

# 11

## **Ship and Airplane Testing: Physics for High School Mathematics Students**

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### **I. INTRODUCTION**

Here at New Haven at the Sound School we want to integrate the Long Island Sound as a theme into our instruction, to make connections between the material taught in the classroom and what goes on in the Sound. The Sound School is one of the schools of choice offered to high school students by the New Haven Public Schools. Children may choose to come to one of these schools instead of the large, traditional high schools. Long Island Sound leads to topics such as marine biology, boats, and navigation. As a mathematics teacher I look for problems to share with my students. By looking at the methods of solution in one field we may find strategies for solving problems in other fields.

One topic I look forward to is naval architecture, the designing of ships. If we are going to look at boat design, we need to know some physics. What is a center of gravity? What is a center of buoyancy? How do levers work?

#### **A. What Is a Unit?**

A unit is the result of a New Haven Public School teacher's participation in the Yale-New Haven Teachers Institute. The Institute provides an opportunity for New Haven public school teachers to write curricula with the assistance of a Yale faculty member and the resources of Yale University. The Institute runs various seminars related to the subjects one teaches. The units are the record of what one accomplished by being in the seminar. One applies in February and finishes by the first week of August.

When one writes a proposal, one has high expectations. "I'll write an explanation that will make it obvious for any student." After one has tried to do it one sees reality. "It doesn't sound so much different from the references." Yes, that is how I learned that I had learned something. The explanations that seemed complicated at the start now make sense. Unfortunately, that means other students will have to put in the time just as I did. I didn't find the magic explanation.

The seminar on aerodynamics was an opportunity to pursue some of the physics related to boats. Aerodynamics is a division of the larger field of fluid mechanics. The rules that apply to air, when the speeds are well below the speed of sound, also apply to water. It is only necessary to take into the account the different properties of the media. Water is more dense and more viscous than air; both, however, are called fluids. The rules for floating boats and balloons are the same. The rule for submarines and airplanes are the same. Boats, however, work on the surface of the water producing free surface waves; this adds some problems that airplanes and submarines do not have to contend with.

## **B. How to Use This Unit**

I want my students to see uses of mathematics. One way students can see uses of mathematics is by reading about engineering projects and their solutions.

I see this unit as something the student can read as a start on the subject. The unit may be broken into sections for reading, or the whole unit may be given to the class. I want discussion with the students on the writing: It is clear? How could it be improved? What was left out? Did some source over-simplify the story? After dissecting this unit it will be their turn to write a report on some technological project.

Learning is a do-it-yourself job. The required work may not be enjoyable. I hope, however, the collateral readings will be enjoyable and fascinating.

I see the student projects centering on the history of engineering, ship building, naval architecture, and technology. Books that are readily available are the Time-LIFE Books, *Ships*, by Lewis et al. and Thomas C. Gillmer's *Modern Ship Design*. Both are public library books.

The most important part of the process will be the discussion. Students are all too willing to sit passively as if they were jugs waiting to be filled up. I see the reading as the motivation for the discussion. If the books are interesting enough the students will be willing to share them with one another.

I use this material when we discuss variation in Algebra II. This unit itself mentions direct and inverse proportion. The naval architecture readings give

examples of variation when they discuss laws of mechanical similitude, such as the wetted surface varies as the square of the length on the water line. There are other places that the unit could be used, but I need a starting place. If the students show interest, the project will expand.

## II. MECHANICS

There are many stories that educated people are “supposed” to know. These are the stories that tell us the axioms or postulates of our culture. Two examples are Archimedes running naked through the street of Syracuse yelling, “Eureka,” and Galileo dropping two cannonballs of different weights from the Leaning Tower of Pisa. Although historians of science may question the accuracy of the stories, there is no doubt that the stories help us remember the physical principles involved. The first is buoyancy, why boats float, and the second tells us that objects fall at a constant acceleration that is not dependent on their weight.

What have I learned that I would like to share with my students? There are a number of stories that illustrate physical principles that everyone is supposed to know. Let us tell them.

There are fundamental principles that need to be known as well. Let us mention them even if we do not teach the topics completely so the students will be prepared to listen more carefully when they meet the principles the next time an instructor presents them.

One topic that I would like to share with my students is called dimensional analysis. Physics is what we can hold, see, and measure. Just what can we measure? We can use a ruler to measure the length of something, we can use a balance to find the mass of something, we can use a watch to time something, and we can use a thermometer to measure the temperature of something. All other measurements are combinations of length, mass, and time. In fact, a thermometer is using length to measure temperature; the length of the column of mercury tells us the temperature. The words “dimensions” and “units” have distinct meanings. When I say a board is six feet long, I have given the units feet to the dimension length.

### A. Archimedes: The Lever, Density, and Buoyancy

Archimedes was a Greek mathematician. He was killed by a Roman soldier during the invasion of Syracuse in 212 B.C. You may read about this in James R.

Newman's collection, *The World of Mathematics*. Two concepts that Archimedes worked on are of interest to naval architects: The lever and buoyancy.

The lever is the first machine. It is the principle of the beam balance, which allows us to weigh things. Scientists look for formulas to describe phenomena. The formula for a lever can be best explained by looking at a seesaw. Two children take a board and support it near the center. Each one sits on an end with the support between them. They now can go up and down. The support is called the fulcrum. What if one child weights more than the other? The heavy end will go down and only go up with much effort on the part of the heavy child. What can be done? Move the board on the support so the heavy child is closer to the fulcrum and the lighter child is farther from the fulcrum. Let  $w_1$  be the weight of the first child, let  $d_1$  be the length of the seesaw from the fulcrum to the first child, let  $w_2$  be the weight of the second child, and let  $d_2$  be the length of the seesaw from the fulcrum to the second child. The formula for this law of mechanics is

$$w_1 d_1 = w_2 d_2$$

When physicists multiply a weight or force times a distance they call the product a moment or a torque. So if you move a weight with a lever and you have to move another weight that is twice as heavy as the first, just use a lever that is twice as long and it will be just as easy to move as the first. This led Archimedes to say, "Give me a place to stand and I will move the earth," the principle being that to apply more force, use a longer lever.

The second principle is illustrated by the story of how Archimedes tested the crown of king Hiero for the purity of its gold content. Rumor said the goldsmith replaced some of the gold with an equal weight of silver. So the crown weighed as much as it should. How could it be tested for purity? Archimedes was consulted.

Archimedes' moment of insight occurred while he was taking a bath. He jumped out of the tub and ran throughout the streets yelling, "Eureka!" which means "I have found it!" in Greek. That's the story, anyway. So what had he found?

Let us ask ourselves the same questions. What happened when his body went into the water? Water ran out of the tub. Water was displaced. How much water was displaced? Two things cannot take up the same space at the same time, so the amount of water displaced has the same volume as the submerged body. Do

equal weights of gold and silver take up the same amount of space? No, gold is denser than silver. Equal volumes of gold and silver would have different weights; the gold would weigh more than the silver. So equal weights of gold and silver would have different volumes, with the gold, being denser, taking up a smaller volume.

Archimedes found that the goldsmith, indeed, had cheated. Archimedes measured the displacements of the crown and of two weights, one of pure gold, the other of pure silver, both equal in weight to the crown. Archimedes found the crown's displacement to be between the displacements for the pure gold and the pure silver.

There is more to the story. Archimedes also weighed the crown in air and suspended under water. He found that it weighed less when submerged. How much less? The weight loss was equal to the weight of the displaced silver. What if a body displaced a volume of water that weighed more than the body itself? The body would weigh less than nothing? No, the body would float. That is the principle of buoyancy. A body floats when the weight of the water displaced is equal to the weight of the body.

If we divide the mass of the body by its volume we get its density. Knowing the density of an object and the density of water we can tell if the object will float. If its density is less than the density of water then the object floats, greater than water it sinks.

With these principles it is possible to determine the water line of a boat. Here is a sample problem. I have a block of wood 8 cm x 10 cm x 15 cm, that has a mass of 780 gm. I would like to float it with the 8 cm dimension being the height. How deep would it sink into the water? What is the density of this block?

One convenient feature of the metric system is that one cubic centimeter of water has a mass of one gram, so by definition the density of pure water is one gram per cubic centimeter. The density of the block is

$$\frac{780 \text{ gm}}{8 \text{ cm} \times 10 \text{ cm} \times 15 \text{ cm}} = 0.65 \text{ gm} / \text{cm}^3$$

So the block is a typical piece of wood; its density is less than one. In other words, its density is less than the density of water, so it will float.

Since the block weighs 780 grams it will have to displace 780 cubic centimeters of water in order to float. If the block is to float with the 8 cm dimension as

the height, then the submerged portion will have dimensions of 10 cm by 15 cm by X cm and will have a volume of 780 cubic centimeters. So find X.

$$\begin{aligned}
 10 \text{ cm} * 15 \text{ cm} * X \text{ cm} &= 780 \text{ cm}^3 \\
 150 \text{ cm}^2 * X \text{ cm} &= 780 \text{ cm}^3 \\
 X \text{ cm} &= 780 \text{ cm}^3 / 150 \text{ cm}^2 \\
 \therefore X \text{ cm} &= 5.2 \text{ cm}
 \end{aligned}$$

Consequently the block will float with 5.2 cm of the 8 cm vertical dimension submerged.

This is a demonstration of the principle that allows a naval architect to determine the water line of a boat before it is built. Of course, the submerged portion will be more complex than a block. Calculus will then be needed to calculate the volume of the submerged portion. The architect will also need to know the densities of the materials out of which the boat is to be so that the weight of the boat can be calculated. Again by calculus the volume of the whole boat can be determined.

Archimedes was one of the Greeks who contributed to classical geometry. In fact, they called themselves philosophers, no matter what they were studying—mathematics, science, or what we call philosophy today. The axiomatic method of thinking was their legacy to us. Consider the statement, “Two things cannot take up the same space at the same time.” That is a self-evident truth, that is an axiom! So the laws of science are axioms that we accept because our experience says they seem to be self-evident. If the axioms are true then the consequences of the axioms are true as well. So investigate by doing experiments to see if the consequences are true. If the consequences do not work out, then our axioms do not apply to the real world. The investigations are called experiments.

What Archimedes was doing in our discussion is known as statics, the study of bodies at rest, in equilibrium. Statics is a branch of the larger field of physics called mechanics which includes the study of motion.

## B. Other Contributors

Among the people who contributed to the field of mechanics we find Copernicus, Brahe, Kepler, Galileo, and Newton. Yes, those are the names associated with the concept of the solar system. Mechanics is the study of motion, any kind of motion, even the motion of celestial bodies.

Here is a short version of the story. Copernicus said the planets revolved around the sun. Brahe made astronomical observations. Kepler analyzed Brahe's data and came up with three laws:

1. Planets follow elliptical paths with the sun as a focus.
2. A line from the sun to a planet will sweep out equal areas in equal times.
3. The square of the period of revolution about the sun is proportional to the cube of the major axis of the orbit.

Some friends went to Isaac Newton (1642-1727) with the idea that gravitational attraction between two bodies was inversely proportional to the square of the distance between the bodies. The story goes that he had the solution already written out for them. The law of gravity with Newton's laws of motion result in proving Kepler's laws as theorems. Newton had used the calculus that he had invented to do the problem. His friends induced him to publish his results in a book called Newton's *Principia*.

### C. Newton's Three Laws:

1. A body at rest will remain at rest and a body in motion will remain in straight line motion, unless it is acted upon by a force to change that motion.
2. The force causing the motion of a body is equal to the product of the mass of the body and the acceleration of the body. ( $F = ma$ ) where  $F$  is the force,  $m$  is the mass of the body, and  $a$  is the acceleration of the body. Acceleration is defined as the rate of change of the velocity, that is, how fast the speed changes.
3. For every action there is an equal and opposite reaction.

Newton also proposed the Universal Law of Gravity. He stated that the force of attraction,  $F$ , between two bodies varies jointly as the product of their masses,  $m_1$  and  $m_2$ , and inversely as the square of the distance,  $r$ , between them:

$$F = \frac{Gm_1m_2}{r^2}$$

where  $G$  is the universal gravitational constant.

There is much more to the story. Who was Leibniz, and what did Galileo have to do with the story?

So Newton's scheme worked; it was applied to celestial bodies and earthly bodies. His work in the area is now called Newtonian Mechanics or Classical Mechanics. When we apply the rules of mechanics to fluids we have fluid mechanics.

### III. FLUID MECHANICS

#### A. The Continuity Equation

One reason I participated in the Aerodynamics Seminar was to find examples using high school mathematics that I could share with my students. The continuity equation is such an example.

When trying to solve a problem we are told to look for things that stay the same. They then can be set equal to each other, giving an equation to solve. Rules like that are easy to state, but examples are easier to understand.

If we have water flowing through a pipe, the water will enter at a certain rate and leave at the same rate; if the quantity pushed in per unit time remains constant, an equal amount of water per unit time will be expelled. That seems like an obvious and reasonable observation. What are we taking for granted in this argument? We are assuming that water is incompressible and that its flow is steady or unchanged with time. This reasoning is using the concept of conservation of mass, namely that in nature mass cannot be gained or lost in a system. A simple, characteristic example of scientific reasoning. So what can we do with it? Let's write a formula.

How would you express the rate of flow? If it were a bilge pump you would say so many gallons per hour. What do gallons measure? Volume. Volume is expressed as cubic units, cubic feet, cubic centimeters, and so forth. Time can easily be changed from hours to minutes or seconds. So the rate of flow could be expressed as cubic centimeters per second. Cubic centimeters per second could also be expressed as square centimeters times centimeters per second. What do square centimeters and centimeters per second each measure? Area and velocity, respectively. So the rate of incompressible flow could be expressed as volume per time equals area times velocity. The velocity would be an average velocity since we are using the total volume per unit time. The water in the center of the pipe goes faster than the water at the pipe wall. Can you tell what area and what velocity to use? The pipes have cross sectional areas and the water has an average velocity. So we could use the area of the entry or the exit opening. Does the argument make sense if we start with the area of the pipe and the average velocity of the water? If we multiply the cross sectional area of the pipe (cm squared) by the velocity of the water (cm/sec), what do we get? Cubic centime-

ters per second, volume per unit time, a rate of flow. It makes sense both ways. Does it matter what cross sections we use? If our principle that the quantity of matter flowing in is the same as the amount flowing out, then it must also be true everywhere in the pipe. Let us write this as an equation

$$AV = k$$

where  $A$  is the cross sectional area at a point in the pipe,  $V$  is the average velocity of the water at the same point, and  $k$  is a constant, the rate of flow of the pipe, in our units cubic cm/sec. Since the equation is true for any two points in the pipe we could also write

$$A_1 V_1 = A_2 V_2$$

where  $A_1$  and  $V_1$  are the area and velocity at one point in the pipe, and  $A_2$  and  $V_2$  are the area and velocity at some other second point in the pipe. This equation is called the continuity equation.

The way I visualize the continuity equation is by thinking of a paper wrapper full of pennies. Instead of closing it, fill it full to the ends so no more will fit in. Now try pushing three more pennies in at one end. What happens? Three pennies are pushed out the other end.

We have just shown that the velocity of water at some place in a full pipe is inversely proportional to the cross sectional area of the pipe at that point. That means when the pipe narrows down the velocity of the water goes up. This is something you probably noticed when you put your thumb over your garden hose.

When fundamental principles are stated one should consider them for some time. Ask what the ramifications of the principle are, what else can be proven by it? Think on it long enough so you too can see that it is "self-evident." After all, how often do you go around repeating the self-evident? If it is self-evident, your listeners can figure it out for themselves.

## B. Dynamic Similarity

In High School Geometry we study the concept of similarity. If two geometric figures are of the same type with corresponding angles congruent and corresponding sides proportional, then the figures are said to be similar. In physics, if the points

in corresponding positions at corresponding times have proportional velocities and proportional accelerations, then the systems are said to be dynamically similar.

What could the corresponding points be? The ideas of geometric and dynamic similarity can be extended to three dimensional bodies, such as models of ships, of aircraft, of canals, or of river systems. We could do model testing.

Why would we want to do model testing? If we want to design a new ship, we could go ahead and build it and see what happens. This could be very expensive, especially if the design were poor or had some dangerous feature. Instead we can make scale models that are geometrically similar to the full size ship we will build. It is cheaper to run tests on the models since they are smaller and we can build more of them to test more features. We must have a way of transforming the measurements we take on the models to get the values we would have on the full size ship. To be able to transform the drag on a model, for example, to the drag on the full size ship, we need to have dynamic similarity.

Corresponding time should be explained. If the problem being modeled is cyclical in nature, such as the revolutions of propellers, the time for the model will be the fraction (number of prototype rpm over the number of model rpm) of the real time. If there are no cyclical features, find the times for the systems to trace out similar curves. The ratio of those times is the scale factor for time. If we can model time, it is possible to tell how long a machine will last. If we run its model at 20 model cycles to one prototype cycle, the model will wear out in one twentieth the time the prototype will take to wear out.

Model testing is useful; it may save lives. Both lack of model testing and ignorance of model results have led to major loss of life and property. Although there was no loss of life, one famous example is the Tacoma Narrows Bridge. It twisted itself apart; the wind pushed it one way, and each time the bridge reacted, the wind continued to push it in the direction of reaction, making the effect bigger. This is known as resonance, the amplification of the effect.

Mario Salvadori in his book, *Why Buildings Stand Up*, reports that on May 17, 1854, the Wheeling Bridge over the Ohio River collapsed in a wind storm in the same way as the Tacoma Narrows Bridge did on November 7, 1940. John Roebling, who designed the Brooklyn Bridge, knew of the failure and designed his bridges with diagonal stays so as to prevent the twist. Figure 10 in Paul A. Hanle's *Bringing Aerodynamics to America* is a photograph of a model of the Tacoma Narrows Bridge in a wind tunnel at the University of Washington; the date, however, is 1941, too late to avoid the disaster. The picture does show the wind-induced waves in the bridge deck. One reason to go to school is to learn from the experiences of those who came before us. Some models may be full size examples that are improved upon in the next version of the design.

How do we keep models dynamically similar? By running the model at the same value of the similarity constant as the prototype. What does that mean? What are similarity constants? Let us answer those questions. There are many similarity constants. Similarity constants are pure numbers, they are dimensionless. They are ratios of various kinds of forces. Three important ones are the Reynolds number, the Froude number, and the Mach number. So if you want your model to be representative, run it at the same Reynolds number or the same Froude number as the prototype. The following paragraphs should make this clearer.

The sources for the following are texts in Aerodynamics and Hydrodynamics, I make no claims to originality. Sources I found most helpful were von Kármán, Prandtl, Rouse, and Wegener, which are listed in the bibliography.

There are various kinds of forces. There is the force of gravity which attracts things to the center of the earth. There are inertial forces. Inertia is the property of bodies in motion to stay in motion until operated upon by another force. Some of the opposing force is due to the viscosity of the medium. When a bead is dropped into a jar of honey it does not fall as fast as it would in a jar of water. We say the honey is more viscous than water. We could form ratios of these forces in various ways. Two of the ratios are the ratio of inertial forces to viscous forces, which is called the Reynolds number, and the ratio of inertial forces to gravitational forces, which is called the Froude number.

The word "constant" needs some comment. The numbers we are talking about have different values for different velocities, different media, and different prototypes. If we are doing testing, we must have the value of the number of the similarity parameter for the model situation equal to the value of the number for the full-size situation. We must keep the numbers equal, keep them "constant."

Dimensionless variables are a significant topic. Let us first learn what are units and what are dimensions. When we say a board is six feet long, the units are feet and the dimension is length. So the dimension length can be measured in many units: feet, inches, miles, meters, etc. In mechanics there are three basic dimensions: length, mass, and time, symbolized as  $[L]$ ,  $[M]$ , and  $[T]$  respectively. These three are combined algebraically to make the dimensions of other physical variables. For example, velocity has units such as feet per second so its dimension is  $[L/T]$ , where feet are units for length, per means division, and seconds are units for time. Acceleration is how fast the speed changes; for example, if the speed increased 3 feet per second every second we could say the acceleration was 3 feet per sec per sec or 3 ft/square sec, the dimension of which would be  $[L/T^2]$ . In Newtonian physics force is mass times acceleration so the dimension for force would be  $[ML/T^2]$ . So the dimensions are multiplied and divided as in Algebra 1. When a new physical variable is presented, the dimension may also be stated along with it. See if you can determine

the dimensions for rate of flow and density as mentioned in the previous section. Can you tell what is the dimension for area?

Before defining the Reynolds, Froude, and Mach numbers, we need to define some notation. The letter  $g$  stands for the acceleration due to gravity [ $L/T^2$  squared]. It is a constant. Its value on earth is 9.8 m/sec squared or 32 ft/sec squared. That is the point of the story about Galileo and the leaning tower. The Greek letter ( $\rho$ ) stands for the density of the material under discussion. It is the mass of a unit volume of the material [ $M/L^3$  cubed]. Density is a property we use with buoyancy to determine the water line of a ship. The Greek letter ( $\mu$ ) stands for the dynamic viscosity of the medium [ $M/(LT)$ ]. The dynamic viscosity of a medium divided by its density is called its kinematic viscosity, symbolized by the Greek letter ( $\nu$ ) [ $L^2/T$ ]. Did you get the dimension for kinematic viscosity?

### C. The Reynolds Number

This ratio known as the Reynolds number,  $R_e$ , is

$$R_e = \frac{VL}{\nu}$$

where  $V$  is the velocity,  $L$  is a representative length such as the diameter of a pipe, the length of a ship, or the chord of an airfoil, and  $\nu$  is the kinematic viscosity. The Reynolds number is named after Osborne Reynolds (1842-1912), a British engineer who did the first work on laminar versus turbulent flow in pipes.

What is the dimension of the Reynolds number? Substitute the dimensions for the defining variables of the number and reduce:

$$R_e = \frac{VL}{\nu} = VL * \frac{1}{\nu} = \frac{L}{T} * L * \frac{1}{L^2/T} = \frac{L^2T}{TL^2} = 1$$

The result is that the dimensions of the fraction cancel out, they reduce to one. The Reynolds number is said to be dimensionless, as would any other variable whose dimensions reduce to one.

Let us use the Reynolds number in an example. We want to test a 1/3 scale model of an automobile in a wind tunnel to determine its aerodynamic drag. How fast should the winds blow? The physical relationships are the same

whether the car moves or the wind moves; it is easier to blow air against a fixed car model. Let the subscript m mean "of the model" and the subscript p mean "of the prototype." If the Reynolds numbers of the model and the prototype are to be equal then

$$\frac{V_m L_m}{\nu} = \frac{V_p L_p}{\nu}$$

Since the kinematic viscosity of the air is the same in the tunnel as on the highway we may cancel it. Substituting  $L_m = L_p/3$  we get

$$V_m \frac{L_p}{3} = V_p L_p$$

Solving for  $V_m$ , we get

$$V_m = 3V_p$$

So the model velocity must be three times the velocity of the full-size car. To model 55 mph the wind tunnel will be run at 165 mph. This is a reason to use large sized models when keeping the Reynolds numbers of the model and the prototype equal. Since the velocity is inversely proportional to its size, the smaller the model the faster the wind will have to be, perhaps too fast.

If the model were small enough we could be required to run the tunnel at supersonic speed. Even if we could (there are such tunnels) we would then have to worry about effects on the model that do not occur at subsonic speeds, such as the air being compressible and thermodynamic effects. The similarity constant for supersonic speeds is the Mach number. We will say more about the Mach number later. When there are forces that have significant effect on the model but negligible effect on the prototype, we have a scale effect. Working with larger scale models helps avoid scale effects.

#### D. The Froude Number

The Reynolds number is not the only similarity constant. After all, viscous forces are not the only forces that act on other systems. When the inertial and gravitational forces predominate then the similarity constant of concern is the Froude

number. This number is also named after its discoverer, William Froude. It is the ratio of the inertial forces to the gravitational forces.

William Froude (1810-1879), a nineteenth century English scientist, was one of the first to use a towing tank to test the designs of ships. Pictures of his apparatus can be found in Rouse and Ince, *History of Hydraulics* (see the bibliography). He wanted to discover the relationship between the forces on a model and the full-size ship. By towing variously sized models of the same design and comparing them to each other, he found that there was no proportional relationship between the resistance of similar models and their sizes. He decided that the resistance was the sum of a frictional force and a residual force. His technique was to determine the frictional force, subtract it from the total force, and get the residual force, which he said was the wave-making force. He determined the frictional resistance by towing boards of various lengths and finishes under water without making waves. He assumed that the frictional resistance ( $R_f$ ) was proportional to the wetted surface area of the ship ( $S$ ) and a power of the velocity ( $V$ ) of the ship.

$$R_f = fSV^n$$

His experiments with the boards were to determine the proportionality constant ( $f$ ), which he called the form factor, and the exponent ( $n$ ) for velocity.

While experimenting with different size scale models he noticed that the wave patterns were the same in number along the hull when the ratios of speed to square root of length were the same. This leads to the dimensionless parameter

$$F_n = \frac{V}{\sqrt{gL}}$$

where  $V$  is the velocity of the boat,  $g$  is the acceleration due to gravity, and  $L$  is the length of the boat.  $F_n$  is now called the Froude number, in his honor. Froude did his work before Osborne Reynolds, so he did not know about the Reynolds number.

Let us use dimensional analysis to show that the Froude number is dimensionless. Substitute the dimensions of the defining variables (look back) and simplify to get

$$F_n = \frac{V}{\sqrt{gL}} = \frac{L/T}{\sqrt{(L/T^2)L}} = \frac{L/T}{\sqrt{(L^2/T^2)}} = \frac{L/T}{L/T} = 1$$

So the dimensions reduce to one and the Froude number is indeed dimensionless.

Froude's program was to determine the frictional resistances from the wetted surface areas of the model and the ship, determine the residual resistance of the model by towing the model, scale the model's residual resistance up to the full size ship, and add it to the ship's frictional resistance to get the ship's total resistance. The scaling up is where Froude's number comes into the discussion. The residual resistances varied as the displacements when the Froude number for both the model and the prototype were equal. The displacement is the volume of water equal to the volume of the boat below its water line. The boat moves the water, it displaces the water.

If this theory is correct, it can be tested against a finished ship. That is what was done when the Admiralty made the battle ship *H.M.S. Greyhound* available. Froude took the measurements for a model and for the full size ship; the results matched.

If viscous and inertial forces are to be similar, the Reynolds number of the model and the prototype must be equal. If the inertial forces and the gravitational forces are to be similar then the Froude number of the model and the prototype must be the same. Is it possible to have both numbers equal at the same time?

Let the subscript m mean "of the model" and the subscript p mean "of the prototype." If the Reynolds numbers are equal then

$$\frac{V_m L_m}{\nu} = \frac{V_p L_p}{\nu}$$

If the model and the ship are both in the same kind of water (salt or fresh) the viscosity terms being equal divide out leaving

$$V_m L_m = V_p L_p$$

and

$$V_m = \frac{V_p L_p}{L_m}$$

If the Froude numbers are equal

$$\frac{V_m}{\sqrt{gL_m}} = \frac{V_p}{\sqrt{gL_p}}$$

Since  $g$  is a constant it may be canceled and

$$V_m = V_p \sqrt{\frac{L_m}{L_p}}$$

Setting the two equations for  $V_m$  equal we get

$$L_m^{3/2} = L_p^{3/2}$$

This equation says the only way the Reynolds and the Froude numbers can be equal is if the length of the model and the length of the ship are equal, that is, if the model is full sized. Does this mean that model testing is impossible? No, just that there are limitations to the modeling process; they are approximations in some aspects. The engineer needs to know what is being modeled and make corrections for the effects that are not being modeled.

Whether the Reynolds or the Froude numbers are kept equal depends on which effects are considered significant. Waves involve gravity, so if waves are involved the Froude numbers are kept equal. If we are submerged in a medium such as air for airplanes and cars or water for submarines and propellers, the Reynolds numbers are kept equal provided the speeds are moderate. If speeds are not moderate then we must consider the Mach number.

Froude leads us to the topic.

## E. How to Test Model Ships

At the Sound School we try to integrate maritime topics into our curriculum, to use Long Island Sound as a theme for our teaching, hoping that students will be motivated to greater study of a topic if they see a use for it. Here is some history of naval architecture related to aerodynamics. Aerodynamics and hydrodynamics are both part of one field called fluid mechanics. Many of the principles of fluid mechanics were known before airplanes existed. These principles had been discovered in hydrodynamics. One apparatus of investigation was the towing tank.

Much of the hydrodynamics was motivated by the building and maintenance of canals. In fact Benjamin Franklin built a towing tank to test the observation he had made on the canals in Holland that boats go slower in shallower water. The tank was in fact a narrow wooden trough; the model was towed by a weight falling over a pulley at one end. An illustration of the apparatus is found in the Naval Historical Foundation booklet, *The David Taylor Model Basin, A Brief History*, which is my source for this section.

Even though Ben Franklin was a founder of the United States, the U.S. Navy did not get Congress to approve funding for a tank until 1896. The tank was built at the Washington Navy Yard under the supervision of Naval Constructor David Watson Taylor who directed it for the next fifteen years. Much significant work was done.

The claims of the advocates of model testing were substantiated early on. In 1902 two armored cruisers of 14,500 ton displacement were designed at the Model Basin. They were 820 tons heavier than similar predecessors but were able to cruise at 22 knots with less horsepower while consuming less fuel.

Taylor instituted the practice of using wooden models instead of wax models as used by other naval architects. This gave more accurate measurements, and avoided models melting in Washington, D.C. summers. It was much more expensive, however, \$80 against fifty cents for wax that could be melted down and used again. He was responsible for the bulbous bow to dampen the bow wave, thus decreasing wave resistance. The bulbous bow is like a torpedo sticking forward from the bow just at or below the water line. This type of bow was first used on the *USS Delaware* in 1907 with great success. On field trips in New Haven harbor with Sound School students, we have observed the bow waves of freighters and tankers. The crest of the bow wave is in front of the bow, not at the bow. The bow does not “cut” the water. There is a trough, a depression in the water, at the bow.

Doing this project I had hoped to answer the question, "What does a towing tank measure?" Here is what I learned.

If we were to attach two models to the ends of a rod and tow the rod from a line attached to its center, what would happen? Most likely, one model would move forward and one would hold back. The model that held back would have more drag than the one that went forward. If the two models were equal in drag, the rod would be perpendicular to its tow line. We are back to Archimedes and a balance, the lever.

That is the idea of a towing tank. Instead of using two models, we put weights at one end of the rod to balance the force of the model. Furthermore, more balances are used because there are more forces to be measured. Our hypothetical example only measured the force in the direction of the towline. There are six motions for a boat, three linear and three rotational: surge, sway, heave, pitch, roll, and yaw. Surge is the linear motion fore and aft. Sway is linear motion in a sideways direction. Heave is the linear motion in a vertical direction. Pitch is rotational motion in which the bow goes down and the stern goes up, or vice versa. Roll is the sideways rotation. Yaw is rotational motion about the vertical axis.

So a system is designed to suspend the model with linkages for the six motions. Since the system is going to move, the balances will need to be dampened so the motion of the apparatus does not disturb them.

## **F. The Mach Number**

The Froude number is of concern when we have gravity waves. When we deal with an elastic medium we have pressure waves. Sound is a pressure wave.

The ratio known as the Mach number is

$$M = V/a$$

where  $V$  is the velocity of flow and  $a$  is the speed of sound in the medium. The Mach number, named after the Austrian physicist Ernst Mach (1838-1916), is the square root of the ratio of inertial forces to elastic forces. The Mach number is of concern when the medium is not considered incompressible, as when we assume that at high speed an object's motion will compress the air.

The Mach number tells us when we have supersonic speed. If something is going faster than the speed of sound we have a Mach number greater than one. The object is moving faster than the sound it makes, so it arrives before it is

heard. The speed of sound in air is approximately 1100 feet per second while in water it is approximately 4700 feet per second.

Supersonic motion appeared prior to the recent invention of high speed aircraft. Bullets and artillery shell can move at supersonic speed. In fact Benjamin Robbins (1707-1751), the inventor of the ballistic pendulum, found that increasing his powder charge increased his range, but only so far; at some point it became inefficient. His projectiles were approaching the speed of sound and the drag was increasing significantly.

#### IV. EXAMPLES FROM HISTORY

The reason for this historical material is my desire to give my Sound School students some knowledge about ships and the problems of building them. We learn from what has gone on before our time. This is material for the students' frames of reference and background information. A math teacher can afford to show students that he is interested in other topics, and the material ties in with the theme of the school.

##### A. The *Great Eastern*

The technological "giant steps" of any historical period determine the topics of scientific research for that period. The *Great Eastern* was one such giant step. It was a ship "before its time."

Construction of the *Great Eastern* started in 1854, her launching began in November, 1857, and she finally floated at the end of January, 1858. She was 680 feet long; the next longest ship of her day was 380 feet. Her length was not exceeded until the *Oceanic* in 1899, and her tonnage was not exceeded until the *Lusitania* in 1906. The reason it took three months to launch her was that metal rails were used for the ways and the cradles were iron shod. So much heat was generated by the friction of iron against iron that the cradle shoes and the rails welded themselves together in November at the first attempt. The ship was jacked down the ways an inch at a time after jacks were designed and built.

She was the only vessel ever built that had sails, paddle wheels, and propellers, with the paddle wheels and propellers having their own independent engines. She burned a ton of coal per mile. She had a capacity of 12,000 tons of coal. She was under powered; she had about 2600 horsepower with a top speed of 14.5 knots on a displacement of 27,000 tons. Remember, no one had ever

done this before; there were bound to be mistakes and unforeseen problems. All of her problems pointed out the need for even more scientific investigations. Her builders had been successful in their previous ventures building other ships, railroads, and bridges.

One engineer associated with the *Great Eastern* was William Froude. From his experiences grew his life's work, the study of the powering of ships. Another person to research is J. Scott Russell who designed her and built her in his ship yard. The name that most people associate with the *Great Eastern* is Isambard Kingdom Brunel, the owners' technical advisor and probably the top engineer of the time. It is a question of historical research as to how much Brunel contributed to the design; it is called "Brunel's great ship" and its misfortunes are said to have killed him.

One success of the *Great Eastern* was the laying of the transatlantic telegraph cable after the Civil War. To learn more, see the article by Chiles in the Fall 1987 issue of *Invention & Technology*.

Another reference to see is *The Great Iron Ship* by James Dugan. Here you will learn that the *Great Eastern* was launched broadside. She was double hulled to six feet above the water line. In fact, her double hull saved her. On her first voyage to the United States she hit an uncharted rock pinnacle which tore a hole eighty-three feet long and nine feet wide in the outer hull. The rock is still known as the Great Eastern Rock after the ship that found it, the only ship big enough to find it. The hull was repaired by using a cofferdam so the work could be done while the ship was afloat, another feat of engineering.

## **B. Turbinia and Cavitation**

In search of more speed, more efficient engines were built and placed in ships. The first turbine-powered ship was Sir Charles Parsons' *Turbinia* built in 1894. Parsons had built the first successful turbine to power a dynamo, an electric generator. He had done model tests and had great expectations for the ship, since the turbine was so powerful.

The results were disappointing. The highest speed recorded was less than 20 knots. The problem was cavitation, a phenomenon recognized and named by William Froude. The propellers were spinning at 18,000 rpm, so fast that the water pressure decreased, forming bubbles, a cavity. The power was going into making bubbles instead of pushing the boat.

The remedy was to operate at lower rpm with more turbines and propellers. The original design was one turbine with one shaft of three propellers. The suc-

successful design used three turbines which each had a shaft turning three propellers, and it achieved the speed of 34.5 knots in 1897.

More information can be found in the literature listed in the bibliography, especially the LIFE book *Ships*.

### ***Student Bibliography***

I claim my students will find these books and articles to be readable. I believe the students will also find them interesting.

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### ***Teacher Bibliography***

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