Curriculum Units by Fellows of the National Initiative
2023 Volume III: Transitions in the Conception of Number: From Whole Numbers to Rational Numbers to Algebra

Money, Money, Money: Decimal Fractions in $ and ¢

Curriculum Unit 23.03.03, published September 2023
by Lisa Yuk Kuen Yau

Introduction

I firmly believe that instilling good math and money skills in students will lead them to greater happiness and better health.

Teaching students about the benefits of money doesn't have to be "the root of all evil." Many researches have shown that wealth and social prestige are often the driving forces to greater happiness and even better health.[1] In fact, money does matter, especially for many marginalized groups in the United States where the striking wealth and health disparities between the haves and have-not continue to rise.[2] In a 2021 published study, about 6,000 Americans of diverse backgrounds were surveyed about their wealth and happiness, and also were given a math test with 8 questions that varied in difficulty to get a sense of their math skills.[3] The study shows that *on average* people who are better at math make more money, and are happier than people who are less mathematically inclined.[4] Because money can make a person's life easier, in terms of providing better shelter, education, healthcare, and access to more opportunities, I would argue that good money skills must be taught at an early age, especially in schools with a high population of underprivileged and underserved students.Let's prepare students to gain control of money instead of money controlling their destinies.

With the use of U.S. coins and decimal fractions, my curriculum unit aims to strengthen 3rd to 5th grade students' grasp of equivalency and place value among whole numbers, decimals, fractions, and percentages. For instance, a quarter can be represented as $0.25 or 25¢, and it is also equivalent to the fractions: 25/100, 5/20, ¼, as well as the percent: 25% of one dollar.

The unit is developed for 5th Grade U.S. History, and addresses the Math Common Core Standards 5.NBT.A.1 (place value), 5.NBT.A.3 (decimals), and 5.NF.A (fractions). The goal is to deepen students' concepts of rational numbers and better prepare them to transition to more abstract thinking involving the use of algebraic signs, symbols, number variables, number expressions, and equations. This curriculum unit is designed for 3rd to 5th grades with two main learning objectives:

- familiarize students with the history of U.S. coins;

- guide students to use coins like quarters, dimes, nickels, and pennies to develop greater understanding of place values.

Rationale

Nowadays, shoppers don't even need wallets to carry cash or credit cards because of mobile phones and online shopping. So why even bother using coins to teach math?

Firstly, most young learners like the idea of playing with money, and using coins to teach math is a great way to engage students. Coins (real or fake) are useful manipulatives and visual models that allow students opportunities to engage with meaningful real-life tasks, and talk about the concepts of addition, subtraction, multiplication, and division as they "do math" and "see math" with hand-on activities like sorting coins by sizes, colors, and denominations.

I've taught 5th grade math in a self-contained classroom for over 12 years, and every year there have been students who didn't know the value of quarters, or nickels, or cannot count by coins to make a dollar. The COVID-19 pandemic had disrupted coin production and circulation for almost 3 years, and also spurred an increased use of digital transactions instead of cash across the globe. When my school returned to in-person learning in 2021, many of my students were unfamiliar with the value of coins; some had never seen a nickel before. That year my school was selling pretzels for $0.75 each as a weekly fundraiser, a female student wanted to buy 4 pretzels. She gave me a five-dollar bill, and just walked away. She didn't know that 4 pretzels at $0.75 each would only cost $3, not $5. This incident reminds me of the 1976 hit song "Money, Money, Money" by the Swedish Europop group ABBA. The song is sung from the perspective of a woman who works hard but still struggles with money, and the only way to survive is to look for a rich man to rescue her financially.[5] In schools, math-gender stereotypes such as "girls are bad at math" and been shown to reduce girls' confidence, motivation, interests and performance in math, and lead to women being less likely to pursue careers in the STEM fields.[6]

Later the following year, in 2022, during our weekly pretzel fundraiser, a male student thought a quarter (25 cents), a dime (10 cents), three nickels (15 cents), and five pennies (5 cents) were enough to buy a pretzel for 75 cents. When I told him that he needed 20 cents more, he didn't know why. When I asked him to count and add all the values of his coins, he said with embarrassment that he didn't know how. Yet, this male student had shown me on many occasions that he can articulate his math thinking with confidence, but at other times when he failed to calculate a math problem correctly, he would look defeated and sometimes expressed his frustrations with: "I'm too dumb to do this." The harmful stereotype that "Only smart people are good at math" is so prevalent in our schools as well as in our society. Research studies have shown that students who were motivated, hardworking and skilled with good math strategies had gained significant math improvements in comparison to apathetic students with higher IQ.[7]

Throughout my teaching career, I've learned to be more empathetic to students who exhibit high math anxiety (an emotional reaction to math) and possible signs of dyscalculia (a math learning disorder akin to dyslexia). Dyscalculia often goes undiagnosed because it is not well-known and understood among educators; about 5-7% of elementary school aged children are estimated to be affected by dyscalculia.[8] I've seen 5th grade students who used their fingers and chins to add two numbers, unable to remember addition facts, and/or unable to understand simple equations.

Mathematics, more than any other subject, has the power to break students' spirits and crush their self-esteem at a young age. Many adults have never overcome their fear of math. In regard to coin values, most adults would have problems explaining mathematically how 4 quarters are equivalent to one dollar with a simple equation like $4 \times \frac{1}{4} = 1$. The concept of adding quarters to make a dollar can be quite straight forward or a complete mystery to some children as well as some adults. The value of a quarter can be written as ¼ of a dollar, 25/100 of a dollar, $0.25, 25¢, or 25 cents. It can be confusing to understand how these digits: 1, 4, 25, 100 are related. Some adults also may not be able to explain it mathematically using multiplication or division: 4 quarters multiplied by $0.25 = $1.00, or $1.00 divided by $0.05 equals to 20 nickels. With coin awareness, students can explain why the decimal point of a number will *move to the right when we multiply* by 10, 100, and 1,000, and why the decimal point will *move to the left when we divide* by 10, 100, and 1,000.

Furthermore, money knowledge and healthy money habits are great starting points to introduce financial literacy. Money can help students to familiarize themselves with large numbers like millions and billions, and very small numbers like decimals. According to a 2019 study about math ability and math confidence, almost one-third of American adults (73 million people) cannot select the health plan with the lowest cost based on annual premiums and deductibles, nor can they figure out how to pay off their credit card debt based on minimum monthly payment and the annual percentage rate.[9] In hospitals, overdosing and underdosing of medicines have happened so often that the mistake has been labelled as *death by decimals*.[10.] Coins offer a real-life incentive for learning mathematics for elementary as well as high school students, and I hope this unit would also help teachers to sharpen their student's math skills in money management.

School Demographics

Beginning in September 2023, it will be my 23rd year teaching in the School District of Philadelphia. Currently, I teach 5th grade at the Francis Scott Key School in South Philadelphia. The school serves students from K to 6th grades. My school demographics include a highly diverse community with a wide range of cultural and language backgrounds. The languages spoken by this diverse group of multilingual students, teachers, administrators, and parents include: Arabic, Burmese, Chinese, French, Hindi (India), Indonesian, Italian, Karen (Myanmar and Thailand), Khmer (Cambodia), Korean, Laos, Malay, Chichewa (Malawi), Nahuatl (Aztec/Mexica), Nepali, Pashto (Afghanistan and Pakistan), Poqomchi (Guatemala), Q'eqchi'(Central America), Spanish, Swahili, Thai, and Vietnamese.

In the school year 2020-2021, we had an enrollment of 419 students: 41% Hispanic, 38% Asian, 10% White, 8% Black, and 3% Multi-Racial; the student population is made up of 43% female students and 57% male students.[11] About 67% are English Language Learners (ELL), 5% had exited out of ELL services, and 15% are children of immigrants who are American-born (these students were NOT classified to receive ELL services, even though a language other than English is primarily spoken at home).[12] That's an estimate of 85% of the student body are recent immigrants and/or children of immigrants. Based on the 2022 Pennsylvania state test results, only 5% of students were either advanced or proficient in math, and only 12% in reading.[13] According to Niche.com, the median household income is $68,314, median rent us $1,385, and median home value is $233,155 for the neighborhood of my school.[14] According to the 2022 Low Income Data report from

Pennsylvania Department of Education, estimated 78.5% of students were qualified for Title One.[15]

Content Objectives

In the USA (as in many other countries), fractions are taught before decimals and percentages, and I believe this is a big reason why so many elementary school students are struggling to transition their knowledge of whole numbers to rational numbers.[16] With this curriculum unit, I'm advocating to math teachers to teach "decimal fractions" in connection to whole numbers, decimals, fractions, and percentages in order to mitigate students' difficulty in learning fractions. From my experiences, "decimal fractions" are easier for students to grasp, and students can transfer their hybrid knowledge to better understand decimals, fractions, and percentages. I believe, it doesn't matter which type of rational numbers is taught first, but more importantly, the connections and relationship among all rational numbers need to be addressed and included in our math instructions. Professor Roger Howe's seminar titled "Transitions in the Conception of Number: From Whole Numbers to Rational Numbers to Algebra" strengthened my knowledge about decimal fractions, and how I can apply this unit to relate it to arithmetic and algebra.

I teach 5th grade math in a 90-minute block daily. Even though six out of seven of the 5th Grade Math Units of Study from my school district focus on how to add, subtract, multiply, and divide fractions and decimals, the topics of decimals and fractions are taught separately with little connections to each other. In a 2004 longitudinal study done by researcher Vicki Steinle, University of Melbourne, about 3,000 students from 12 schools in Australia completed 10,000 tests over a 4-year period to evaluate their conceptual understanding of decimal notation.[16] The findings indicated that students' confusion about decimals often related to what they had been taught in the lower grades, and these prevalent errors are often not well-known to teachers.[17]

These misconceptions mainly stem from overgeneralization of place value understanding with whole numbers. For instance, it's true that the whole number 3,456 > 356 while it's false that the decimal 3.456 > 3.56. It seems likely that students do not understand the role of the decimal point versus the comma. In math, we use commas ("thousand-separators") to make it easier to read long numbers (equal or greater than 1,000). When writing decimals, we must specify that the location of a decimal point is between the "ones" and "tenths" place. When writing whole numbers, we have a convention that the decimal point is not shown, but is understood to be just to the right of the "ones" place (the rightmost of its last digit). For instance, the whole number 1 is understood as 1. [notice the decimal point to the right of the digit 1], and it is equivalent to the decimals: 1.0, 1.00, or 1.000, and so on. Because a decimal point serves as a dividing line between a whole number to its left and a decimal to its right, we don't write whole numbers as decimals because whole numbers are *not* decimals.

Yet, when decimals are first introduced in elementary school, many students mistakenly think that the decimal point separates two whole numbers – one whole number to the left, and another whole number to the right of the decimal point. In addition, how decimals are read aloud can be very misleading. For instance, the decimal 34.56 is generally read as "thirty-four point five six" which negates the concept of decimal place value; it is more accurate to read it as "thirty-four and fifty-six hundredths." Here I'm using the word **"and"** instead of "point," "fifty-six" instead of "five six," and adding the word **"hundredths"** to indicate the place value of the decimal. I believe that it is more helpful to teach students to read each decimal as a decimal fraction (example: 34 and 56/100 as thirty-four and fifty-six hundredths). It is important to clarify these practices and their implications for students.

Although “decimal fractions” obey the base-10 notational system of rational numbers, there are major differences between whole numbers and decimals that often lead students to misconceptions of decimals as well as fractions. Here are two common misconceptions about decimals and fractions:

- More digits mean greater magnitude (value). Examples of such common mistakes: the decimal 456 > 3.56 or the fraction 12/100 > 12/10;
- Adding zeroes to the right of a decimal or the denominator of a fraction will increase its magnitude. Examples of such common mistakes: 6.10 > 6.1 or 1/100 > 1/10.

Descriptions and Historical Aspects of U.S. Coins

The names and naming of each U.S. coin reveal many fascinating origins and important history of the United States. For more detailed descriptions, visit the U.S. Mint’s website: https://www.usmint.gov/.

Dollar or Thaler and Half-Dollar Coins:

In 1776, a committee led by Thomas Jefferson recommended to the Continental Congress to create a basic unit of currency for the United States called the “dollar.” The U.S. dollar derived its name from a Spanish and Austrian coin originally called the “thaler”— named for *Joachimsthal*, a silver-mining town in Bohemia. The U.S. “half-dollar” (head: President John F. Kennedy and tail: Presidential Seal) was conceived on the day of JFK’s assassination on November 22, 1963, in Dallas, Texas.

The U.S. “dollar coins” are now parts of a program that include: Native Americans (Sacagawea minted in 2000, Elizabeth Peratrovi in 2020, and Ely Parker in 2022), Presidents (Dwight Eisenhower in 2015, Ronald Reagan in 2016, George H. W. Bush in 2020), and American Innovation which began in 2018 and scheduled to run through 2023, with the obverse (head) featuring the Statue of Liberty and the reverse (tail) honoring innovation and innovators for each of 50 states, territories, and the District of Columbia. Other dollar coins feature Susan B. Anthony issued from 1979 to 1981 and 1999, Negro Leagues Baseball in 2022, Morgan dollar (named after its designer George T. Morgan, minted from 1878 to 1904, 1921 and 2021), and Peace dollar (designed by Anthony de Francisci as a result of a 1921 competition to find an emblem of peace). Even though “true gold dollars” are no longer minted, the dollar coins are often referred to as “golden dollars” because of their color.

Quarter Dollar:

The term "quarter dollar" refers to a quarter-unit of currencies of several different countries that are named "dollar." One dollar ($1.00) is normally divided into subsidiary currency of 100 cents, so a quarter dollar is equal to 25 cents. These quarter dollars (aka quarters) were denominated as either coins or banknotes depending on the country. The U.S. quarter (head: President George Washington, and tail: the bald eagle) was produced in 1932 to commemorate Washington’s 200th birthday. From 2010 through 2021, 56 quarters were issued as part of the America the Beautiful Quarters Program with the reverse designs depicting a national park or other national site for each of the 50 states, District of Columbia, and the five U.S. territories - Puerto Rico, Guam, American Samoa, the U.S. Virgin Island, and the Northern Mariana Islands.

Beginning in 2022 through 2025, the American Women Quarters Program celebrates the accomplishments and contributions made by women of the United States; five new reverse design are issued each year. The obverse will feature a portrait of Washington facing right. The reverse will honor the following women. See webpage https://www.usmint.gov/learn/coin-and-medal-programs/american-women-quarters and the list below for more details.

- 2022: Maya Angelou (writer), Sally Ride (astronaut), Wilma Mankiller (Cherokee Nation Chief), Nina Otero Warren (suffrage leader), and Anna May Wong (film star).
- 2023: Bessie Coleman (pilot), Edith Kanaka'ole (composer), Eleanor Roosevelt (human rights activist and first lady), Jovita Idar (educator), Maria Tallchief (ballet icon).
- 2024: Pauli Murray (civil rights activist), Patsy Mink (Congresswoman), Mary Edwards Walker (Civil War era surgeon), Celia Cruz (singer), Zitkala-Sa (writer).
- 2025: Honorees to be determined.

Decimus, Disme, or Dime:

"Dime" is based on the Latin word "decimus," meaning "one tenth." The French used the word "disme" in the 1500s when they came up with the idea of money divided into ten parts. In America, the spelling changed from "disme" to "dime." The designs of the early dimes showed a woman symbolizing liberty. The U.S. dime (head: President Franklin D. Roosevelt, and tail: olive branch, torch, and oak branch) replaced the Statue of Liberty in 1946, the year FDR died.

A Demon named Nickel:

A group of frustrated miners blamed Nickel, a demon in German mythology for playing a dirty trick, and prevented the extraction of copper from the mine. The miners started to call the demon "*kupfernickel"* which means copper demon in German. After the American Civil War, 5 cents were made out of copper and nickel alloy rather than silver, and gradually the name nickel became popular. Modern nickels are now made from 75% copper and 25% nickel. Silver coins called "half-dime" were produced from 1792 to 1873. The U.S. nickel (head: President Thomas Jefferson, and tail: Monticello) was debuted in 1938, five years before Jefferson's 200th birthday.

Pennies or Cents:

The U.S. Mint's official name for the penny is "cent." The term "penny" derived from the British coin with the similar name, but different value. "Pence" refers to the British unit of currency and is sometimes confused with the term "pennies." In the early 1960s Great Britain still used a pre-decimal currency system of 12 pence to a shilling, and 20 shillings to a pound. Therefore, there are 240 pence in a pound. On August 2023, one pound can be exchanged to about 1.27 U.S. dollars. The U.S. penny (head: President Abraham Lincoln, and tail: Lincoln Memorial) was debuted in 1909, 44 years after Lincoln's assassination.

U.S. Mints in Philadelphia, Denver, San Francisco, and West Point

Over 200 years ago on April 2, 1792, the first U.S. Mint was established in Philadelphia (Pennsylvania) by the U.S. Congress as part of the Coinage Act, also known as the Mint Act. Currently, there are also three other locations: Denver (Colorado), San Francisco (California), and West Point (New York). The U.S. Mint is a bureau of the U.S. Department of Treasury, and is responsible for producing coinage for trade and commences, designing collectible commemorative coins, and distributing coins to banks. The U.S. Bullion Depository at Fort Knox, Kentucky is another U.S. Mint facility, but its primary purpose is for the storage and protection of gold and silver bullion (in bars or ingots). The U.S. Mint makes only coins and not dollar bills. The Bureau of Engraving and Printing (with facilities in Washington, D.C. and Fort Worth, Texas) is responsible for the printing of dollar bills.

Figure 1. Top row: 1) number lines in quarters, dimes, and pennies; 2) rolls of stacked coins in different monetary values and denominations; 3) a coin collection of the 2023 American Women quarters. See webpage: https://www.usmint.gov/learn/coin-and-medal-programs/american-women-quarters. Bottom row: Cover of the National Park Quarter Collector Map (2010-2021), and a U.S. map designed to collect quarters from 50 states. See webpage https://www.usmint.gov/learn/coin-and-medal-programs/america-the-beautiful-quarters.

The public can visit the Philadelphia Mint located at 151 N. Independence Mall E. for free to see the different process of coin production from 40 feet above the factory floor and walk through an exhibit with many historical details and educational activities. This mint is the fourth and current building (1969 to present). The basic coin production process includes: *designing* by in-house artists, *blanking* round disks, *annealing* (in a furnace over 1,600 degrees in Fahrenheit, then cool in a large quench tank), *washing* to remove discoloration, *upsetting* to make a raised rim that protect the images of the coin and make it stackable, *striking* to create designs on the coins, *counting*, *bagging* bulk bag that weighs up to one ton, and finally *shipping* to different Federal Reserve locations across the United States. The public can also see the legendary (embalmed) bald eagle named Peter the Mint Eagle who lived at the Philadelphia Mint from c.1830 until 1836, the first coining press, and other historical artifacts. Seven 5-foot tall Favrile glass mosaics designed by the Tiffany Studios, based on a wall painting unearthed at Pompeii, illustrate figures of children performing the ancient Roman coin-making processes of melting, granulating, drying, weighing, stamping, and finishing. A variety of virtual tours at the U.S. Mint webpage (https://www.usmint.gov/), and many educational YouTube videos are available online (https://www.youtube.com/watch?v=mX73eThpM2k&t=20s).

Five Stages of Place Value Understanding

In the article titled "The Five Stages of Place Value" written by Roger Howe and Harold Reiter, the authors point out that place value is often the most undervalued concept in the elementary curriculum.[18] Place valuation notation represents every whole number as a sum of *numbers of a very special kind*. The authors detail five important stages of place value that can help students develop better conceptual understanding needed for more advanced math thinking like algebra.

$$9{,}786.543$$

$$= 9{,}000 + 700 + 80 + 6 + 0.5 + 0.04 + 0.003$$

$$= (9 \times 1000) + (7 \times 100) + (8 \times 10) + (6 \times 1) + (5 \times 0.1) + (4 \times 0.01) + (3 \times 0.001)$$

$$= (9 \times 10 \times 10 \times 10) + (7 \times 10 \times 10) + (8 \times 10) + (6 \times 1) + (5 \times \tfrac{1}{10}) + (4 \times \tfrac{1}{100}) + (3 \times \tfrac{1}{1000})$$

$$= (9 \times 10^{3}) + (7 \times 10^{2}) + (8 \times 10^{1}) + (6 \times 1^{0}) + (5 \times 10^{-1}) + (4 \times 10^{-2}) + (3 \times 10^{-3})$$

Stage 1 (First Expanded Notation):

9,000 + 700 + 80 + 6 + 0.5 + 0.04 + 0.003 is the first expanded notation. For 1st grade, students start by getting comfortable with two-digit numbers made out of 10s and 1s, and get comfortable to add and subtracts with 10s and 1s independently. For example: 34 = 30 + 4, 34 = 3 tens and 4 ones. This habit should be reinforced in 2nd grade as the basis for understanding computation of any whole numbers.

Stage 2 (Second Expanded Notation):

$(9 \times 1000) + (7 \times 100) + (8 \times 10) + (6 \times 1) + (5 \times 0.1) + (4 \times 0.01) + (3 \times 0.001)$ is the second expanded notation. Students should learn the 2nd expanded notation when they are introduced to multiplication (usually in 3rd grade).

Stage 3 (Third Expanded Notation):

$(9 \times 10 \times 10 \times 10) + (7 \times 10 \times 10) + (8 \times 10) + (6 \times 1) + (5 \times \tfrac{1}{10}) + (4 \times \tfrac{1}{10} \times \tfrac{1}{10}) + (3 \times \tfrac{1}{10} \times \tfrac{1}{10} \times \tfrac{1}{10})$ is the third expanded notation. Students should learn the 3rd expanded notation in 4th grade as part of understanding the relationship between the base ten units where each is 10 times larger than the digit to its left, and 1/10 times smaller than the digit to its right. For 5th grade, this is part of preparation for extending the base ten system from whole numbers to decimal fractions. This skill is related to the 5th grade Common Core Standard 5.NBT.A.1 on recognizing that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right, and 1/10 of what it represents in the place of its left.

Stages 4 and 5 (Polynomial Form):

$(9 \times 10^{3}) + (7 \times 10^{2}) + (8 \times 10^{1}) + (6 \times 1^{0}) + (5 \times 10^{-1}) + (4 \times 10^{-2}) + (3 \times 10^{-3})$ is the fourth expanded notation in polynomial form. In 6th grade, the polynomial form should be discussed and worked with when exponential notation is introduced, and continue to develop in 7th grade with the Law of Exponents. In 8th grade, the extension to negative exponents (for decimal fractions) should be made explicit. Stage 5 is when the polynomial form should be reviewed and related to polynomial algebra when it is studied in high school.

In summary, it is important to reinforce these five stages of place value understanding starting from 1st grade through high school. For my 5th grade students, I make it a standard practice for them to read a price like \$5.20 as 5 dollars and 20 cents (not 2 cents), and explain why "5 dollars and 2 dimes" is also correct. Then I want students to understand why the expanded notation can be expressed as 5 + 0.2, and be cognizant of the reason that the digit zero in the hundredth place is needed when representing money. I teach explicitly how to say decimals in word forms to relate them to decimal fractions, examples: 5.2 reads as 5 $^{2}/_{10}$ (five and two-tenths), and 5.20 reads as 5 $^{20}/_{100}$ (five and twenty-hundredths). Students will want to say 5.2 as "five point 2" and 5.20 as "five point two zero" and may mistakenly think 20 cents is equivalent to 2 cents rather than the value of the decimal fraction: $^{20}/_{100}$. It is also beneficial to students to learn to the decimal fractions into other equivalent fractions, decimals and percentages such as $^{1}/_{5}$, $^{2}/_{10}$, $^{3}/_{15}$, $^{4}/_{20}$, $^{5}/_{25}$, $^{6}/_{30}$, $^{7}/_{35}$, $^{8}/_{40}$, $^{9}/_{45}$, $^{10}/_{50}$, 0.2, 0.20, and 20%.

Beware: Money as a False Analogy of Decimals

Teaching decimal fractions with references to U.S. coins and money is a good start, but it may limit students' understanding to only two place values after the decimal points because they only have experiences with dollars and cents. Therefore, knowledge of using decimal fractions to represent dollars and cents does not necessary help students to understand decimals beyond the hundredths place.

One task to mediate such misconceptions is to use a number line, and ask students how would they describe the relative size of decimals with more than two decimal places. For example, I will have students to compare the difference between 1.987 and 1.986 by plotting points on a horizontal or vertical number line, and ask high order thinking questions such as: Which decimal is closer to 1.98? I would present number lines of different intervals, and ask students where is zeroes in which number line. This will support students in deciding how to construct a helpful number lines to solve future math problems.

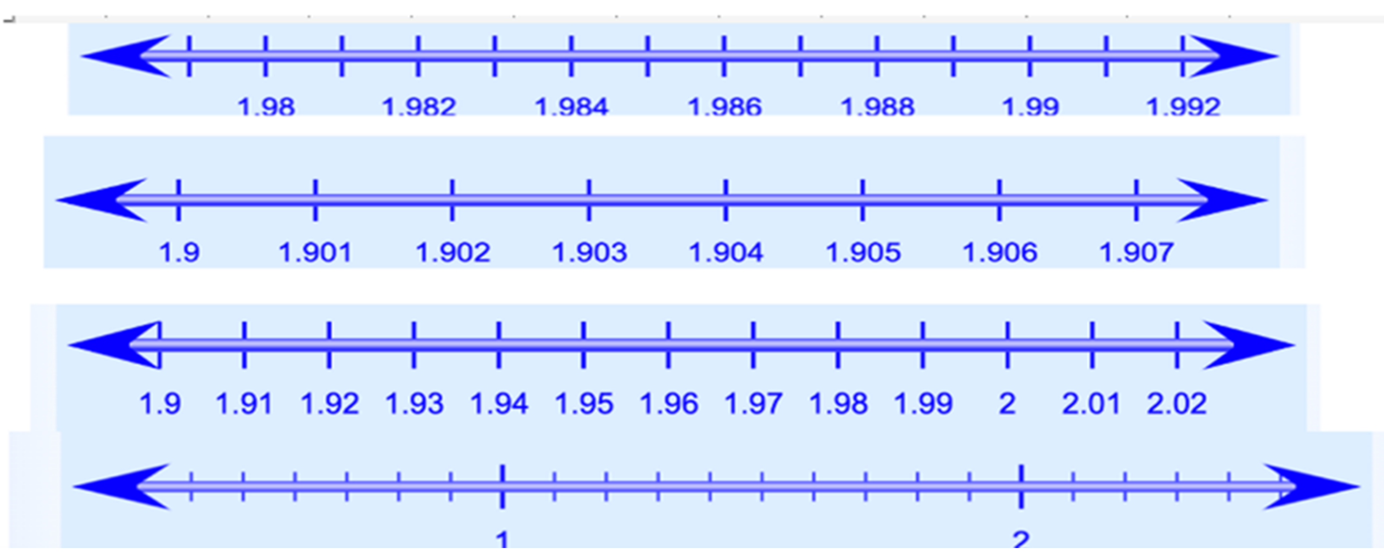


Figure 2. The different number lines (above) were generated from the website: https://www.mathsisf

un.com/numbers/number-line-zoom.html.

Teaching Strategies

This curriculum unit integrates Social Studies Pennsylvania State Standards about the United States (history and geography), and 5th grade Math Common Core Standards about place value, decimals and fractions. Teachers could choose to teach all or the part that is relevant to their students. The unit is designed for grades 3rd to 5th , but can be modified for grades 1st to 8th .

Progression of the Unit

The 1st group of lessons will focus on observing, classifying and researching U.S. coins, locating National Parks from the 50 states, and historical individuals, especially women trailblazers. Real and fake coins as well as paper illustrations will encourage students to be more active and hands-on when learning about decimal fractions. For instance, students can model $0.30 with 3 dimes and the equation 3 x $0.10 = $0.30, or with 6 nickels and the equation 6 x $0.05 = $0.30. As part of the unit, I will encourage and challenge students to collect as many different types of U.S. coins as they can, and coins from other countries as well. These class challenges are relevant, especially when schools have a diverse student body like my school [See School Demographics].

The 2nd group of lessons will focus on using coins to learn more about decimal fractions, the base-10 number system and place values. Different number systems will be discussed to illustrate why understanding the concept of place value is so critical. For instance, the two most commonly used number system are: the decimal (Base-10), and binary (Base-2) mainly because both are easy to manipulated for human use and computer applications. Starting in the school year 2023-2024, my school district is implementing Illustrative Math (IM) as a district-wide math curriculum. The IM framework uses the following teaching strategies: *opening routine, formal assessment, whole group guided practice, small group instruction, independent work,* and *whole group reflective closure.* I believe that it is important to use direct instruction, guided instruction, peer tutoring, and cooperative learning, to develop a community of math learners, and independent learning centers for students to practice and master what they are learning. In addition, number lines, tape diagrams, and graphic organizers are excellent tools to enable students to visualize their math thinking as well as build their academic math vocabulary.

Independent Learning Centers

Embedded throughout the unit will be opportunities for students to understand how numbers are written more logically in certain languages, and more misleadingly in some other languages, and discuss the major advantages and disadvantages of the different number systems. For example, I've witnessed Spanish-speaking students who struggle to remember the English words for numbers from "11 to 19," and often say numbers such as 13 as "thirty" instead of "thirteen." One of my former students used to say "one three" for 13, "one four" for 14, and so on. The words "eleven" and "twelve" give little clues about the based-10 number system and place value. In other languages like Chinese, 11 is written 十一(tens one), and 12 is 十二 (tens two) while twenty is 二十 (two tens). In Chinese, most fractions are written with the denominator before the nominator, example: 1/3 (one-third) is written as 三分之一 (three portions of one) where three is not a whole number, but three parts that make up a whole. Once students learn how to say whole numbers in Chinese, they are able to talk about rational numbers in decimals, fractions, and percentages with the addition of a few more vocabulary words.[19] I would argue that most Asian languages have a built-in system of relative linguist clarity to make numbers easily to learn and manipulate mathematically. Other examples are: in Khmer, the number six is written as five-one (ប្រាំមួយ); and in Swahili, two-digit number like eleven (kumi na moja) are written with the word “na’ meaning “and” between the two digits.

I plan to set up a learning station for students to work collaboratively or independently with coin collection, and another learning station for students to learn how to write and say the numbers 0 to 20 from different languages. Below are some resources for the two learning centers.

- https://polyglotclub.com/wiki/Language/Multiple-languages/Vocabulary/Count-from-1-to-10-in-many-languages
- https://mathlair.allfunandgames.ca/languages.php 1 to 10 in different languages https://mathlair.allfunandgames.ca/languages2.php 11 to 20 in different languages
- https://www.mentalfloss.com/article/31879/12-mind-blowing-number-systems-other-languages

Opening Routines and Reflective Closures

In recent years, there has been a big push for teachers to use “teacher noticing” and “leading discussion” as an opening routine to allow students multiple entryways to the lesson.[18] Encouraging students to use the strategy of mental math (without the use of pencil and pen) to make reasonable approximation is a powerful tool. Opening routine questions can also be modified to serve as reflective closure activities at the end of a lesson to assess the precision of what students have learned. Below are some approximating questions to help students to build fluency with decimal fractions using coins:

- What is the least number of coins you need to make a total of $1.20?
- What is the greatest number of coins you need to make a total of $1.20?
- Estimate what fraction of 20¢ is 15¢? What fraction of $0.75 is $0.25?
- What is half of $0.50 versus $0.50 divided by one-half?
- Using the fewest number of coins possible, how close can you come to 1/3 of dollar?
- Is 1/3 (one-third) of $1.00 the same as 33% of $1.00? Which value is greater?

Formative Assessment Versus Reflective Closure

I think both formative assessment (FA) and reflective closure serve similar purposes. Their core role is to enable teachers to assess students’ understanding of the learning objective of the lesson. While FA is continuous, typically informal and constant throughout a lesson with teachers giving as many immediate feedbacks as possible to as many students as realistically doable, reflective closure (usually at the end of a lesson) provides time for all students to absorb what should have been learned, and how the concepts are learned through individual writing (sometimes sharing by verbalizing with peers or in group discussion). Reflective closure is an effective way to debrief the lesson in order for teachers to plan the next instructional steps. Sometimes it can be productive to assign homework for reflective closure in order to give students ample time to process what they have learned.

Guided Instruction: Whole Group and Small Group

Guided instruction can be done with a whole group or in small groups. It is a structure whereby a teacher supports each child's development of math proficiency at increasing levels of difficulty in order to reach all students where they are at, and to take them to the next level. During guided instruction, teachers can use guided questioning, manipulatives like coins, base-10 blocks, as well as visual representations like number lines, tape diagrams and hundred**th** grids (not the same as hundred grids for whole numbers) to deepen student understanding.

Classroom Activities

I will introduce this curriculum unit to students with coins, visuals, videos, articles, and websites about the U.S. coins. Here is a list of materials and helpful resources I expect to use: Fake and real U.S. coins, paper copies of coins, magnifying lens, rulers, chart paper, notebooks, books on the history of U.S. coins, and the U.S. Mint's webpage: https://www.usmint.gov/learn/kids.

U.S. Coin History and Observation

For the 1st group of lessons, I will have students work in pairs to observe real pennies, nickels, dimes, quarters (optional: half dollar coins and dollar coins) with the chart below *(Figure 3: U.S. Coin Specifications).* I will have students refer to https://www.usmint.gov/learn/kids/coins-life/parts-of-a-coin (Parts of a Coin), https://www.usmint.gov/learn/kids/resources/coloring-pages (Coloring Pages), and/or use a magnifying lens to see the magnified details of each denomination. Below is a short list of probing questions I expect to use during this activity:

- Why are coins in the shape of a circle? (Easier to roll and use).
- Does the edge of a coin feel smooth (pennies) or rough (quarters)?
- Why is the rim of a coin raised? (Protect design and allow coins to stack)
- What is written on the head (obverse) and the tail (reverse) of a coin?
- Why words or phrases like "Liberty", "In God We Trust," "E Pluribus Unum," and a mint mark (D, P, S, or W) are inscribed on the coin you are observing?

Research Project on U.S. Coin History:

Quarters are excellent tools for students to start their research about U.S. History. I will have students work in pairs to write a report about the history behind one of the quarters that honor an individual or a national park. Here are a few webpages to get the research started:

- https://www.usmint.gov/learn/coin-and-medal-programs/american-women-quarters
- https://catalog.usmint.gov/coins/coin-programs/american-innovation-dollar-coins/
- https://parksexpert.com/national-park-quarters/
- https://www.usmint.gov/learn/coin-and-medal-programs/50-state-quarters

U.S. Coin Specifications for Student Observations and Notes						
Denomination (Coin Name)	**Penny**	**Nickel**	**Dime**	**Quarter**	**Half-Dollar**	**Dollar Coin**
Obverse (Head)				Washington		
Designer/Sculptor				Fraser/Fraser		

Reverse (Tail)				Bessie Coleman		
Designer/Sculptor				Damstra/Gordon		
Color				Silver		
Material Composition				8.33% nickel, balance copper		
Weight				5.670 grams		
Diameter				0.95 in./24 mm		
Radius				0.475 in./12 mm		
Edge				reeded		
Place of Mint				San Francisco		
Mint Mark				S		
$ Notation	$0.01	$0.05	$0.10	$0.25	$0.50	$1.00
¢ Notation	1¢	5¢	10¢	25¢	50¢	100¢
Decimal Fraction	1/100	5/100	10/100	25/100	50/100	100/100
# of coins to make $1.00	100 pennies	20 nickels	10 dimes	4 quarters	2 half-dollars	1 dollar coin
Additional Observations, Drawings & Notes						

Figure 3. The above chart will help students as they explore the coin specifications for the penny, nickel, dime, quarter, half-dollar and dollar coin.

Decimal Fractions

For the 2nd group of lessons on decimal fractions, start with a quick pre-assessment called Decimal Comparison Test.[21] The purpose is to diagnose how students are thinking about decimal notations; it is a quick, easy, and revealing way to assess what students know. The original test was part of a study from the University of Melbourne, and it asked each student to simply choose the larger decimal number. The research study concluded that only about half of 900 students understand whole number place value well and many of them understand the place value of decimals even less.[20] Some children misinterpret decimals by making false analogies with whole number concepts. The most common sources of confusion are with money, sport, and remainders in division. Other misconceptions include: longer-is-larger whole number thinking; shorter-is-larger thinking; zero-makes-larger thinking; left to right comparison while ignoring place value.

Below is a similar test that I constructed based on the original study. Here students are asked to use the mathematical symbols >, < or = to correctly complete the comparison. I would use the following approach for a class period of 45 minutes. For the first try, I would give students 5 minutes to quickly complete as many comparisons as they can. After the initial assessment, I would group students in pairs for a 2nd try, and give them 10 minutes to discuss and rework their answers. For the 3rd try, I would group the two pairs into a group of four students, and give them another 10 minutes to compare their answers and discuss any discrepancies. For the remaining class period, I would gather students into a whole group for a reflective closure, and ask each group to share one misconception that they were able to spot and correct it together.

Direction: Below are 30 pairs of decimals. For each pair of decimals, write the correct symbol >, < or = to complete the comparison.

1.	0.5 ____ 0.457	11.	1.053 ____ 1.06	21.	4.4502 ____ 4.45
2.	0.86 ____ 1.3	12.	4.08 ____ 4.7	22.	17.350 ____ 17.35
3.	0.300 ____ 0.3	13.	3.72 ____ 3.073	23.	8.24 ____ 8.2456
4.	1.85 ____ 1.84	14.	2.621 ____ 2.068798	24.	3.2618 ____ 3.26
5.	3.67 ____ 3.76	15.	8.0526 ___ 8.056	25.	0.350 _____ 0.035
6.	4.8 ____ 4.63	16.	5.62 _____ 5.736	26.	2.186 _____ 2.954
7.	0.03 ____ 0.0300	17.	0.5 _____ 0.75	27.	0.872 _____ 0.813
8.	0.75 ____ 0.8	18.	0.426 ____ 0.3000	28.	0.006 _____ 0.53
9.	0.37 ____ 0.216	19.	2.516 ____ 2.8325	29.	3.741 _____ 3.746
10	3.92 ____ 3.4813	20.	7.942 ____ 7.63	30.	0.038 _____ 0.04

Figure 4. Decimal Comparison Assessment: Students will compare 30 pairs of decimals.

Decimal Points and Place Values

When teaching decimals, I would make sure students understand the purpose of a decimal point by asking them guiding questions, and then give time discussion time to think-pair-share. Possible answers: A decimal point separates the whole number from the fractional part; acts as "a point of reference" between the ones and tenths place; the right side of a decimal point represents value less than one whole. Below is a short list of videos that explain decimal place value, conversion of decimals into fractions:

- https://www.youtube.com/watch?v=KG6ILNOiMgM on decimal place value
- https://www.youtube.com/watch?v=_jcW-ZgpRbM&t=209s on decimal fractions
- https://www.youtube.com/watch?v=do_IbHId2Os on convert any fraction to a decimal

1	1.	1.0	1,000	1.00
10.	100.	0.1	0.01	0.001

Figure 5. I would use the above decimals to discuss what are the place values of the digit 1.

Here are some guiding questions I expect to: What is the greatest value of the digit 1? What is the least? Is the decimal 0.1 greater or less than 0.01? (Greater since 0.1 = 10 x 0.01). How many tenths are needed to make one? (10 because 10/10 = 1) How many hundredths are needed to make one whole? (100 because 100 divided by 1/100 = 1) How many thousandths are needed to make one? (1,000 because 1,000 x 0.001 = 1).

One common misconception when multiplying by 10 is teaching students the "tricks": the decimal point moves one place to its right (when multiplying by 10) or to its left (when dividing by 10). First of all, the decimal point NEVER moves. I would have students visualize the decimal point like a fixed column in a building that always stands between the ones and tenths place. The mathematically correct explanation is: the digits slide one place to the right when multiplying 10, and slide one place to the left when dividing by 10.

Another misleading "trick" is: when multiplying by 10, add a zero to the end of the number. Again, the trick works with whole number, but not with decimals or fractions. For instance, 5 (a whole number) x 10 = 50, but 0.5 (a decimal) x 10 does not equal to 0.50. For a fraction, this trick (for multiplying by 10) works when you add a zero to its numerator, but fails when you add to a zero to its denominator. For example: ½ x 10 = 10/2 or 5, but 1/2 x 10 is not 1/20.

I would use a decimal slider (see this video https://www.youtube.com/watch?v=lucM3lR04nc) or a place value chart (see below) to show how the digits slide one place to its right when multiplying by 10, two places when multiplying by 100, three places when multiplying by 1,000. I would also have students practice writing the multiplication or division equations to visualize and explain their reasoning. For instance, 0.5 x 10 x 10 x 10 x 10 = 5,000 and not 0.50000.

Whole Number and Decimal Place Value Chart								
1,000s	**decimal comma**	100s	10s	1s	**decimal point**	1/10ths	1/100ths	1/1000ths
10^{3}		10^{2}	10^{1}	10^{0}		10^{-1}	10^{-2}	10^{-3}
thousands		hundreds	tens	ones		tenths	hundredths	thousandths
5	,	5	5	5	●	5	5	5
=5000		=500	=50	=5		=0.5	=0.05	=0.005
Below is an example multiplying by 10 (123.45 x 10 = 1,234.5) where each digit will move one place to the right. The digit 1 moves to the thousands place (left of the decimal comma), the digit 2 moves to the hundredths place, the digit 3 to moves to the tens place, the digit 4 moves to the ones place (left of the decimal point), and the digit 5 moves to the tenths place								
		1	2	3	●	4	5	
		⬅1	⬅2	⬅3	●	⬅4	⬅5	
1	,	2	3	4	●	5		

Figure 6. Decimal place value chart and an example of 123.45 x 10 = 1,234.5

Decimal Grids in Ones, Tenths, Hundredths, and Thousandths

When transitioning students from whole numbers to decimals and fractions, I find it critical to discuss how manipulatives like the 3-dimensional base-10 blocks and visual model like a 2-dimensional hundreds grid (where one unit is 1, one rod is 10, one flat or a hundred grid is 100, and one block is 1,000 units) are used to represent whole numbers. Even though the same manipulatives and worksheets are suitable to represent decimals or decimal fractions, there is one major change: a flat (like a hundred or hundredth grid) is now a whole or 1 like a dollar that is equivalent to 100 pennies, a rod now represents 0.1 or 0.10 which is equivalent to a dime or 10 pennies, and a unit has changed to represent 0.01 or a penny. Most importantly, a block of 1,000 is not 0.001 but has changed to represent 10 flats or 10 dollars which are equivalent to 1,000 pennies. In addition, a thousandth grid has 1,000 units of 0.001, and therefore, a thousandth grid is actually one whole, that is equivalent to a hundredth grid, a tenths grid, or one grid. For my own teaching experience, I have **not** seen a math lesson that clarify this major shift, but have seen many students struggle with this major change.

I will explain this major change of representation with blank tenths grids, hundredth grids, thousandths grids, decimal number lines, decimal place value charts, as well as worksheet drills on decimal expanded notations, rounding, comparing, conversion of decimals and fraction, addition, subtraction, multiplication and division. Below is a short list of resources.

- https://www.math-drills.com/decimal.php#decimal-grids-charts
- http://www.decimalsquares.com/Repromateri/
- https://curriculum.illustrativemathematics.org/k5/teachers/grade-5/unit-5/lesson-2/lesson.html

To facilitate this shift to use a hundred grid as a hundredths grid, I find it critical to have students explain why decimals such as 0.8 and 0.80 are equivalent, but 0.8 is not equivalent to 0.08. For the equivalence of 0.8 and 0.80, we should observe that both have 8 in the tenths place; extra zeros to the right do not change the value. The zeroes in $0.80 need to be taught explicitly where the digit 0 in the ones place represents zero dollar, the digit 8 in the tenths place means 8 dimes, and the digit 0 in the hundredth place represents zero cents. Therefore, the decimal 0.8 is equivalent to 0.80 = 0.800 = 0.8000 and so on. For non-equivalence of 0.8 and 0.08, we can argue that $.80 is eighty cents, while $.08 is only eight cents. I would use guided instruction (whole or small group) to have students compare these decimals in the following ways:

- Draw a number line from 0 to 1 with 10 equal partitions (intervals of 0.1 or 0.01), and label the locations of the decimals 0.80 and 0.08.
- Use a tenths (10 x 1) grid to shade 0.8, and a hundredth (10 x 10) grid to shade 0.08. Ask students to explain how they shaded their grids to describe any similarities and/or differences. Then reverse the decimals: Use a tenths (10 x 1) grid to shade 0.08 and a hundredth (10 x10) grid to shade 0.8.
- Use quarters, dimes, nickels, and pennies to show the values of 0.80, 0.8, and 0.08. I will have students to draw circles to visually represent the value, and label a quarter with the letter Q or 25¢, a dime with D or 10¢, a nickel with N or 5¢, and a penny with P or 1¢.

Guessing Jar of Coins

I would set up a jar of fake coins of different denominations for students to guess the total dollar amount once a month. This webpage https://www.supercoloring.com/paper-crafts/set-of-usa-coins-printable-template#gsc.tab=0 has coin illustrations that teachers can make paper copies for students to cut out. I would use different colors of paper for different denominations. If the class decide to use real coins, the guessing jar can serve as a donation jar at the end of the year. I would visit the website https://coinstar.coV/charitypartners/ and enter our zip code after clicking “Find a Kiosk” to allow students to choose a class charity. Charity programs such as “The Coinstar Coins that Count”will donate the coins directly to a participating charity such as the American Red Cross, Feeding America, Make-A-Wish Foundation of America, NAACP, UNICEF Relief Fund, and the World Wildlife Fund.

To help students to estimate more precisely, I will have students work together to figure out different ways to make one dollar and make a collaborative poster of visual representation. I will challenge students with the fact that there is a total of 293 ways to make a dollar with only quarters, dimes, nickels, and pennies! The following video explains and illustrates different ways to make a dollar with different coins: https://www.youtube.com/watch?v=dU0adeaEjIY&t=13s. Additional prompts for student inquiries:

- Make a total of $1.20 (any amount greater than $1.00) with as many ways as you can with dollar coins, half-dollar coins, quarters, dimes, nickels, and pennies.
- Make a total of ____________ with the fewest number of coins.

  Make a total of ____________ with the greatest number of coins.

- Make a total of ____________ with only dimes, nickels, and pennies.

Example #1	Example #2
If x = the dollar value of a nickel, and y = dollar value of a dime, what is the dollar value of z? What would be the name of this U.S. coin?	If x = the dollar value of a dime, and y = dollar value of a nickel, what is the dollar value of z? What would be the name of this U.S. coin?
$3x + 4y = 2z$ $3(\$0.05) + 4(\$0.10) = 2z$ $\$0.15 + \$0.40 = 2z$ $\$0.55 = 2z$ $\$0.55/2 = 2/2z$ $\$0.275 = z$	$3x + 4y = 2z$, $3(\$0.10) + 4(\$0.05) = 2z$ $\$0.30 + \$0.20 = 2z$ $\$0.50 = 2z$ $\$0.50/2 = 2/2z$ $\$0.25 = z$
Answer: z is not a possible U.S. coin that worth $0.275	**Answer:** z is a quarter 3 dimes + 4 nickels = 2 quarters

Figure 7. The above chart shows two examples with the algebraic equations $3x + 4y = 2z$ using coins such as nickels, dimes, and quarters as units of measurement.

Decimal of the Day

I will write the phrase "Decimal of the Day" on a graphic organizer or a chart paper posted in the classroom. Example of a Decimal of the Day: **3.095**. This activity will be a routine at the beginning or ending of a math lesson. Students will complete selected items from the list below. I will provide an assignment sheet with all the tasks listed below at a later date:

The Decimal of the Day (DotD) is ☐

- Write DotD as a decimal fraction: ____________________________________.
- Write three equivalent fractions: _____________________________________.
- Write DotD in word form: ____.
- Write DotD as an expanded form: ____________________________________.
- Add DotD to: ____________ Write the addition equation: ___________________.
- Subtract DotD from:________ Write the subtraction equation: ________________.
- Multiply DotD by:_________ Write the multiplication equation: ______________.
- Divide DotD by:_________ Write the division equation: __________________.
- Compare DotD to: ________ Write the comparison: _______________________.
- Represent DotD with the least number of coins.
- Represent DotD on a number line.
- Represent DotD with a tape diagram.
- Represent DotD with a hundredth grid.

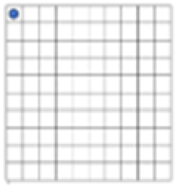
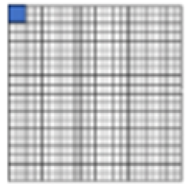
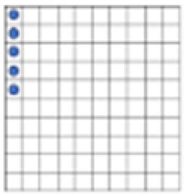
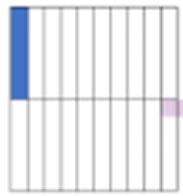
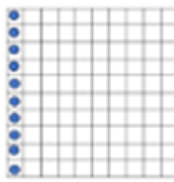
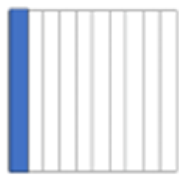
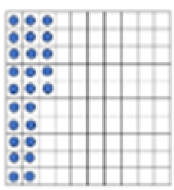
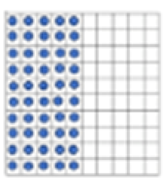
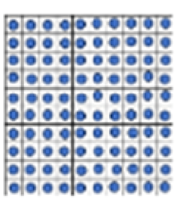
Tenths, Hundredths, and Thousandths Grids		I know... I wonder... or draw the grid another way
One penny 1¢	 = 1/100  = 10/1000	
One nickel 5¢	 = 5/100  = 1/20	
One dime 10¢	 = 10/100  = 1/10 one tenth	
One quarter 25¢	 = 25/100  = 1/4 one fourth	
One half-dollar 50¢	 = 50/100  = 1/2 one half	
One dollar-coin 100¢	 = 100/100  = 1/1 = 1 One whole	

Figure 8. The above chart shows the decimal fractions 1/100, 5/100, 10/100, 25/100, 50/100, 100/100 using grids in tenths, hundredths, thousandths as well as fourths, halves, and a whole.

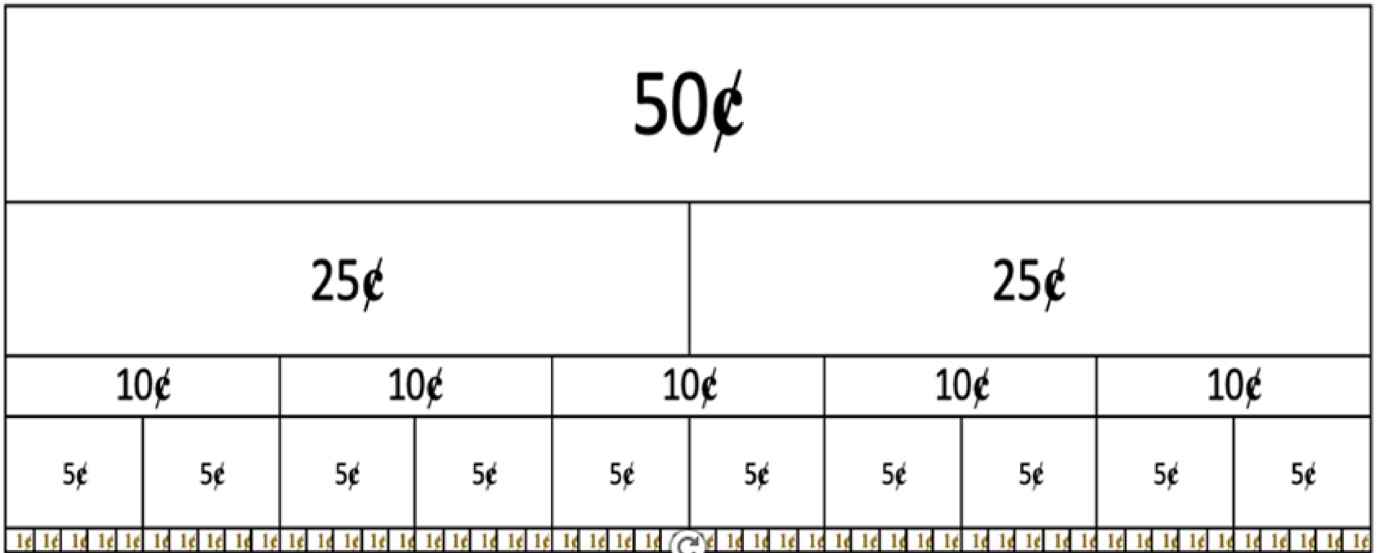
COIN COUNTING RECORD SHEET	Decimal Notation	Fraction Notation
1 quarter = \$0.25 = \$25/100 *How many quarters?* _____ 		
1 dime = \$0.10 = \$10/100 *How many dimes?* _____ 		
1 nickel = \$0.05 = \$5/100 *How many nickels?* _____ 		
1 penny = \$0.01 = \$ 1/100 *How many pennies?* ______ 		
Total \$ Write an equation in decimal **or** fraction notation __________ + __________ + ___________ + __________ = ____________		

Figure 9. Above is a record sheet for students to count the number of coins in their guessing jar. Students can practice how to convert decimals into fractions.

Accordion Books Using Number Lines and Tape Diagrams

It would be helpful for students to have a large number line with equivalent decimals like 0.3, 0.30, and 0.300 (all at the same place on the number line) to show equivalence among decimals, fractions, and percentages. Because a large classroom number line can be cumbersome and/or inflexible, sometimes I will present this challenge in the following way:

I will use the format of an accordion book to make a large classroom number line; this would allow space to add more numbers, and write labels with explanations. This way the large number line can be easily folded back into a manageable book. In addition, students can easily make mini-number lines and staple them to their notebooks. Tape diagrams are similar to number lines, but give a qualitative way to represent and visualize calculation. A tape diagram is a visual model in the shape of a rectangle resembling a piece of tape. It is used to assist students with the calculation of addition, subtraction, multiplication, and division. It is also known as a divided bar model, fraction strip, length model or strip diagram. See the following sequences of tape diagrams that explain the differences in terms of place value: 100 divided by 4, 10 divided by 4, and 1 divided by 4.



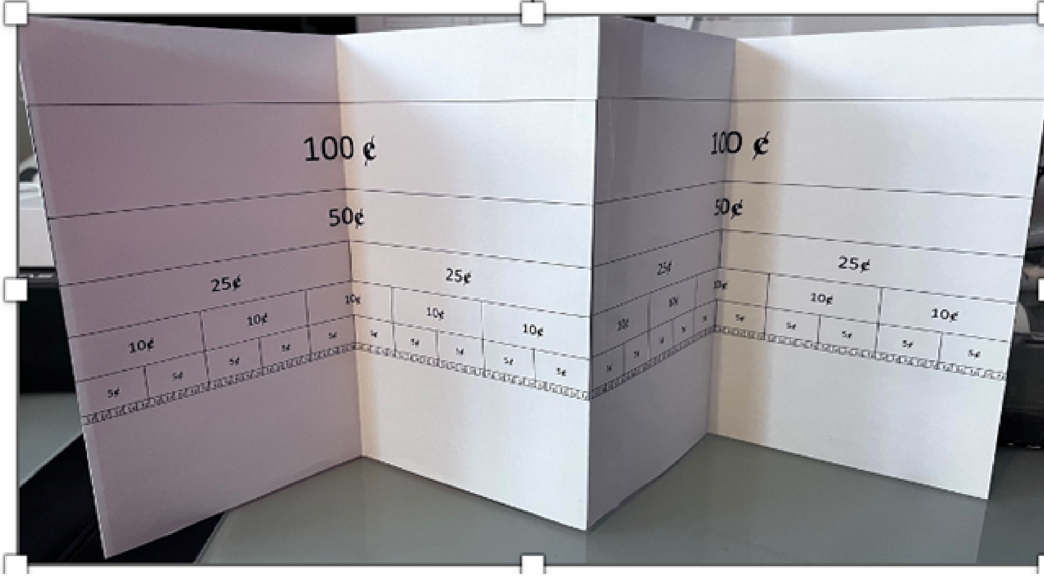


Figure 10. The above tape diagrams were used to make an accordion book.

Total Whole(s)	$100			
Parts of the whole(s)	$100 \div 4 = 25$	$100 \div 4 = 25$	$100 \div 4 = 25$	$100 \div 4 = 25$

Total Whole(s)	$10.00			
Parts of the whole(s)	2.5	2.5	2.5	2.5

Total Whole(s)	$1.00			
Parts of the whole(s)	0.25	0.25	0.25	0.25

Total Whole(s)	$\$1.00 = \frac{4}{4}$			
Parts of the whole(s)	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$	$\frac{1}{4}$

Total Whole(s)	$\$1.00 = \frac{4}{4} = 4 \div 4 = 100\%$			
Parts of the whole(s)	$\frac{1}{4} = \frac{25}{100} = 0.25 = 25\%$	$\frac{1}{4} = \frac{25}{100} = 0.25 = 25\%$	$\frac{1}{4} = \frac{25}{100} = 0.25 = 25\%$	$\frac{1}{4} = \frac{25}{100} = 0.25 = 25\%$

Figure 11. Different tape diagrams (above) showing how to divide $100, $10, and $1 by 4, or into fourths

Another model for a number line is a “meter stick” because it already has tick marks to indicate equal intervals; also, one side has units of measurement in centimeters and the other side has units of measurement in inches. As much as possible let students have time to define the starting and ending points of their number lines, as well as the equal distance of each interval (partition between two numbers).

Appendix on Implementing District Standards

1st group of lessons: My curriculum unit will address the following Social Studies Pennsylvania State Standards on U.S. History and geography of places. Students will collect U.S. coins, complete research, write reports, and make presentations about quarters.

- 7.1.6.B. Describe and locate places and regions. Students will study quarters that celebrate the National Parks of the 50 states in order to locate places on a U.S. map. See webpage:

https://www.usmint.gov/learn/coin-and-medal-programs/america-the-beautiful-quarters

- 8.2.6.C.8. Women's Movement. In particular, students study quarters that honor women leaders in order to explain a variety of their political and cultural contributions. See https://www.usmint.gov/learn/coin-and-medal-programs/american-women-quarters.

2nd group of lessons: My curriculum unit will address aspects of the following 5th Grade Math Common Core Standards about place value, decimals, and fractions.

- 5.NBT.A.1: Recognize that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right, and 1/10 of what it represents in the place to its left.
- 5.NBT.A.3: Read, write, and compare decimals to thousandths.
- 5.NF.A.1: Add and subtract fractions with unlike denominators, including mixed numbers.

Annotated Bibliography

Bjälkebring, Pär, and Ellen Peters. "Money matters (especially if you are good at math): Numeracy, verbal intelligence, education, and income in satisfaction judgments." *Plos one* 16, no. 11 (2021): e0259331. Accessed August 18, 2023. https://journals.plos.org/plosone/article?id=10.1371/journal.pone.0259331. This research article shows that findings are consistent with the importance of numeric intelligence to income and life satisfaction.

Devon Primary Maths Team. “The Gaps and Misconceptions Tool - Why do fractions and decimals seem difficult to teach and learn?” *Babcock LDP* (2016). http://www.gapsandmisconceptions.co.uk/ This webpage has resources like an interactive diagnostic tool to help identify student gaps and misconceptions in math.

Ghose, Tia. “Like Math? Thank Your Motivation, Not HQ.” *LiveScience* (2012). https://www.nbcnews.com/id/wbna50315642. Accessed August 17, 2023. A study that suggests a person’s motivation to work hard, and good study techniques, not IQ, lead to better math skills.

Howe, Roger, and Reiter, Harold. "The Five Stages of Place Value." *Teachers of India.* (2012). The authors explain with examples the five stages of place value in which true understanding of place value can be developed from 1st grade to high school.

Hong, Kho Tek, et al. “Primary Mathematics 5A.” (1995): 1-96, and “Primary Mathematics 5B.” *Curriculum Development Institute of Singapore*. (1995): 1-104. 5th grade student workbooks based on the Singapore Math teaching approach where students mastering fewer math concepts at greater details using a learning process of concrete, pictorial, and abstract representations including tape diagrams and number lines.

Jacobson, Rae. “How to Spot Dyscalculia.” *Child Mind Institute* (2022). Accessed August 1, 2023. https://childmind.org/article/how-to-spot-dyscalculia/ This article is a quick read to understand the math learning disorder called dyscalculia.

Martinie, Sherri. “Decimal Fractions: An Important Point.” *Mathematics Teaching in the Middle School* 19, no. 7 (2014): 420–29. https://doi.org/10.5951/mathteacmiddscho.19.7.0420. A veteran math classroom teacher of 19 years, the author reveals research that shows when teachers are skilled in recognizing students' misconceptions about decimal fractions, they are better equipped to make instructional decisions that build on these ideas.

Mehmetlioğlu, Deniz. “Misconceptions of Elementary School Students about Comparing Decimal Numbers.” *Procedia - Social and Behavioral Sciences* 152 (October 2014): 569–74. https://doi.org/10.1016/j.sbspro.2014.09.245. This study was conducted with 183 students from grades 6th to 8th and it concluded that student misconceptions of decimal comparison may have stemmed from previous learned misconception, lack of connectivity of math concepts on place value, and differences with teacher approaches.

Oishi, Shigehiro, Youngjae Cha, Asuka Komiya, and Hiroshi Ono. "Money and happiness: the income–happiness correlation is higher when income inequality is higher." *PNAS Nexus* 1, no. 5 (2022): pgac 224. This article discusses studies that explain the income-happiness correlation.

Peters, Ellen, and Shoots-Reinhard, Brittany. “How math skills plus confidence equals judgment on health and money.” *Washington Post*. September 29, 20219. https://www.washingtonpost.com/heallth/how-math-skills-plus-condence-equals-better-judgment-on-health-money/2019/09/27/b5f04bb8-d3eb-11e9-86ac-0f250cc91758_story.html (accessed August 1, 2023). Description of studies that claim: math ability and confidence can greatly impact a person’s health and income.

Pollack, Craig Evan, Catherine Cubbin, Ayesha Sania, Mark Hayward, Donna Vallone, Brian Flaherty, and Paula A. Braveman. "Do wealth disparities contribute to health disparities within racial/ethnic groups?” *J Epidemiol Community Health* 67, no. 5 (2013): 439-445. This article discusses the importance of measuring wealth in health studies to inform public policy and medical practices.

Steinle, Vicki. “Changes with age in students' misconceptions of decimal numbers.” *University of Melbourne, Department of Science and Mathematics Education* (2004). This 326-pages report includes a longitudinal study of 3,000 students in Australia to assess students’ understanding and misconceptions about decimals and decimal notations.

Strauss, Jacob R. “U.S. Dollar Coins: History and Current Status.” *Congressional Research Service*. IF1190. January 6, 2023. Accessed August 18, 2023. https://crsreports.congress.gov/search/#/?termsToSearch=U.S.%20coins&orderBy=Relevance. This article from the Congress gives a brief history of the U.S. dollar coins with . The website also has reports about other circulating coins.

Su, Qiu Gui. "Rational Numbers in Chinese: How to Talk about Decimals, Fractions and Percents in Chinese." ThoughtCo. Updated March 22, 2019. ttps://www.thoughtco.com/mandarin-fractions-2279408 (accessed August 16, 2023). This article describes how the Chinese language has a relative higher linguistic clarity that able native speakers to about rational numbers.

Tian, J., & Siegler, R. S. “Which type of rational numbers should students learn first*?” Educational Psychology Review. (2018). 30(2), 351-372*. p 351. doi: 10.1007/s10648-017-9417-3. This research article reviews different teaching approaches of rational numbers and concludes that the goal of instruction is to achieve a robust and flexible understanding of all rational number notations (decimal, fraction, and percentage)

Wang, Ming-Te, and Jessica L. Degol. "Gender gap in science, technology, engineering, and mathematics (STEM): Current knowledge, implications for practice, policy, and future directions." *Educational Psychology*

Review 29 (2017): 119-140. This article describes findings of gender inequality in STEM.

Yeoman, Richard S., and Garrett, Jeff. *Handbook of United States Coins 2024*. Whitman Publishing, 2023. This annual “Official Blue Book” is a reference used by most U.S. coin dealers and coin collectors. To order coins online, visit catalog.usmint.gov.

Endnotes

1. S. Oishi, "Money and happiness," in *PNAS Nexus,*10.
2. C. E. Pollack, "Do wealth disparities,” in *J Epidemiol Community Health*, 439.
3. E. Peters, “How math skills plus confidence,” in *Washington Post*. Accessed August 1, 2023.
4. Ibid.
5. *Genius*, https://genius.com/Abba-money-money-money-lyrics. Accessed August 1, 2023.
6. M. Wang, "Gender gap," in *Educational Psychology Review*, 119.
7. T. Ghose. “Like Math?” in *LiveScience*. https://www.scientificamerican.com/article/like-math-thank-your-moti. Accessed August 1, 2023.
8. R. Jacobson,“How to Spot Dyscalculia,” in *Child Mind Institute*. https://childmind.org/article/how-to-spot-dyscalculia/. Accessed August 1, 2023.
9. Ibid, Peters.
10. V. Steinle, “Changes with age in students' misconceptions of decimal numbers,” in *University of Melbourne*, 2.
11. “Key Francis Scott School,” *U.S. News.* Accessed August 5, 2022. https://www.usnews.com/education/k12/pennsylvania/key-francis-scott-school-236825.
12. Ibid.
13. Niche.com, https://www.niche.com/k12/francis-scott-key-school-philadelphia-pa/.
14. Ibid.
15. “Low Income Data by School” in *PA Department of Education*. Accessed August 17, 2023.
16. 16. Ibid, Steinle, 17.
17. Ibid, Steinle, 221.
18. R. Howe, "The Five Stages of Place Value." *Teachers of India.* 4.
19. Q.G. Su, “Rational Numbers in Chinese,” in *ThoughtCo*. Accessed August 11, 2023. https://www.thoughtco.com/mandarin-numbers-2279638
20. Ibid, Steinle, 73.
21. Ibid, Steinle, 48-50.