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2024 Volume V: Evolutionary Medicine

Exponential Functions in Evolutionary Disease

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Introduction

The goal of my original curriculum unit, *Exponential Functions in Evolutionary Disease*, is for students to develop a conceptual understanding of the difference between linear and exponential functions; I plan to facilitate students' reaching this understanding through analysis of exponential growth (and decay) in acute infectious disease.

Rationale

According to William C. Overfelt (WCO) High School's 2023 School Accountability Report Card, in the 2021-22 school year only 11% of students met or exceeded math standards; in 2022-23, only 9.25% of students met or exceeded these same standards (The report card details that "The "Percent Met or Exceeded" is calculated by taking the total number of students who met or exceeded the standard on the Smarter Balanced Summative Assessment plus the total number of students who met the standard (i.e., achieved Level 3-Alternate) on the CAAs divided by the total number of students who participated in both assessments.").[1]

The CAASP website details that over two thirds of WCO students scored Below Standard in Concepts & Procedures[2] , meaning that "The student[s] do… not yet demonstrate the ability to explain and apply mathematical concepts or the ability to interpret and carry out mathematical procedures with ease and accuracy."[3] Furthermore, nearly 40% of students did not meet proficiency in Problem Solving and Modeling & Data Analysis[4] , meaning that "The student[s] do… not yet demonstrate the ability to solve a variety of mathematics problems by applying [their] knowledge of problem-solving skills and strategies[; t]he student[s] do… not yet demonstrate the ability to analyze real-world problems, or build and use mathematical models to interpret and solve problems."[5] Over 40% of students did not meet proficiency in Communicating Reasoning,[6] meaning that "The student[s] do… not yet demonstrate the ability to put together valid arguments to support [their] own mathematical thinking or to critique the reasoning of others."

My curriculum unit is intended to mitigate these statistics by encouraging students to establish their understanding of exponential functions through tangible, real-world examples of evolutionary biology and medicine. I plan to incorporate these examples of exponential functions at the beginning of and throughout the unit, rather than at its culmination, so that they can serve as the foundation of students' understanding of the difference between linear and exponential functions.

In my Math I classes in particular, I notice that students understand linear and exponential functions when introduced and discussed separately from one another; however, when asked to identify which of the two types of functions are representative of given graphs, word problems, and tables, students struggle. They have difficulty not only determining the mathematical equation that describes the given medium, but in recognizing the rate of growth or decay as well. Furthermore, they lack the confidence and mathematical vocabulary to justify their use of numbers and variables in place of real or possible situations.

My curriculum unit will try to bridge this gap of understanding by introducing students to exponential functions through instances of their occurrence in biology. Not only will this facilitate student success in working with exponential functions, both as word problems and as algebraic expressions, but this will further prepare students when asked to work with both linear and exponential functions as tables, graphs, and word problems after the planned unit is taught.

Background Knowledge & Content

Exponential growth is a process "in which the rate of increase of a quantity is proportional to the present value of that quantity."[7] In other words, exponential growth occurs when a quantity grows by an ever-increasing rate over time. A linear growth pattern, in comparison, grows by a constant rate over time; for instance, an employee at a store makes a fixed rate of money per hour.

Virus replication is an example of exponential growth. A virus invades a cell; the virus then duplicates itself, creating 100 or more progeny particles within this host cell. The virus eventually spreads to other cells where the same replication process occurs. The speed with which virus pathogens grow within human cells while mutating to create new genotypes causes their rapid evolution, forcing us to constantly create new iterations of vaccines in an effort to combat the spread of viral diseases. However, the inability of our vaccine formulations to keep pace with virus evolution can further the virus's continued spread of infectious disease. This inability of hosts, such as humans, to evolve at the same pace as their pathogens is a key example of evolutionary medicine, which applies evolution thinking to better understand how and why health and disease problems occur.

While scientists and mathematicians may be familiar with the concept of exponential growth in evolutionary medicine and disease, the vast majority of people are not; they misunderstand exponential functions and, instead, tend to assume a linearized pattern of growth. This misunderstanding, referred to as exponential growth bias, can have dire consequences, as seen with the COVID-19 pandemic.[8]

Previous studies of epidemiology show that the initial spread of infectious disease, including COVID-19 caused by SARS CoV-2, often follows an exponential growth pattern.[9] Li et al documented the initial spread of COVID-19 in Wuhan, China by analyzing the first 425 confirmed cases of the disease. In this analysis, Li and his colleagues concluded that the rate of infection grew exponentially over the first month or so of its presence in humans.[10] (While the R-value, or "the number of individuals that the average infected person will infect," changes over the course of the spread,[11] it appears to remain somewhat constant at the onset of infectious disease).[12]

Despite this documented proof of exponential growth during the initial phase of viral infection, many did not understand or appreciate the rapidity with which the disease would spread. In the United States, for instance, many political figures downplayed the spread of COVID-19 due to their exponential growth bias, either failing to respond to the pandemic or outright refusing to follow guidelines set by the U.S. Centers for Disease Control (CDC).[13] Exponential growth bias discouraged the general public from following non-pharmaceutical interventions, such as masking, social distancing, and quarantining; the lack of following these interventions had dire consequences on public health.[14]

Another virus pandemic is a very real possibility, and the potential to misunderstand its initial pattern of spread is quite high. Therefore, the need to inform the next generation about exponential growth bias and its consequences is essential. It is imperative that students learn the difference between exponential and linear growth, not only so that they can take appropriate action in the event that another pandemic occurs in their lifetime, but also so that they can be better informed about general trends that follow these patterns.

To facilitate student comprehension about both linear and exponential functions, I will utilize the Herbartian instructional model. This model focuses on "the creation and development of conceptual structures that would contribute to an individual's development of character." Johann Friedrich Herbart, a German philosopher, found both student interest and conceptual understanding to be foundational components of teaching.

There are two types of interest: (1) that "based on direct experiences with the natural world" and (2) that "based on social interactions."[15] In my unit, I plan to engage the first type of student interest through examination of a disease they are all too familiar with: COVID-19. Students will explore the exponential growth patterns associated with advent of the COVID-19 pandemic in Wuhan, China through hands-on activities, tables, and graphs. While the cause of the pandemic may or may not have been what is considered to be a natural occurrence, it spread naturally, therefore facilitating student learning by evoking memories of direct experiences that students had with the natural world.

The second type of student interest will be engaged through socialization, which will be imparted via collaborative learning (collaborative learning will be expounded upon in the next section, Teaching Strategies). Through this collaborative learning, students will work together to digest and discuss various depictions of exponential functions, including tables, graphs, and word problems. Furthermore, toward the end of the unit, students' collaborative learning will facilitate their understanding of the difference between linear and exponential functions.

The conceptual understanding component of Herbart's philosophy deals with the coherence of ideas; new learning should be directly related to students' prior knowledge. The COVID-19 pandemic, along with evoking student interest, will directly connect students' learning of exponential functions to an event they all experienced and can relate to.

These foundational components of teaching, interest and conceptual understanding, are reflected throughout the Herbartian instructional model's four phases: Preparation, Presentation, Generalization, and Application. This unit will consist of two cycles of the Herbartian model up to the Generalization phase; the first cycle will cover exponential growth and the second cycle will cover exponential decay. Students will be assessed on both types of exponential functions in the Application phase.

Preparation & Presentation

The Preparation and Presentation phases will both occur during the first lesson of my unit.

In the Preparation phase, the teacher brings prior knowledge to the forefront of the students' learning experience; the practice of activating prior knowledge has been found to be beneficial to students' comprehension of new, unfamiliar mathematical concepts.[16] In the exponential growth cycle of the Preparation phase, students will be shown a video that splices news clips together from the onset of the COVID-19 pandemic.[17] In the exponential decay cycle of this phase, students will be shown a news clip that discusses the need for the general public to get the COVID-19 vaccine.[18] These clips will evoke personal memories that students had during their experience with the pandemic; the more personally meaningful and relevant the content is for students, the more likely they are to engage in learning.[19]

The Presentation phase focuses on connecting prior learnings to new learnings. During the Presentation phase of the exponential growth cycle of this unit, students will be introduced to exponential functions, particularly exponential growth, through a class simulation illustrating the initial spread of COVID-19. In the Presentation phase of the exponential decay cycle, students will visualize the effects of vaccines on the spread of COVID-19 through tree diagrams. An epidemic that students have great familiarity with, COVID-19 will, as seen in the Preparation phase, evoke prior knowledge and facilitate students' connection to new learning about exponential functions. Students will collaborate to graph and analyze the data gathered and observed during both the simulation and visualization activities respectively, thereby introducing them to the mathematical application of exponential functions.

Generalization

During the Generalization phase, the teacher clarifies and facilitates development of students' conceptual understanding. The Generalization phase of this unit will cover exponential functions through explicit (direct) instruction (which is discussed further in the Teaching Strategies section of this unit). Explicit instruction will more clearly define the following mathematical formulas for students:

- *Exponential Growth and Decay Formula*: $f(x) = ab^x$, where a is the initial (starting) quantity, b is the multiplicatory rate of change, and x is the amount of time passed (In this formula, exponential growth is represented by a b-value greater than 1; exponential decay is represent4ed by a b-value between 0 and 1).
- *Exponential Growth Formula (with Percent)*: $f(x) = a(1 + r)^x$, where a is the initial (starting) quantity, r is the rate of growth (which has to be changed from a percent into a decimal or fraction before being substituted in the formula), and x is the amount of time passed.
- *Exponential Decay Formula (with Percent)*: $f(x) = a(1 – r)^x$, where a is the initial (starting) quantity, r is the rate of decay (which has to be changed from a percent into a decimal or fraction before being substituted in the formula), and x is the amount of time passed.

Throughout the generalization phase of this unit (during both cycles of the Herbartian model), students will practice identifying exponential growth and decay rates through tables, graphs, and word problems and will translate these mediums into equations and evaluate said equations using the above formulas. Furthermore, they will compare and contrast exponential and linear functions in an effort to alleviate exponential growth bias within the student population. Students will answer questions about specific data points and general data trends within these functions, so that they can more successfully transfer this understanding to the Application phase.

Application

In the fourth and final phase, the Application phase, students demonstrate their newfound comprehension through application of concepts to new contexts. In this phase, students will research graphs depicting real-life scenarios in which exponential growth or decay is occurring. Students will have to analyze these graphs to determine what their exponential patterns mean and will communicate this meaning and its importance through a presentation to their classmates. In addition to this presentation, students will complete a summative assessment to show their overall comprehension of exponential (and linear) functions.

Teaching Strategies

Exploratory Tasks & Learning

An exploratory task in mathematics is one which leads to students “interpreting situations in mathematical terms, formulating mathematics questions, making conjectures… and making generalizations.”[20] Exploratory tasks will primarily be used in the Presentation and Application phases of this unit when analyzing various tables and graphs.

Explore-Before-Explain

An explore-before-explain strategy means “situating learning in real-life situations and problems and using those circumstances as a context for learning.”[21] With the simulation activity, students will have a chance to explore the mathematical concept of exponential functions before the technical vocabulary and formulas are directly taught to them, facilitating coherence in their learning and generally encouraging their mathematical empowerment.

Explicit Instruction

Explicit instruction is a direct, structured method of teaching. While an overabundance of direct instruction can inhibit student learning, the calculated use of this strategy can greatly benefit all students, especially those who “learn and think differently” than their peers.[22] Explicit instruction will be used exclusively during the Generalization phase of this unit to teach students relevant academic vocabulary as well as to teach them the formulas for exponential functions.

Cooperative Learning (Group Work)

Cooperative learning is “the instructional use of small groups so that students work together to maximize their own and each other’s learning.”[23] In cooperative learning, however, it is imperative to hold students individually accountable; otherwise this learning will not occur.[24] Throughout the Generalization and Application phases of the unit, students will be assigned to small, heterogenous (in academic attainment) groups, so that they can learn with and from one another when analyzing both exponential and linear data. Within these groups, students will be assigned roles and responsibilities, so that individual learning does not fall by the wayside.

Classroom Activities

Cycle One, Lesson One

The first lesson, which takes place during the Preparation and Presentation phases of the unit's first learning cycle, will focus on students' general comprehension of exponential growth via examples of its occurrence in evolutionary biology. Students will watch a video depicting spliced news clips taken from around the advent of the COVID-19 pandemic.[25] Students will then participate in a whole-class "pandemic" activity, in which one student is selected at random to be "Patient Zero." Patient Zero will have the chance to infect three other students in a game of tag; then those three students will infect three more students each, etc. From there, we, as a class, will create a graph of the number of infections (*frequency*) over the number of "days" (*time*), so that students can see the curved, hockey stick pattern of a graph depicting exponential growth. We will then return to the video clip, during which there is an image of a graph illustrating the onset of the COVID-19 pandemic depicting exponential growth. Students will have a better contextual understanding of what this graph means and how it directly applies to the spread of COVID-19.

From there, students will be given a list of examples and non-examples of exponential growth. Students will work in partner pairs to determine whether each item is or is not an instance of exponential growth. These partner pairs will then combine with other partner pairs to compare their results. As a class, we will go through each example and will debate and discuss whether or not these examples are, in fact, depicting exponential growth. Upon completion of discussing these examples, students will, first in partner pairs and then independently, practice determining whether or not tables are showing exponential growth.

Cycle One, Lesson Two

In the second lesson, which takes place during the Generalization phase of the unit's first learning cycle, students will be formally introduced to the formula for exponential growth, $f(x) = ab^x$ (the formula for exponential growth without percent). Students will utilize this formula in partner pairs to translate given tables and word problems into equations. These partner pairs will then partner with others to determine if they got the same answers. Each newly formulated group will be instructed to put one translated equation on the board, so that the class can check their own and each other's understanding of this concept. Once students are comfortable translating tables and word problems depicting exponential growth into equations, students will be tasked with another set of tables and word problems; however, this set will include linear growth patterns as well (which students have covered in a prior unit). Students will have to be able to differentiate between these two types of functions in order to successfully complete this task; this differentiation will facilitate students' better understanding of exponential growth bias and will (hopefully) encourage students to apply this understanding to any future pandemic they may experience in their lifetimes.

Cycle One, Lesson Three

During the third lesson, which still takes place during the Generalization phase of the unit, students will be introduced to the exponential growth equation that deals with percent increase, $f(x) = a(1 + r)^x$. Students will first practice translating a given percent into a decimal or fraction, so that they can confidently plug this value in for r. Upon successful understanding of how to determine and plug in the r-value, students will translate word problems depicting exponential growth with percent into equations; students will simplify these equations by adding 1 together with r (rather than keeping them as two separate entities). Once students are confident in utilizing the exponential growth formula with percent, students will be provided with an assortment of word problems which will depict both regular exponential and percent increase exponential functions as well as linear functions. Students will have to determine which formula to use based on the contents of the word problem. This will further protect students against harboring exponential growth bias toward what are undoubtedly incidences of exponential growth that they encounter in the future.

Cycle Two, Lesson One

The first lesson of cycle two sees a return to the Preparation phase. Students will be introduced to exponential decay through the concept of effective vaccination; during the first lesson, students will watch a news clip about the advent of the COVID-19 vaccine.[26] Despite the resounding efficacy of the COVID-19 vaccines, the advent of new variants has not truly allowed for the exponential decay of the disease to occur; however, if the vaccine were to work perfectly and if 75% of the population were to get vaccinated, we would absolutely see a pattern of exponential decay at work.[27] To best illustrate this concept, students will observe exponential growth of disease spread through a tree diagram; this same diagram will be continued after indicating where vaccination of the public occurred to show exponential decay of that same disease.

Similar to the happenings of the first lesson of cycle one, students will be given a list of examples and non-examples of exponential decay. Students will work in partner pairs to determine whether each item is or is not an instance of exponential growth. These partner pairs will then combine with other partner pairs to compare their results. As a class, we will go through each example and will debate and discuss whether or not these examples are, in fact, depicting exponential decay. Upon completion of discussing these examples, students will, first in partner pairs and then independently, practice determining whether or not tables are showing instances of exponential decay.

Cycle Two, Lesson Two

In the second lesson, which takes place during the Generalization phase of the unit's second learning cycle, students will form the understanding that the exponential growth formula, $f(x) = ab^x$, is also used for exponential decay; they will learn through explicit instruction that the b-value, in the case of exponential decay, must be between the values of 0 and 1 and, therefore, must be written either as a fraction or as a decimal.

Students will utilize this formula in partner pairs to translate given tables and word problems into equations. These partner pairs will then partner with others to determine if they got the same answers. Each newly formulated group will be instructed to put one translated equation on the board, so that the class can check their own and each other's understanding of this concept. Once students are comfortable translating tables and word problems depicting exponential decay into equations, students will be tasked with another set of tables and word problems; however, this set will include tables and word problems that depict linear growth and exponential growth as well, so that students can learn to effectively differentiate between each type of function.

Cycle Two, Lesson Three

During the third lesson, which still takes place during the Generalization phase of the unit, students will be introduced to the exponential decay equation that deals with percent decrease, $f(x) = a(1 - r)^x$. Students will translate word problems depicting exponential decay with percent into equations; students will simplify these equations by subtracting the *r*-value from 1 (rather than keeping them as two separate entities). Once students are confident in utilizing the exponential decay formula with percent, students will be provided with an assortment of word problems which will cover all types of functions touched on thus far: linear growth, exponential growth, exponential growth with percent, exponential decay, and exponential decay with percent. Students will have to determine which formula to use based on the context of the given word problem and appropriately plug values into said formula in order to successfully prove their understanding of these functions.

Final Assessment Lessons

In the lessons that cover the Application phase of the unit, students will have two tasks to complete: (1) researching and presenting on a graph they've found depicting either exponential growth or decay and (2) taking a summative assessment to show what they've learned about exponential (and linear) functions.

Research and Presentation

For the first task, students will research and identify a real-world instance of an exponential function (with teacher assistance to ensure the information is coming from a reliable source). Students will work in groups of three to create posters that (A) identify and define various aspects of the graph shown, including the x and y-axes and the interpretation of a single point shown on the graph, (B) explain (in words) the story the graph is telling, and (C) explain at least 3 reasons as to why the graph is or may be important to understand. These students will be assigned the following roles:

- *Researcher/Presenter 1*: This student (who also serves as presenter 1) is in charge of the computer during the research portion of the task. They are taking direction from their classmates (as well as themselves) to inform what type of exponential relationship they want to explore. Additionally, this student will present the information listed in part A of the poster.
- *Poster Writer/Presenter 2*: This student creates an artistically appealing poster with the input of the group that provides the above-mentioned information (A-C) to the reader; they will present the information from section B of the poster.
- *Poster Illustrator/Presenter 3*: This student creates an illustration with the input of the group of what the graph is about; this illustration will be included on the poster; they will present the information from part C of the poster.

The presentations given as part of the students' final assessment will likely improve achievement in Communicating Reasoning (as laid out by CAASP), as it will encourage students' "ability to put together valid arguments to support [their] own mathematical thinking..." Furthermore, students' use of critical thinking in part C to determine why the graph may be important will likely bolster student success in Problem Solving and Modeling & Data Analysis, as it will facilitate their "ability to analyze real-world problems, or build and use mathematical models to interpret and solve problems" as they relate to exponential functions. (The rubric for the students' posters and presentations will be provided in this section of this unit).

Summative Assessment

For the second task, the summative assessment, students will be evaluated on their understanding of exponential (and linear) functions in terms of the following:

- Identifying instances of exponential or linear growth or decay in graphs
- Translating x/y tables into linear or exponential equations
- Translating word problems into linear or exponential equations
- Evaluating exponential functions

(The final assessment for this unit will be provided in this section).

Annotated Bibliography

Berg, Siv Hilde et al. "Exponential growth bias of infectious diseases: Protocol for a systematic review." *JMIR Research Protocols* 11, no. 10 (2022).

Berg and his colleagues conduct a systematic review of exponential growth bias in epidemiology. They posit that effective communication regarding the exponential growth pattern of epidemiology can facilitate public compliance with mitigation strategies.

Li, Qun et al, "Early Transmission Dynamics in Wuhan, China, of Novel Coronavirus-Infected Pneumonia." *The New England Journal of Medicine* 328, no. 13 (2020): 1199-1202.

In this report, Li and his colleagues study the epidemiological characteristics of the initial stages of the COVID-19 pandemic in Wuhan, China. They find that the growth in the initial stages of the pandemic was exponential with an R-value of about 2.2. They conclude by stating that measures to curb transmission should be executed in populations at risk.

Johnson, Norman A. "Going Viral." *Darwin's Reach.* Boca Raton: CRC Press, 2022.

In this chapter, Johnson explores the COVID-19 pandemic through comparison to the flu, discussion of virus mutation and human transfer, and the necessity for vaccines. He concludes with examination of the AIDS pandemic, its evolution into HIV, and how this evolution informs treatment options.

Schonger, Martina and Daniela Sele. "How to better communicate the exponential growth of infectious diseases." *PLOS ONE* 15, no. 12, (2020).

Schonger and Sele discuss exponential growth bias as it pertains to infectious disease. They explain that framing, or alternate methods of communicating the same scenario, significantly impact how well people adhere to non-pharmaceutical interventions that serve to curb infectious disease spread.

Sidney, Pooja G. and Martha W. Alibali. "Making Connections in Math: Activating a Prior Knowledge Analogue Matters for Learning." *Journal of Cognition and Development* 16, no. 1 (2015).

In this study, Sidney and Alibali observe the activation of prior knowledge in learning a new mathematical concept, in this case division of fractions. They find that activating prior knowledge does facilitate this learning, so long as the prior knowledge is conceptually relevant and well-guided.

Priniski, Stacy J., Cameron A. Hecht, and Judith M. Harackiewicz. “Making Learning Personally Meaningful: A New Framework for Relevance Research.” *The Journal of Experimental Education*, 86, no. 1 (2017).

Priniski and her colleagues explore personal relevance as it pertains to motivation in learning. They discuss relevance as it pertains to prominent, existing theories in motivation, as well as relevance constructs, which are synthesized to suggest direction for further research.

Wijaya, Ariyadi. “Empowering Mathematics Learners through Exploratory Tasks.” *Empowering Mathematics Learners Yearbook 2017*, Edited by Berinderjeet Kaur and Lee Ngan Hoe. Singapore: World Scientific Publishing Co. Pte. Ltd., 2017.

Ariyadi argues that students become empowered mathematicians through learned skills and problem-solving strategies; she cites scaffolded exploratory mathematics tasks as instances for students to facilitate this empowerment.

Ferlazzo, Larry. “Four Teacher-Recommended Instructional Strategies for Math.” *Education Week*, July 11, 2021. https://www.edweek.org/teaching-learning/opinion-four-teacher-recommended-instructional-strategies-for-math/2021/07.

Larry Ferlazzo lays out four teacher-recommended instructional strategies for math; these strategies include concrete representational abstract, encouraging discourse, explore-before-explain, and a whiteboard wall.

Greene, Kim. “What is explicit instruction?.” Understood, accessed July 10, 2024, https://www.understood.org/en/articles/what-is-explicit-instruction.

In this article, Kim Greene defines explicit instruction and provides educators and families how to utilize explicit instruction, both at school and at home.

Johnson, David W. and Roger T. Johnson. “An Overview of Cooperative Learning.” Cooperative Learning. The Cooperative Learning Institute, accessed July 10, 2024. https://www.co-operation.org/what-is-cooperative-learning.

Johnson and Johnson define cooperative learning; elaborate on the three types of cooperative learning- formal cooperative learning, informal cooperative learning, and cooperative base groups; and identify the basic elements of cooperation.

Slavin, Robert E. “Co-operative learning and Student Achievement.” *Cooperative Learning* . New York: Longman Inc., 1983.

In this chapter, Slavin discusses the necessity for both group goals and individual accountability in cooperative learning.

Skew The Script. “Algebra 5.4 (Version A)- Exponential Decay.” YouTube. https://www.youtube.com/watch?v=xXN2XptB97U.

In this video, Aidan Gonzales elaborates on exponential patterns through the lens of the COVID-19 pandemic. She explains how the spread of the virus illustrate a pattern of exponential growth, while the effects of vaccination illustrate exponential decay.

Appendix on Implementing District Standards

My school follows the California Common Core State Standards for Mathematics (CCSSM). The following standards for Integrated Mathematics I are touched upon throughout this unit:

Quantities (N-Q):

Reason quantitatively and use units to solve problems.

1. Use units as a way to understands problems…; choose and interpret units consistently in formulas…

Students will have to identify units to successfully plug values into exponential formulas from word problems.

Creating Equations (A-CED)

Create equations that describe numbers or relationships.

2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.

Students will be creating exponential functions with the variables y or f(x) and x. Students will graph these functions by solving for individual points which they will compile into a table and graph.

Interpreting Functions (F-IF)

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch the graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*

Students will identify key features of exponential functions; students will use academic vocabulary to explain the relationship between the x and y-values of exponential functions.

6. Calculate and interpret the average rate of change of a function (presented…as a table) over a specified interval…

Students will use the tables they created and will identify the rate of change from these tables; they will identify both the change in x and the change in y to determine the overall rate of exponential growth or decay.

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph…

   e. Graph exponential…functions, showing intercepts and end behavior…

Students will graph exponential functions; they will be able to identify intercepts and end behavior, and will be able to explain what this means for the function in its given context.

Linear, Quadratic, and Exponential Models (F-LE)

Construct and compare linear…and exponential models to solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.

Students will have to identify whether a given word problem represents a linear or exponential function; it is imperative that they come away from the unit with this understanding to mitigate the effects of exponential growth bias in public health and other consequential matters.

2. Construct…exponential functions…given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Students will have to construct exponential functions, using the three exponential function formulas, from word problems, tables, and graphs.

3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly…

Students will graph both linear and exponential functions on the same graph; students will realize that the exponential function, despite growing at a much slower rate, will eventually surpass the linear function.

Notes

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[1] "William C. Overfelt High School 2022-23 School Accountability Report Card (Published During the 2023-2024 School Year)," East Side Union High School District, accessed June 18, 2024, https://www.esuhsd.org/documents/Board%20-%20Admin/SARC/SARC%2022-23/2023_School_Accountability_Report_Card_William_C._Overfelt_High_School_20240118.pdf.

[2] "English Language Arts/Literacy and Mathematics Smarter Balanced Summative Assessment Detailed Test Results for: School: William C. Overfelt High," California Assessment of Student Performance and Progress, accessed June 18, 2024, https://caaspp-elpac.ets.org/caaspp/ViewReportSB?ps=true&lstTestYear=2023&lstTestType=B&lstGroup=1&lstGrade=13&lstSchoolType=A&lstCounty=43&lstDistrict=69427-000&lstSchool=4335428&lstSubject=m.

[3] "English Language Arts/Literacy and Mathematics Smarter Balanced Summative Assessment Understanding Smarter Balanced English Language Arts/Literacy and Mathematics Summary Reports," California Assessment

of Student Performance and Progress, accessed June 18, 2024, https://caaspp-elpac.ets.org/caaspp/UnderstandingSBResults#cm2

[4] "School Accountability Report Card."

[5] "Understanding Smarter Balanced ELA/Literacy and Mathematics Summary Reports."

[6] "Detailed Test Results for: William C. Overfelt High."

[7] Russell K. Hobbie and Bradley J. Roth, "Exponential Growth & Decay," in *Intermediate Physics for Medicine and Biology* (New York: Springer Science + Business Media, LLC, 2007), 31.

[8] Berg et al, "Exponential growth bias of infectious diseases: Protocol for a systematic review," *JMIR Research Protocols* 11, no. 10 (2022).

[9] Berg et al, "Exponential growth bias."

[10] Li et al, "Early Transmission Dynamics in Wuhan, China, of Novel Coronavirus-Infected Pneumonia," *The New England Journal of Medicine* 328, no. 13 (2020): 1199-1202.

[11] Norman A. Johnson, "Going Viral," in *Darwin's Reach* (Boca Raton: CRC Press, 2022), 21.

[12] Berg et al, "Exponential growth bias."

[13] Berg et al, "Exponential growth bias."

[14] Martin Schonger and Daniela Sele, "How to better communicate the exponential growth of infectious diseases," *PLOS ONE* 15, no. 12, (2020), 1-2.

[15] Bybee et al, *The BSCS 5E Instructional Model: Origins and Effectiveness*, (Colorado Springs: BSCS, 2006), 5.

[16] Pooja G. Sidney and Martha W. Alibali, "Making Connections in Math: Activating a Prior Knowledge Analogue Matters for Learning," *Journal of Cognition and Development* 16, no. 1 (2015), 179.

[17] NBC News, "2020 Timeline: The Year Of The Covid Pandemic | NBC News NOW," YouTube, 0:00-2:05, December 23, 2020, https://www.youtube.com/watch?v=8rFg-FvZZhY.

[18] NBC News, "COVID-19 Vaccine Milestone | NBC Nightly News," YouTube, https://www.youtube.com/watch?v=DlX-xLkrvZE.

[19] Stacy J. Priniski, Cameron A. Hecht, and Judith M. Harackiewicz, "Making Learning Personally Meaningful: A New Framework for Relevance Research," *The Journal of Experimental Education*, 86, no. 1 (2017), 3.

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