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From English to Algebra: Solving Linear Equations with Word Problems

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by Kristina Kirby

Introduction

As the great mathematician George Pólya once said, “It is better to solve one problem five different ways, than to solve five problems one way.” I’ve found this to be especially true with word problems. A given algebraic word problem can be solved in a multitude of ways; for instance, one can solve a linear equation algebraically (with “variables”), arithmetically (with constants), or illustratively (with pictures or use of manipulatives). When students engage in all three methods of solving linear equations, they build on prior knowledge and develop a more comprehensive understanding of variables and manipulation of algebraic equations.

Rationale

This curriculum unit focuses on fostering students’ conceptual understanding of the role of variables and their units in linear equations- specifically in writing linear equations from word problems, justifying these equations, solving these equations, and evaluating their answers. In the past, I have taught translating and solving linear word problems *after* teaching solving, evaluating, and graphing linear equations; I noticed that, rather than gaining a comprehensive understanding of linear equations, this sequence resulted in students’ overreliance on rote procedures and did not culminate in the “Aha!” moment I was hoping they would reach.

Research out of UC Santa Barbara found that “unsuccessful problem solvers base their solution plan on numbers and keywords that they select from the problem (the direct translation strategy), whereas successful problem solvers construct a model of the situation described in the problem and base their solution plan on this model (the problem-model strategy).”[1] Many U.S. textbooks teach word problems in conjunction with a very specific type of mathematical concept, thereby promoting the direct translation strategy. For instance, after learning about velocity, students will read word problems about velocity, identify the necessary values in the problem to plug in (distance and time), and determine where these values go in the velocity equation.

However, this rote proceduralism does not foster meaningful learning and, instead, results in students' inability to solve the same kinds of problems outside of the classroom context.

According to the California Assessment of Student Performance and Progress results from 2021-22, more than two thirds of all 11th grade students at Overfelt High School did not meet state standards in math. Over two out of every five students scored "below standard" in "Problem Solving and Modeling & Data Analysis," meaning that they "[do] not yet demonstrate the ability to solve a variety of mathematics problems by applying… knowledge of problem-solving skills and strategies. [They do] not yet demonstrate the ability to analyze real-world problems, or build and use mathematical models to interpret and solve problems;" nearly a third of students scored "below standard" in "Communicating Reasoning," meaning that they "[do] not yet demonstrate the ability to put together valid arguments to support [their] own mathematical thinking or to critique the reasoning of others." Finally, nearly three out of every five students scored "below standard" in Concepts & Procedures, meaning that they "[do] not yet demonstrate the ability to explain and apply mathematical concepts or the ability to interpret and carry out mathematical procedures with ease and accuracy." Other categories of scoring include "near standard," in which some ability is perceived, and "above standard," in which a thorough ability is perceived.[2]

My Math I students reflect these same struggles in their comprehension of linear equations. They lack the conceptual understanding that equations, graphs, and tables can all represent the same real-world scenario or problem despite presenting very differently. My students also lack the confidence as well as the mathematical vocabulary to justify their use of numbers, symbols, and variables in place of words. My curriculum unit will try to bridge this gap of understanding by introducing students to linear equations through word problems, so that they can truly comprehend the meaning (or a possible meaning) behind each equation as it is translated, justified, and then solved. My hope is that this unit will not only will this facilitate student success in working with linear equations, both as word problems and as algebraic expressions, but this will further prepare students when asked to work with linear equations as tables and graphs after the planned unit is taught.

Students' use of the critical thinking that is needed when creating algebraic expressions and equations from English sentences will likely bolster their success in Problem Solving and Modeling & Data Analysis, as it will facilitate their "ability to analyze real-world problems, or build and use mathematical models to interpret and solve problems."[3] Additionally, the explanations they provide around why they wrote the expression or equation as they did will likely improve achievement in Communicating Reasoning, as it will encourage their "ability to put together valid arguments to support [their] own mathematical thinking…"[4] Furthermore, having students evaluate and simplify expressions and solve equations will improve student performance in Concepts & Procedures, as they will be more apt at "interpret[ing] and carry[ing] out mathematical procedures with ease and accuracy."[5]

Content Objectives:

My unit utilizes the BSCS 5E Instructional Model, or the Biological Sciences Curriculum Study 5E method, as defined by Rodger Bybee. This method consists of five learning phases: engage, explore, explain, elaborate, and evaluate.[6]

Engage Phase

In the engage phase, students will, either individually or in small groups, solve a given word problem. Once all have solved it, the whole class will delve into the different methods individual students or small groups used to solve the problem (in Japan, this is called Kikan-jyunshi or Kikan-shido; this concept is further explained in the section on teaching strategies). Students will determine solutions to these word problems via illustrations, arithmetic, or algebra. For instance, take the following word problem:

> Natalia ordered twelve items from Starbucks for her friends. Each friend ordered two items. How many friends did Natalia order for?

Translation of Word Problems into One-Step Equations

Some students, based on their prior learning of translating word problems into mathematical expressions, may formulate a one-step equation without being prompted to do so.

One option students may come up with is twelve items divided between the number of friends (x) is equal to two items for each friend.

$$\frac{12}{x} = 2$$

Another option students may come up with is two items times the number of friends (x) is equal to twelve total items.

$$2x=12$$

It is imperative that the whole-class discussion expound upon both of these methods, so that students who solved it illustratively or arithmetically can transfer this understanding to solving it algebraically.

Despite the potential for students to successfully translate word problems into equations, many students have never solved a one-step equation before and, therefore, may be unsure of how to get the answer from their newly constructed equations.

Arithmetic Translation of Word Problems

Other students may not come up with an algebraic equation at all. Instead, they may use arithmetic to solve the word problem. One option for solving the problem arithmetically is the use of subtraction.

Twelve total items minus two items Natalia bought for each friend shows that she bought items from Starbucks for six friends.

$$12 - 2 - 2 - 2 - 2 - 2 - 2 = 0$$

The repeated addition of the two items that Natalia bought for each friend shows that she bought items from Starbucks for six friends.

$$2 + 2 + 2 + 2 + 2 + 2 = 12$$

Illustrations or Use of Manipulatives in Solving Word Problems

Still, other students may rely on illustrations or manipulatives to solve the problem.

This illustration depicts twelve Starbucks items broken up into two items for each friend, which means that Natalia bought Starbucks for six friends.

Explore and Explain Phases

In the *explore* phase, students will grapple with their understanding of writing and solving algebraic expressions and equations through increasingly difficult word problem sets. The *explain* phase, in the case of my unit, is married to the *explore* phase; in the *explain* phase, students are asked to identify the units of each term and then justify their use of selected operations via their identified units when translating word problems into expressions and equations.

Elaborate Phase

In the *elaborate* phase, I will ask students to come up with scenarios that could represent given equations, providing them with opportunities to transfer their understanding of algebra to settings outside of the math classroom to establish more meaningful learning.

Evaluate Phase

Finally, in the *evaluate* phase, students will prove both their conceptual understanding of and their procedural fluency in working with solving equations on an assessment.

This sequence of learning begins with expressions and repeats itself three more times, with one-step equations, two-step equations involving multiplication and addition, and two-step equations involving multiplication and subtraction. This unit culminates in a final exam that determines students' level of understanding around real-world application of up to two-step linear equations and measures their preparedness to apply this learning to tables and graphs.

Teaching Strategies:

Pre-Teaching

I will pre-teach the use of *x* or another "variable" as a placeholder for an unknown value (The word *variable* appears in quotations, as the letters used for unknown variables in the word problems given throughout the unit are fixed). This will further prepare students to translate word problems into expressions and equations, as they will recognize an unknown quantity in the word problems to be *x*. Vocabulary terms that I will pre-teach students include unit, term, expression, and equation, so that students can successfully complete the problem sets which use this terminology.

Inquiry-Based Learning

This unit plan relies heavily on inquiry-based learning, which is learning through formulating new ideas and solutions to given problems. This learning will be conducted individually, in pairs, or in small groups- depending on the complexity of the given word problems- so that students can deliberate and help one another understand the proper use of a variable and its units in writing expressions and equations from word problems.

Kikan-jyunshi/ Kikan-shido

The Japanese concept of Kikan-jyunshi or Kikan-shido is the "purposeful scanning or monitoring" of the class to "elicit [individual] students' ideas and bring them forward for whole-class discussion."[7] Japanese teachers very much prefer to question and elicit response rather than to lecture. In this unit, it is important to use Kikan-jyunshi or Kikan-shido to identify student work that promotes algebraic, arithmetic, *and* illustrative means of solving given word problems; this may facilitate transfer of knowledge from images or arithmetic to algebra for some learners. As Yale mathematics professor Roger Howe suggests, "…teaching symbolic algebra, and its use in solving word problems, might profit from the linguistic perspective that students might benefit from seeing, studying, discussing and working through the translation into algebra of many examples of verbally formulated situations, including work of solving these problems both with and without algebra, and comparing the algebraic and the arithmetic solutions to these problems."[8] The practices of Kikan-jyunshi and Kikan-shido encourage pathways of learning that not only follow Howe's suggestion but provide personal relevance to the students and give learning a deeper meaning.

Culturally Responsive Teaching

I plan to incorporate word problems that are relevant to students' day-to-day lives. Too often, word problems are irrelevant to my students; they convey scenarios that my students either don't care about or have never experienced. Therefore, I plan to create word problems that not only are relevant but that are actual problems students may have interest in solving.

Cooperative Learning (Group Work)

Students will work together in pairs and small groups to support each other's learning. Studies have shown that "Use of co-operative learning almost always improves affective outcomes."[9] Yet, "group goals" and "individual accountability" are necessary for this improvement to occur.[10] Therefore, I will explicitly show students how group work is intended to operate; this way, students will have a better understanding of their individual roles in a group setting.

Classroom Activities: Problem Sets

Upon completion of pre-teaching English synonyms of mathematical operations along with the use of "variables" in the place of unknown values, students will be introduced to expressions through word problems. Students will begin by translating word problems into expressions. Then, they will defend their choice of operation, define each term and its unit, and evaluate the expression. It is imperative that students define each term and its unit when working with word problems;[11] this ensures that students have a true understanding of the problem at hand. For the same reason, "students should be required to provide the justifications for solution steps."[12] Students will follow the same cycle of inquiry for one-step, two-step, and multi-step equations. Students will conclude each exercise by confirming their answer is correct via substitution. After answering all questions in a set accurately, students will practice solving and evaluating algebraic equations to ensure procedural fluency. Once they have established and proven this fluency, they will be asked to make up their own word problems for given equations; these word problems will serve as further evidence of students' strong conceptual understanding of algebraic equations.

Students will first complete a problem set around creating algebraic expressions:

Problem Set 1: Translating Word Problems into Expressions

1. You have a package of 18 pencils in your backpack. Some of your friends borrowed pencils during English class.
    a. Write an expression that represents how many pencils you have left.
    b. Defend your reasoning as to why the operation you chose to include in your expression makes sense.
    c. What does each term in the expression represent? What are their units?
    d. If 14 friends borrowed one pencil each, how many pencils do you have left? (Don't forget to include your units!)
2. Carlos purchased 24 conchas and wants to share them equally between his classmates (including himself).
    a. Write an expression that represents how Carlos would share the conchas.
    b. Defend your reasoning as to why the operation you chose to include in your expression makes sense.
    c. What does each term in the expression represent? What are their units?
    d. If there are 8 students in Carlos' class, (small class!) how many conchas does each get? (Don't forget to include your units!)
3. Angelina bought a ticket to see the Barbie movie for $22. She also spent money on popcorn.
    a. Write an expression that represents the total amount she spent.
    b. Defend your reasoning as to why the operation you chose to include in your expression makes sense.
    c. What does each term in the expression represent? What are their units?
    d. If Angelina spent $10 on popcorn, how much did she spend in total? (Don't forget to include your units!)
4. Juan Diego drove from San Jose to Oakland at 30 miles per hour.
    a. Write an expression that represents how long it took Juan Diego to get to Oakland.
    b. Defend your reasoning as to why the operation you chose to include in your expression makes

sense.

c. What does each term in the expression represent? What are their units?
d. If Juan Diego drove for 1 ½ hours, how many miles did he drive? (Don't forget to include your units!)

5. Mari spends $5 on boba each day.
    a. Write an expression that represents how much Mari spends on boba.
    b. What does each term in the expression represent? What are their units?
    c. How much does Mari spend in one year (365 days) on boba? (Don't forget to include your units!)
6. Overfelt's swim team had 48 pizza slices to sell at Club Food Day. They sold some of the slices.
    a. Write an expression that represents how many slices of pizza remain.
    b. What does each term in the expression represent? What are their units?
    c. If the swim team sold 37 slices, how many slices are left? (Don't forget to include your units!)
7. Karina earned $86 at work plus tips.
    a. Write an expression that represents how much money she made in total.
    b. What does each term in the expression represent? What are their units?
    c. If Karina earns $53 in tips, how much money did she make?
8. Guadalupe baked tres leches and cut it into 32 pieces. She wants to divide the cake evenly between her family members.
    a. Write an expression that represents how many pieces of tres leches each family member would
    b. What does each term in the expression represent? What are their units?
    c. If there are 16 family members, how many pieces of tres leches does each family member get?
9. Kayla wanted to get McDonalds delivered for lunch. She paid $18 for her food and an additional amount for the delivery fee.
    a. Write an expression that represents how much Kayla paid in total.
    b. If Kayla paid $2.50 for the delivery fee, how much did she pay in total?
10. Sam makes $28 per hour working at In-N-Out as the manager.
    a. Write an expression that represents how much money Sam makes.
    b. If Sam worked for 6 hours, how much money would she make?
11. Michelle rides the bus for 32 minutes each day to Evergreen for class. Once she gets off at the bus stop, she has to spend more time walking to her class.
    a. Write an expression that represents how long it takes Michelle to get to class from the time she gets on the bus.
    b. If Michelle walks 4 minutes to get to class, how long does it take her to get to class?
12. Angel owns an auto mechanic shop. For the month of July, he paid a total of $25,578 to his workers. Each worker received equal pay.
    a. Write an expression that represents how much money each worker received for the month of July.
    b. If there are 7 workers, how much money did each worker make in July?

Upon completion of the first set of word problems, students will practice adding, subtracting, multiplying, and dividing integers, so that they can more successfully solve one-step equations in the second problem set.

Prior to jumping into the second problem set, students will work with small groups on a single one-step word problem. Kikan-jyunshi or Kikan-shido will be used here to identify pairs of students who solved the problem illustratively, arithmetically, or algebraically. These partner pairs will write their solutions on the board and explain their reasoning. It is imperative that a pair who solved the problem illustratively go first, as the illustrative understanding is the most foundational understanding of the problem and makes the most sense to the majority (if not all) of the students. A partner pair who solved the problem arithmetically goes second, as arithmetic bridges the gap between illustrating a problem and solving it algebraically. Finally, a partner pair (if there is one) who used algebra to solve the word problem goes third. If there is not a partner pair who solved it algebraically, I, the teacher, will point students to the algebraic expressions they created from word problems the lesson prior; this should give them enough background knowledge to at least be able to create an equation. From there, since students know the answer already via the illustrative and arithmetic methods of solving the problem, students will try to determine how to reach the solution via the equation they created. Once a pair has been identified that determined how to successfully solve the equation, they will also write their solution on the board and explain their reasoning. Students will then continue to work in small groups to solve word problems in the second set:

Problem Set 2: Translating Word Problems into One-Step Equations

1. Roberto traveled to Hawaii over the summer. He found a relatively inexpensive hotel that cost $106 a night. His total stay at the hotel cost $636.
    a. Write an equation that represents the total cost of Roberto's stay.
    b. Defend your reasoning as to why the operations you chose to include in your equation makes sense.
    c. What does each term in the equation represent? What are their units?
    d. How many nights did Roberto stay at the hotel? (Don't forget to include your units!)
    e. Confirm that your answer is correct through substitution.
2. Kaitlin won money in the lottery! She split her earnings evenly between 4 total family members. Each family member received $3,138.
    a. Write an equation that represents the amount of money each family member received in relation to Kaitlin's winnings.
    b. Defend your reasoning as to why the operations you chose to include in your equation makes sense.
    c. What does each term in the equation represent? What are their units?
    d. How much money did Kaitlin win in the lottery? (Don't forget to include your units!)
    e. Confirm that your answer is correct through substitution.
3. There were a lot of tickets available for the 49ers home opener at Levi's stadium. After 67,959 tickets are sold, there are 541 tickets left.
    a. Write an equation that represents how many tickets are left, in relation to the total number of tickets.
    b. Defend your reasoning as to why the operations you chose to include in your equation make sense.
    c. What does each term in the equation represent? What are their units?
    d. How many total tickets were available before any were sold?
    e. Confirm that your answer is correct through substitution.
4. Juan paid money for gas. He then went into the convenience store and bought $12.82 worth of snacks. He spent $53.55 total.
    a. Write an equation that represents how much Juan spent in total.
    b. Defend your reasoning as to why the operations you chose to include in your equation make sense.
    c. What does each term in the equation represent? What are their units?
    d. How much money did Juan pay for gas? (Don't forget to include your units!)
    e. Confirm that your answer is correct through substitution.
5. Lin drove at a constant speed of 60 miles per hour from San Jose to LA. She drove a total of 330 miles.

a. Write an equation that represents the total number of miles Lin drove, in relationship to her driving time.
b. What does each term in the equation represent? What are their units?
c. How many hours did she drive?
d. Confirm that your answer is correct through substitution.

6. Jackie went to Mexico for 21 days to visit her family. She spent an average of $99.60 each day she was there.
    a. Write an equation that represents the amount of money Jackie spent each day in Mexico.
    b. What does each term in the equation represent? What are their units?
    c. How much money did Jackie spend total?
    d. Confirm that your answer is correct through substitution.
7. Victor's family has pet chickens. One chicken laid 12 eggs and another chicken also laid some eggs. There were 23 eggs laid in total.
    a. Write an equation that represents the total number of eggs laid.
    b. How many eggs did the other chicken lay?
    c. Confirm that your answer is correct through substitution.
8. Andrew had to write an essay for English class. He wrote some words and then took a break. He then wrote another 1,300 words which completed his 3,000 word essay.
    a. Write an equation that represents the total number of words in Andrew's essay.
    b. How many words did Andrew write before he took a break?
    c. Confirm that your answer is correct through substitution.

Upon completion of the second set of word problems, students will practice solving one-step equations to gain procedural fluency. In the next lesson, students will review one-step equations via the game "whip-it." In this game, each student has a flashcard; however, these are not typical flashcards. Each card has a number (with units) on one side, and a problem statement on the other. The answer on one card is the answer to the question on somebody else's card. To play, one student starts off by reading their card's problem, and the person whose card has the answer states the answer and then reads the problem on their card. The goal of the game is to get through all the cards and get back to the person who started by reading their question as quickly and with as few mistakes as possible.

Upon attaining a higher level of fluency in solving one-step equations, students will then move onto the third and fourth problem sets:

Problem Set 3: Translating Word Problems into Two-Step Equations with Multiplication and Addition

1. Joaquin rented a car for $80 per day. He also had to pay a $100 insurance fee. The total cost of the rental was $820.
    a. Write an equation that represents the total cost of the rental car.
    b. Defend your reasoning as to why the operations you chose to include in your equation make sense.
    c. What does each term in the equation represent? What are their units?
    d. How many days did Joaquin rent the car for?
    e. Confirm your answer is correct through substitution.
2. Denzel wanted to buy a car. He made a down payment of $1200 and then paid an additional $500 per month until the car was paid off. The car cost $3,700.

    a. Write an equation that represents the total cost of the car.
    b. Defend your reasoning as to why the operations you chose to include in your equation make sense.
    c. What does each term in the equation represent? What are their units?
    d. How many months did it take Denzel to pay off the car?
    e. Confirm your answer is correct through substitution.
3. Karina started a new teaching job where she makes $4,350 a month over a whole year (12 months). She received an extra $1,500 as a bonus when she signed her contract. Since she began her new job, she has made $49,350.
    a. Write an equation that represents how much money Karina has made at her new teaching job.
    b. What does each term in the expression represent? What are their units?
    c. How many months has Karina been working?
    d. Confirm your answer is correct through substitution.
4. Kingston went hiking in Alum Rock Park. He started at 200 feet above sea level. Each hour, he gained another 350 feet of elevation. When he reached the top, he was at 1,250 feet of elevation.
    a. Write an equation that represents the elevation at the top of the mountain.
    b. What does each term in the expression represent? What are their units?
    c. How many hours was Kingston hiking?
    d. Confirm your answer is correct through substitution.
5. Lydia wants to buy one tube of mascara and as many tubes of lip gloss as she can from Sephora. The mascara she wants costs $22.99. Each lip gloss costs $4.89. She has $37.66 to spend on makeup.
    a. Write an equation that represents the total amount of makeup that Lydia can buy.
    b. What does each term in the expression represent? What are their units?
    c. How many tubes of lip gloss can Lydia buy?
    d. Confirm your answer is correct through substitution.
6. Tia wants more followers on Tik Tok. She currently has 1,237 followers but wants to have at least 2,707 followers like her sister. She gets 30 new followers per video she posts to her Tik Tok account.
    a. Write an equation that represents the minimum number of followers Lydia wants to have.
    b. What does each term in the expression represent? What are their units?
    c. How many videos will Tia have to make to reach 2,707 followers?
    d. Confirm your answer is correct through substitution.
7. Adrian weighed 148 pounds but started getting swole at the gym. Each month, he gained 2.5 pounds in muscle. After several months, he weighed 190.5 pounds.
    a. Write an equation that represents how much Adrian weighed after he started working out.
    b. How many months did Adrian work out in the gym?
    c. Confirm your answer is correct through substitution.
8. Regina was throwing a party and needed balloons. She found 5 unused balloons at her house. She didn’t think this was enough, so she went to the store and bought 4 packages of balloons. She now has 21 balloons.
    a. Write an equation that represents how many balloons Regina now has.
    b. How many balloons are in each package?
    c. Confirm your answer is correct through substitution.

Problem Set 4: Translating Word Problems into Two-Step Equations with Multiplication and Subtraction

1. Camila purchased a 30-pound bag of dog food. She feeds her dog 1.5 pounds of food each day.

a. Write an equation that represents when there is no dog food left.
b. Defend your reasoning as to why the operations you chose to include in your equation make sense.
c. What does each term in the equation represent? What are their units?
d. How many days did it take for Camila to go through the whole bag of dog food?
e. Confirm your answer is correct through substitution.

2. The Overfelt girls' rugby team brought a 16-liter Gatorade cooler full of water to practice. The 13 girls on the team all drank the same amount of water. At the end of practice, there were 0.4 liters of water left in the cooler.
    a. Write an equation that represents the amount of water left in the cooler.
    b. Defend your reasoning as to why the operations you chose to include in your equation make sense.
    c. What does each term in the equation represent? What are their units?
    d. How much water did each girl drink?
    e. Confirm your answer is correct through substitution.
3. Eduardo went to the Casino. He started with $600 and then played and lost 6 rounds of Blackjack. He lost the same amount in each round. He ended the night with $480.
    a. Write an equation that represents the amount of money Eduardo has at the end of the night.
    b. What does each term in the equation represent? What are their units?
    c. How much did each round of Blackjack cost Eduardo?
    d. Confirm your answer is correct through substitution.
4. Lily wanted to buy tickets for the Bad Bunny concert with her friends. Initially, there were 55,288 tickets available. Each minute, 13,822 tickets were sold until they were sold out. Thankfully, she was able to get a ticket at the last minute!!!
    a. Write an equation that represents when there were no more tickets left.
    b. What does each term in the equation represent? What are their units?
    c. How many minutes did it take for Bad Bunny's concert to sell out?
    d. Confirm your answer is correct through substitution.
5. A fire was burning across 12,000 acres in the Santa Cruz Mountains. The CalFire firefighters were able to put out the same number of acres each day and eliminated the fire in 24 days.
    a. Write an equation that represents when the fire was extinguished.
    b. What does each term in the equation represent? What are their units?
    c. How many acres was CalFire able to put out per day?
    d. Confirm your answer is correct through substitution.
6. Dwayne wanted to buy yogurt from the store. He had $18.95 to spend on groceries. He purchased 4 small packages of yogurt and was left with $6.99.
    a. Write an equation that represents how much money Dwayne has left.
    b. What does each term in the equation represent? What are their units?
    c. How much does 1 small package of yogurt cost?
    d. Confirm your answer is correct through substitution.
7. Chloe needed to buy some batteries. She went to the hardware store with $20. She bought 4 D-batteries and was left with $5.
    a. Write an equation that represents how much money Chloe has left.
    b. How much does one D battery cost?
    c. Confirm your answer is correct through substitution.
8. Clarice wanted to get tickets for the Ariana Grande concert. She had $650, so she bought 3 VIP tickets.

After purchasing the tickets, she had $200 left.

a. Write the equation that represents how much money Clarice had left.
b. How much does a VIP ticket cost?
c. Confirm your answer is correct through substitution.

Once the fourth problem set is completed accurately, students will practice solving two-step equations to gain procedural fluency. Students will then play a game of Kahoot! in which they will practice solving these equations in partner pairs. Once students have attained a higher level of procedural fluency in solving two-step equations, students will move on from this unit to learn about the distributive property and combining like terms, so that they can find more success in solving multi-step equations.

Annotated Bibliography

Bybee, Rodger. *The BSCS 5E Instructional Model: Creating Teachable* Moments. Arlington: National Science Teachers Association Press, 2015.

Bybee elaborates on his BSCS 5E Instructional Model (Engage, Explore, Explain, Elaborate, and Evaluate) with research-based evidence of the success of its individual components and application of the model to STEM education.

Fujii, Toshiakira. “Understanding the Concept of Variable Through Whole-Class Discussions: The Community of Inquiry from a Japanese Perspective.” In *Algebra Teaching around the World*, edited by Frederick K. S. Leung, Kyungmee Park, Derek Holton, and David Clarke, 129-148. Rotterdam: SensePublishers, 2014.

In this chapter, Fujii discusses the Japanese teaching practice of Kikan-jyunshi, or Kikan-shido, through a case study involving student understanding of variables.

Hegarty, Mary, Richard E. Mayer, and Christopher A. Monk. “Comprehension of Arithmetic Word Problems: A Comparison of Successful and Unsuccessful Problem Solvers.” *Journal of Educational Psychology* 87, no. 1 (March 1995): 18-32.

authors of this article discuss how the direct translation strategy approach to word problems is largely unsuccessful, as students are merely following a procedure and don’t have the conceptual understanding of the word problem itself. The problem-model strategy, on the other hand, is much more successful, as students create a model to describe the situation at hand and then solve the equation.

Howe, Roger. “From Arithmetic to Algebra.” *Mathematics Bulletin: A Journal for Educators*, 13-22.

Roger makes the compelling case in this article that teaching students algebra through word problems can help students truly understand algebra rather than having a basic, procedural knowledge of the subject. Furthermore, he suggests that showing both the arithmetic and algebraic methods of solving the equations can help students facilitate a conceptual understanding of algebra.

Slavin, Robert. “Co-operative learning: what makes group-work work?.” *The Nature of Learning: Using Research to Inspire Practice*, edited by H. Dumont, D. Istance and F. Benavides, 161-178. Paris: OECD

Publishing, 2010.

In this chapter, Slavin reviews studies around two types of cooperative learning practices in schools- “Structured Team Learning” and “Informal Group Learning”- and compares the effectiveness of the two. Largely positive results were identified in this literature review, particularly when the group goals are clearly stated and when there is individual accountability within each member of the group.

Appendix on Implementing District Standards

My school follows the California Common Core State Standards for Mathematics (CCSSM). The following standards were touched on throughout this unit:

Quantities (N-Q):

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas…”

Students will be required to label a value’s units when answering word problems in the problem sets.

Seeing Structure in Expressions (A-SSE):

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.
    a. Interpret parts of an expression, such as terms, factors, and coefficients.

Students will interpret expressions they create from word problems in Problem Set 1.

Creating Equations (A-CED):

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable…and use them to solve problems.

Students will create equations that describe relationships they observe in the problem sets’ word problems.

Reasoning with Equations and Inequalities (A-REI):

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to jutify a solution method.

Students will justify solutions by substituting and evaluating their found solutions into the original equations.

Solve equations and inequalities in one variable.

3. Solve linear equations and inequalities in one variable…

Students will create and then solve equations from word problems in the problem sets.

Notes

[1] Mary Hegarty, Richard E. Mayer, and Christopher A. Monk, “Comprehension of Arithmetic Word Problems: A Comparison of Successful and Unsuccessful Problem Solvers,” *Journal of Educational Psychology* 87, no. 1 (March 1995): 18.

[2] “English Language Arts/Literacy and Mathematics: Smarter Balanced Summative Assessments,” California Assessment of Student Performance and Progress, accessed June 10, 2023, https://caaspp-elpac.ets.org/caaspp/ViewReportSB?ps=true&lstTestYear=2022&lstTestType=B&lstGroup=1&lstGrade=13&lstSchoolType=A&lstCounty=43&lstDistrict=69427-000&lstSchool=4335428&lstSubject=m.

[3] “Smarter Balanced Summative Assessments.”

[4] “Smarter Balanced Summative Assessments.”

[5] “Smarter Balanced Summative Assessments.”

[6] Rodger Bybee, *The BSCS 5E Instructional Model: Creating Teachable Moments* (Arlington: National Science Teachers Association Press, 2015), 4-7.

[7] Toshiakira Fujii, “Understanding the Concept of Variable Through Whole-Class Discussions: The Community of Inquiry from a Japanese Perspective,” in *Algebra Teaching around the World*, eds. Frederick K. S. Leung, Kyungmee Park, Derek Holton, and David Clarke (Rotterdam: SensePublishers, 2014), 144-45.

[8] Roger Howe, “From Arithmetic to Algebra,” *Mathematics Bulletin: A Journal for Educators*, 13.

[9] Robert Slavin, “Co-operative learning: what makes group-work work?” in *The Nature of Learning: Using Research to Inspire Practice*, eds. H. Dumont, D. Istance and F. Benavides (Paris: OECD Publishing, 2010), 170.

[10] Slavin, “Co-operative learning,” 170.

[11] Howe, “From Arithmetic,” 15.

[12] Howe, “From Arithmetic,” 16.

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