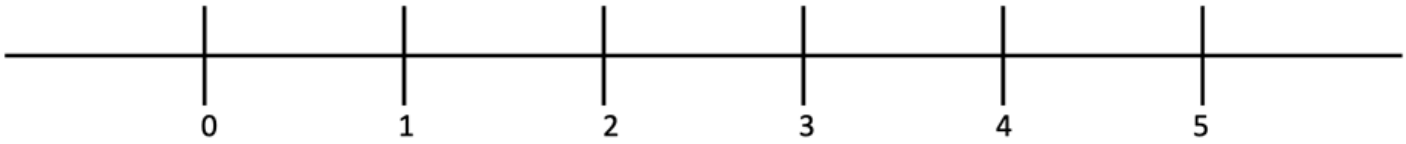


Figure 2 – Standard Number Line

A standard number line looks like this, a line with ticks which have unit intervals between them, which are equal to a unit length like a ruler. The important thing about a ruler is that it measures length or distance. Length is the size of the line, or any long and thin linear object like a rod or a tape.

Figure 3 Thickness of a line does not determine length.

Despite the thickness of an object or a line, they can be measured to the same length. A line could be one inch, and a thin strip could also measure an inch unit interval that has two points that are the same distance apart. Distance refers to the length of the line between two points. The distance between A and B is the same unit distance and the same unit distance between C and D. In general, the length of a straight line is the distance between its endpoints, and vice versa.[12] In order to measure length, a unit of length needs to be chosen, such as centimeter, foot, meter, inch, or any unit that measures length. Length, units of measurements, distance, straight lines, and endpoints are all involved in constructing a number line.

Creating a Number Line

As adults, we have seen and have constructed number lines, but a child who has never seen or has any experience with number lines will need help with the process of constructing a number line. How do you begin and how do you explain it to a third-grade student?

To begin the construction of a number line, you start with an unmarked straight line.

Figure 4 – An unmarked straight line.

The next step is to choose a point of origin or a zero point. Since I will only be working with positive numbers, I will place the zero point on the left side, but not placed as an endpoint. Students need to understand that a number line is endless in both directions.

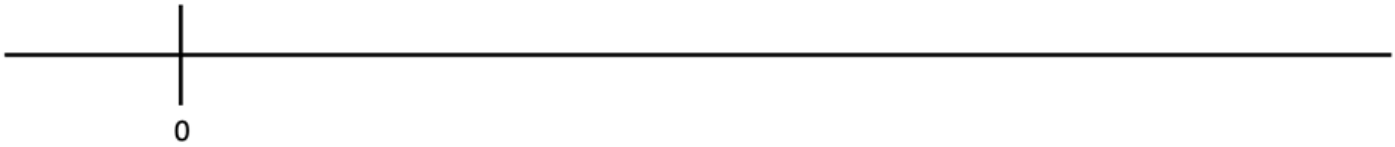


Figure 5 – Choose a point of origin on a number line.

Next, you choose a point to be one:

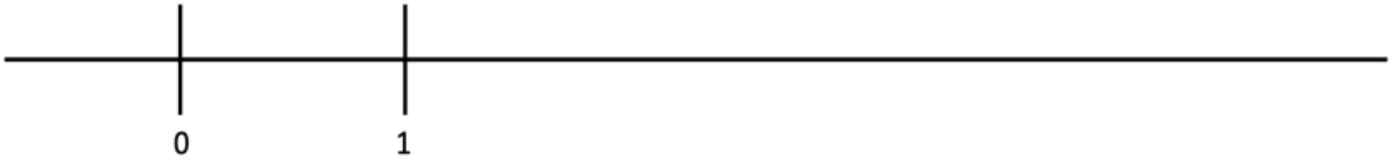


Figure 6 – Establish unit length of 1.

The unit length is established: on the number line, the unit length is length of the interval between 0 and 1.[13] The interval between 0 and 1 establishes the position of every number on the number line. The number labeling some other point tells you about the distance between that point and the origin, as a multiple of the unit distance. For example, the point labeled two is at distance of two units from the origin. In the diagram below, you can see that you can fit two intervals of length, one between 0 and 2. The intervals are shown in assorted colors so that it is easy to tell them apart, but they both have length one.

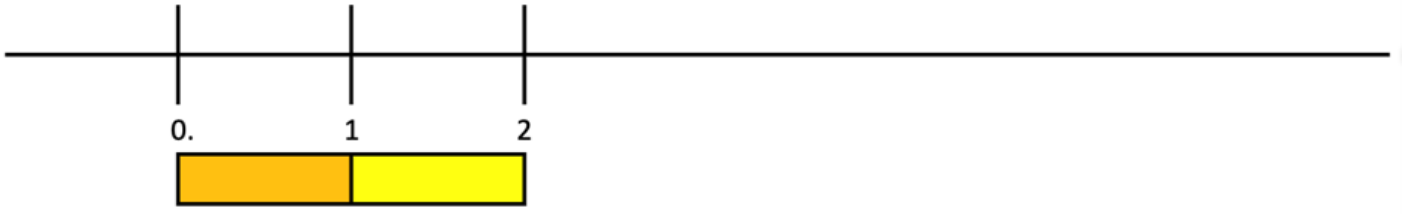


Figure 7 – The unit length between the two intervals is equal length.

As the diagram shows, you can fit three-unit intervals in the space between 0 and 3, and four-unit intervals between in the spaces between 0 and 4, and so on.[14]

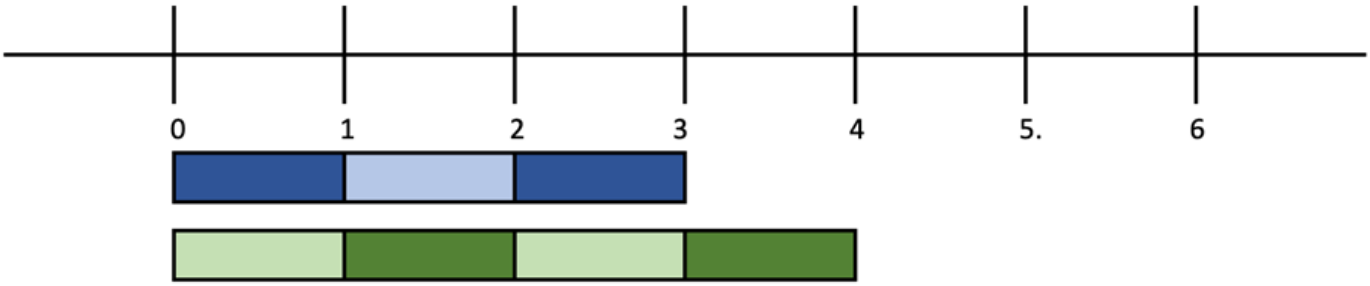


Figure 8 – Unit intervals between 0 and 3, and unit intervals between 0 and 4.

This relationship between unit length and measurements can be applied when we put fractions on a number line. For example, to get an interval of length 1/2, we cut the unit length into two equal pieces, where each one has a length of 1/2:

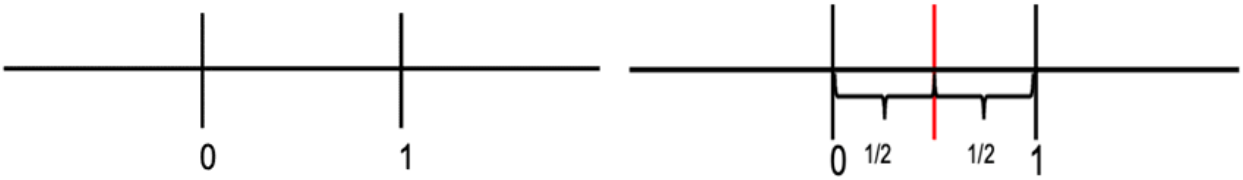


Figure 9 – One interval partitioned into two equal parts to show halves.

According to the rules of relating numbers to distance, 1/2 should label the point at the right end of the interval of length one-half, with its left end at zero.

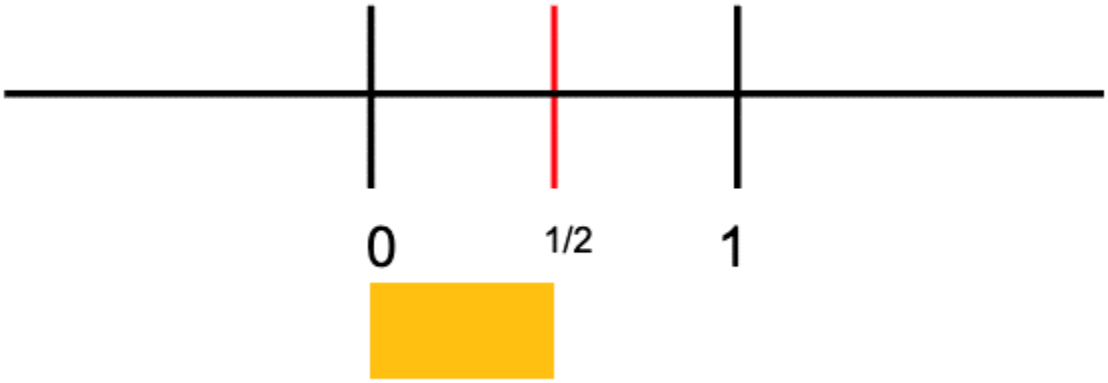


Figure 10 To show an interval of half, the interval begins at 0.

As we continue with one-half, we would put 2/2 at the distance made by two bars of length 1/2, which is the same as one. In arithmetic we learned that 2/2 = 1. As we continue the number line, we would put 3/2 at distance equal to 3 length of 1/2, and 4/2 would go four lengths of 1/2 (which is the same as 2-unit lengths from zero.) This entire process continues, the numbers are arranged in multiples of 1/2, including the whole numbers.[15]

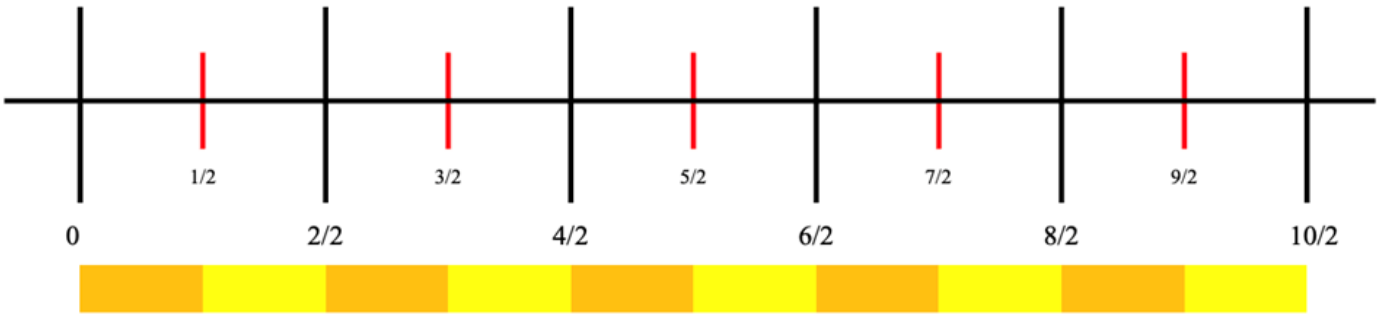


Figure 11 – The number line is arranged in multiples of halves, including the whole numbers.

Fractions with larger denominators follow the same process. For example, if we want to work with thirds, we divide one unit into three equal parts. The first piece starts at the left end or at zero, and the first piece will end at one-third. To get to the next third (2/3), we take two lengths of 1/3 and put them together, with the first one beginning at 0, and the other end will be at two-thirds. This continues the same pattern, each time we add one-third, it reaches the next third.

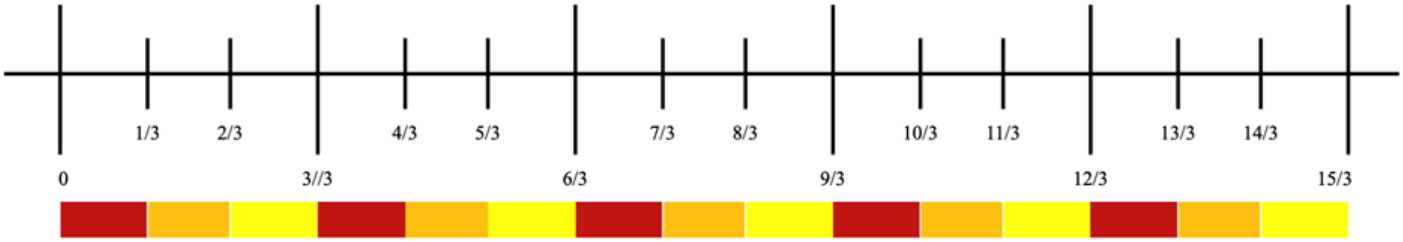


Figure 12 – The number line is arranged in multiples of thirds, including the whole numbers.

The number lines showing multiples of 1/2 and the number line showing multiples of 1/3 are like the number line for whole numbers. The only difference is that the number line for one-half is evenly spaced into two pieces for each whole number. The multiples of 1/3 are only 1/3 as far apart as the whole numbers, with three intervals of length 1/3 in every unit interval.[16] If the same process is applied to ¼, that would imply that the multiples of 1/4 are only 1/4 as far apart as the whole numbers, with four intervals of length 1/4 in every unit interval.

That means that a whole number would be able to be divided into multiples of any unit fraction. If the denominator is d, the multiples of 1/d are an evenly spaced set of points on the number line, and chopping up each unit interval into d equals smaller intervals. Here is a picture when d = 6.[17]

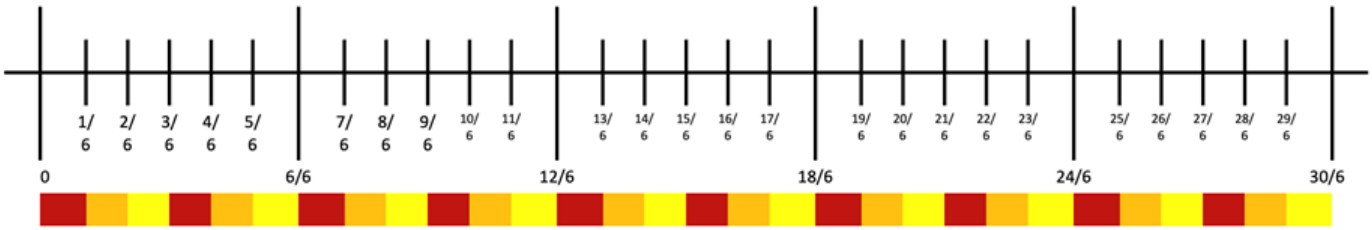


Figure 13 – A number line can be divided into multiples of any unit fractions, including sixth.

Comparing Fractions

Now that I have established what a number line is and how to set it up, comes the application of number lines to teach fractions. One main objective is for students to eliminate the confusion and problems of how large a fraction is. Number lines will help students see that as the denominator goes up fraction gets smaller. If students see fractions as measurement of distance, they will be able to distinguish between the intervals of each fraction. In addition, number lines help students compare fractions and find equivalent fractions.

Relative size of fractions causes the most problems when we start comparing fractions. Up to this point in math, I have been teaching students that larger numbers mean "more" when dealing with whole numbers. The students often transfer the knowledge and concepts of whole numbers to fractions. However, fraction is not dealing with whole numbers, but rather a whole unit that is chopped up into smaller unit intervals. For example, we can say that whole number 4 is larger or more than number 2. The students misuse that knowledge of comparing whole numbers to conclude that fourths should be larger than half. The relationship between number of parts and the size, or in the case of number lines, the length of the parts is a hard skill for students to learn. For many students, it is a developmental skill that takes time and repetition to grasp the concept.

For most of us, the rules of comparing fractions or the algorithm for comparing two fractions is finding the common denominator through the process of cross multiplying. This is a great strategy…if you understand fractions, you can get the right answer, but there is no thought that goes into the actual size of the fractions. If children are taught these rules before they have had the opportunity to think about the relative size of various fractions, there is little chance that they will develop any familiarity with or number sense about fraction size.[18]

So, what do we do? Let us look at some common errors that teachers and students make, especially with fractions that have like denominators. Suppose we compare 1/4 and 3/4. It is common for students to look at only the numerator and decide that 3/4 are larger. They are not wrong, because 3 is larger than 1, and the denominators are the same. But students need to realize that denominators in fractions must also be considered. A visual representation of 1/4 and 3/4 will show students why three-fourths are more than one-fourth.

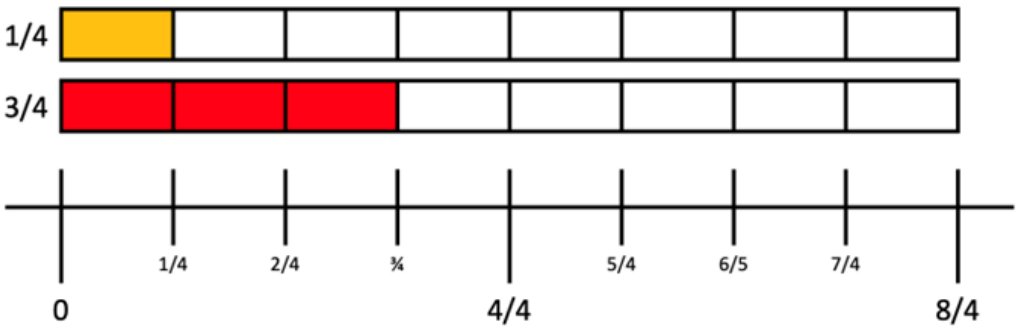


Figure 14 A visual representation to show that 3/4 is larger than 1/4.

What about fractions that have the same numerator, but different denominator? If a whole is chopped into four pieces, each piece will be smaller than when a whole is chopped into three pieces. Students will choose 1/4 over 1/3 only because 4 is larger than 3. Students do not realize that fourths and thirds are different

fractions and length relationships are different when dealing with denominators. It is like comparing grapes and oranges. One grape is smaller than one orange, the students must learn to compare the same fruits or find a tool such as a number line to help them compare length. Students may even reason that the fraction is the same size because the numerator is the same, but they must be taught that both numerator and denominator need to be factored in to determine which fraction is larger. However, if we put both numbers on the number line as in the illustration, children will see there is a difference in length and realize that one-third is larger than one-fourth.

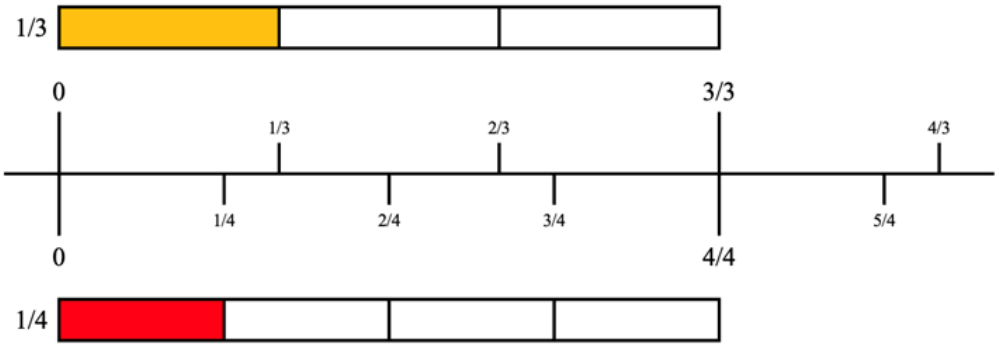


Figure 15 Numerators and denominators need to be factored in to compare fractions.

Comparing Fractions also creates an opportunity for students to see how each fraction grows in length. What I mean is that suppose I compare a whole number to 1/2, 1/3, and 1/4. Students will see that each unit fraction length is a different distance from the origin, 0. What I want students to notice as well is that each number line is chopped into intervals of different lengths. Here you can easily see that as the denominator gets larger, the unit length of the fractions gets smaller: $1/4 < 1/3 < 1/2 < 1$. The other crucial point to make is that the denominator describes how many pieces the (k/d) chops up a unit interval (where k is varying, and d is fixed).

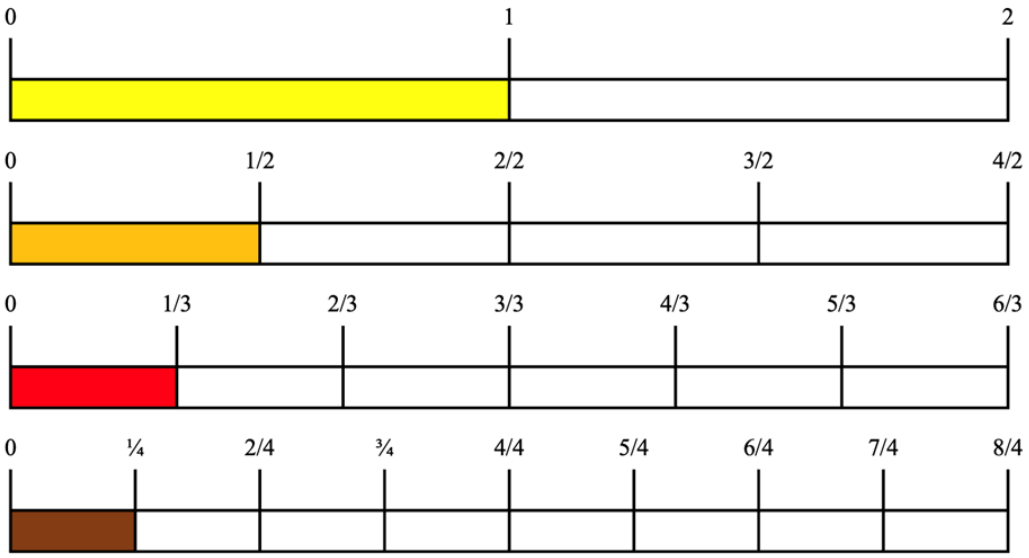


Figure 16 - Each unit fraction length is a different distance from the origin, 0.

But what happens when you add another fraction piece? Now each number line increases by one unit interval. When you connect the intervals, it appears that some unit intervals increase in length faster than the fractions

that are chopped into smaller unit intervals. Furthermore, when you connect each unit from each number line, it creates a special curve. The interesting part is that both make a smooth, and similar curve.

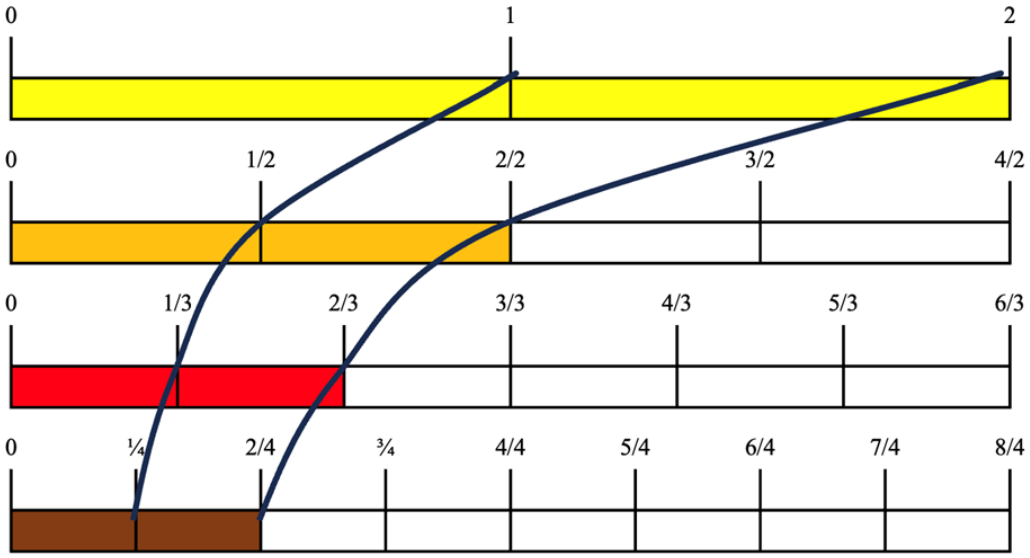


Figure16 A smooth and similar curve appears when you connect each unit fraction from the number line.

Equivalent Fractions

For students to find equivalent fractions on a number line, they must learn to deal with different denominators. Students are used to dealing with whole numbers that are organized using the base ten system. However, fractions do not follow the base ten system. The infinite number of fraction symbols for each number make dealing with fractions redundant. How do we eliminate confusion and redundancy? I will use number lines to help students see what is going on with equivalent fractions and comparing fractions to determine which fraction is larger than another. Now that we have a basic understanding of how to put numbers on a number line and understand that the number line is based on length and distance, let us use these ideas to construct a system of whole number multiples of a given unit fractions.[19]

Teaching Strategies

Manipulatives

Fractions are an abstract concept and often confuse students because they do not work the same way as whole numbers. In primary grades, they are introduced to fractions, but they do not really manipulate them in any way, meaning they do not add them, subtract them, multiply them, or divide them. The students are expected to learn that fractions are parts of a whole.[20] One way to help students recognize the parts, and the size, or in the case of number lines, the length of the fractions is to use manipulatives. Kayenta Elementary is privileged in the sense that the school district provided manipulative kits, enough for each student to have their own set. Building the fractions using the manipulative kits helps students with visual concepts on which to build their fraction knowledge.

Collaboration

Our school uses Instructional Empowerment to promote student collaboration, build engaging learning cultures in classrooms to close achievement gaps.[21] Collaboration groups have four to five students with mixed abilities. Each team member is assigned a role. In a collaborative group, one student is assigned as a Facilitator. The Facilitator's job is to lead the discussion and keep the students on task. In addition, the Facilitator encourages team members to share ideas or questions. The group also has a Learning Monitor. The Learning Monitor helps all students understand what they are learning and encourages team members to show their work. The rest of the group are Team Members. Team Members are expected to contribute to the discussion and bring their thinking to the group. They may also step in to help the Learning Monitor and the Facilitator with coaching team members. Collaboration works for all students because they are more willing to collaborate with their peers. The team members also get an Agree/ Disagree card to aid in their discussion. The card has four parts, the students may turn the card to "Agree" to show they agree with the speaker or turn the card to "Disagree" to show that they disagree with the response of the speaker. If they have a question, they may turn to "I Have a Question" section and pose their question. The last part is "More to Add." If they feel like they have more to add to the discussion, they may turn the card to this section. Each section has a sentence frame to help students have a discussion respectfully.

Small Groups and Station Activities

Small group instruction is the best way to help students who are struggling. Sometimes, students just need that extra attention or extra practice. Small group instruction is pulling 3-4 students at a time with similar abilities and reteach the lesson. Another purpose for small group instruction is to reach out to all students and help them master the lessons.

While attending to students in small group instructions, the other students will rotate through four concurrent math activities. Stations are activities that review and maintain the skills already taught and allows students to refine conceptual understanding and mastery. The stations are independent practice activities to help enhance learning using various modes of learning that include kinesthetic learning, auditory learning, visual learning, and promote conversations among students. Fractions will eventually become a station activity once the students are able to confidently work with fractions.

Classroom Activities

Making a Navajo Sash

Students will be given two sentence strips to create a sash design using fractions. Each sentence strip is twenty-four inches long, which will represent one unit length, so the sashes will be two units long. Students will be assigned a fraction unit. For example, if a student is assigned a fraction unit of 1/2, the student will partition their sentence strips into halves. Each half will have a single shape design, which will be one long triangle, and the second half will have the inverse of the same shape, so the designs become symmetrical. The design for the sash that is partitioned into thirds will have three symmetrical designs, and fourths will have four symmetrical designs. Once all the sashes are designed, the students will compare the fractions. They will be looking for equivalent fractions, the intervals of the designs. Why do some sashes have more

symmetrical designs than others? Students will discuss in collaborative groups the relationship between the denominators and find patterns in fractions.

Vocabulary Lesson

Mathematics has its own language, so it is like students are learning another language. Fraction has so many terminologies that students need to learn besides identifying a fraction representation, drawing a fraction representation using shapes, or just memorizing halves, thirds, fourths, sixths, or eighths. They also need to learn proper terms like numerator and denominator, instead of referring to it as the top number or bottom number. One of the reasons students do poorly on summative and formative assessments is because students neglect to learn the terms associated with the mathematical concepts, in this case fraction lesson. Students will define and use the terminologies in collaborative groups and throughout the lessons. It is important for students to learn mathematical terminologies.

Vocabulary activity will include matching a terminology with the definition, and an example using a game called Tap-Tap. In this activity, students will be given a worksheet with examples of fractions or fraction terminologies. The first step is for students to choose an example and make up a sentence about it. Then, once all the examples are tapped, the students will be given the terminology. They will match the terminology with the examples. The final activity is where the students are given the definition of the terminologies. They match the examples, terminologies, and definitions. Most of the time, students catch on after several practices. Furthermore, it gives them confidence to use the new words in their collaborative group discussions.

Growing Fractions Game

Students will be given fraction pieces that represent 1/2, 1/3. 1/4, 1/6, or one-eighth. In this activity, the students will add one fraction piece at a time and watch as the fractions "grow." The growth will help students see that the fraction pieces that have smaller denominator will grow faster than a fraction that has a larger denominator. This activity is also a good activity for comparing fractions as well as finding equivalent fractions. Collaborative Groups will discuss why denominators are important.

Resources

Kinaaldá*: A Navajo Girl Grows Up* by Monty Roessel is a story about thirteen-year-old Celinda McKelvey who is getting ready for her Kinaaldá, which is a coming-of-age ceremony for Navajo Girls. During the ceremony, she challenges her strength and endurance as she works side by side with family members and takes us through the events that happen in the ceremony which includes making a Navajo cake. It also includes Navajo oral stories about the first Kinaaldá. This book is important because Navajo sash represents a transition into womanhood for Navajo girls. The sash is one of the items that the girl wears during the ceremony. I will read this book to the students to help them understand the importance of Navajo sash to Navajo girls.[22]

Math is a Verb: Activities and Lessons from Culture Around the World by Jim Barta, Ron Eglash, and Cathy Barkley is a great resource for incorporating culture into mathematics. In many cultures, math is something they do as an intrinsic part of their everyday lives. [23] In this book, there are lessons that integrate culture and

math that meet common core objectives.

Appendix on Implementing District Standards

Arizona Mathematics Standards classify fractions under Number and Operations. The standard states that students will understand fractions as numbers. In third grade, students are expected to learn fractions with denominators 2, 3, 4, 6, 8. Standard 3. NF.A.2 wants students to understand a fraction as a number on the number line and represent fractions on a number line diagram.[24] I will cover this standard in my unit when I teach students how to draw a number line and partition the unit into parts. Standard three. NF.A.3, which expects students to explain the equivalence of fractions and compare fractions by reasoning about their size will also be taught in this unit.

Pre-K – 3rd Dine Culture Standard 1, Concept 1, P.O. 3 – I will recognize cultural items and jewelry. This Dine Culture Standard will be used in this unit. The cultural item that I will use is Navajo sash. Navajo sash designs will be utilized to help students learn fractions, compare fractions, and find equivalent fractions.

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Notes

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[2]A 'punitive model' for failing schools: GOP proposal would let charters

[3] Pedro Vallejo and Vincent Werito, *Transforming Diné Education Innovations in Pedagogy and Practice*, p. 19.

[4] Ibid, p. 22.

[5] Jennifer Tsosie, Interview, *Navajo Sash*, July 17, 2023.

[6] Wikipedia, annotated.

[7] Wikipedia, annotated.

[8] Beyond Textbook, Unwrapped Standard (UW,3. M.G.A.02), p.2

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[12] Ibid, p. 30

[13] Ibid, p. 31

[14] Ibid, p. 31

[15] Ibid, p.32

[16] Ibid, p.33

[17] Ibid, p. 33

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[19]*Engaging with Text in Mathematics*, p. 34

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